Cooperative Communications

Lecture 10

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with contributions from Maxime Guillaud

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Outline

Cooperative interference management in multiuser networks

- Interference channel
- Point to point MIMO with channel state information at transmitter
- Multiuser MIMO interference channel
  - Achievable rates with linear precoding
  - Precoding design: selfish and altruistic
  - Interference alignment
- Receiver structures for alignment based precoding
  - Interference decorrelator
  - Performance comparison

Preface

- Today's lecture...
  - ...focuses on the interference channel (IFC)
  - ...deviates from the cooperation strategies encountered so far
  - ...does not consider relaying strategies (amplifying and forward, multihop, compress and forward, decode and forward)
  - ...only focuses on PHY layer

Interference channel - illustration

- Many communication pairs want to exchange messages
  - overhear each others messages, i.e. interference
  - there is no central authority that coordinates the communication
  - "cocktail party" effect, coupling in the network
- "Interference channel" is a mathematical model to capture the competition for limited resources

picture source: http://de.academic.ru/dic.nsf/dewiki/438911

Think about it for today

Interference channel - competition for limited resources of a wireless network

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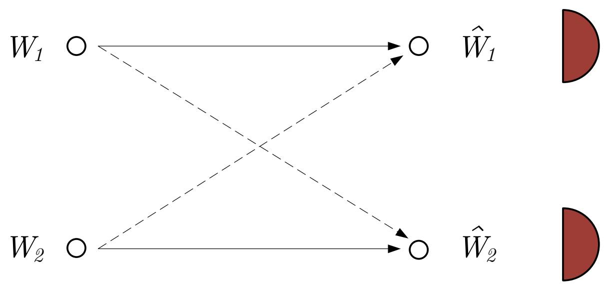
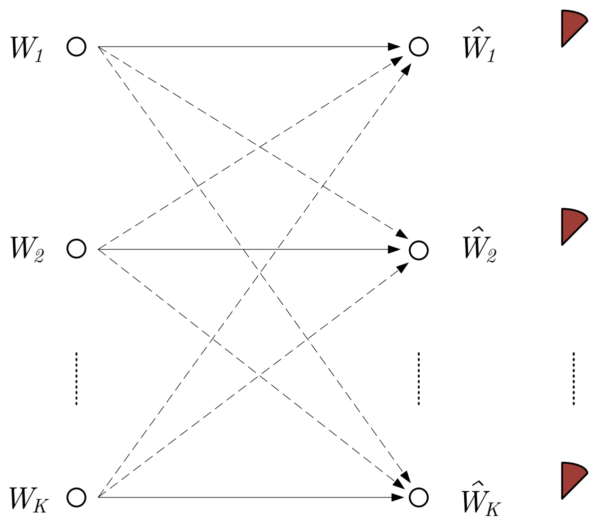


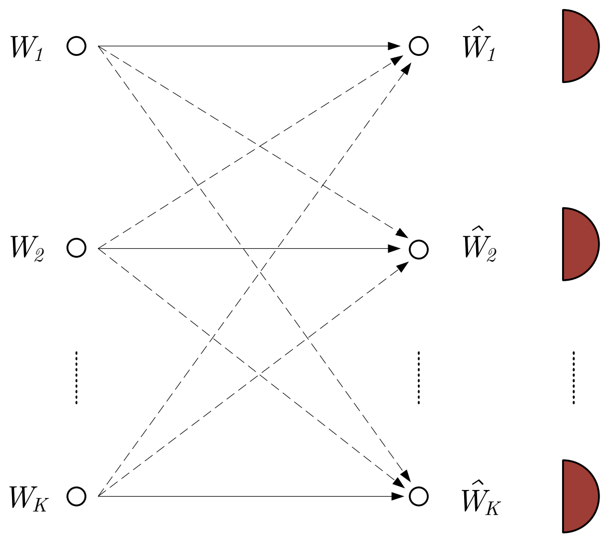
Think about it for today

Interference channel - competition for limited resources of a wireless network



Think about it for today

Interference channel - competition for limited resources of a wireless network



Half the cake result based on assumptions...

- No information exchange about the W_i's among transmitters, i.e. no joint signal processing at transmitter or receiver side
- Altruistic communication strategy
- Based on knowledge about the channels
- Interference limited systems, i.e. noise can be neglected

We use the *directional capabilities* of multi-antenna systems for *interference suppression*

Yet another advantage of MIMO

- Point to point (P2P)
  - Energy efficiency (array gain)
  - Error rate reduction (diversity gain)
  - Spectral efficiency (multiplexing gain)
- Multi-user systems
  - Interference mitigation in spatial domain

The foundation of many results is *linear algebra*

Discrete time transmission on flat-fading MIMO channels

- Focus on OFDM transmission (consider one subcarrier) or narrowband transmission
- Slow time varying channel, block fading model
- Gain between transmit antenna n and receive antenna m is an complex scalar $h_{m,n} \in \mathbb{C}$
- Channel matrix $\mathbf{H} = \begin{bmatrix} h_{1,1} & \dots & h_{1,N_\mathrm{T}} \\ \vdots & & \vdots \\ h_{N_\mathrm{R},1} & \dots & h_{N_\mathrm{R},N_\mathrm{T}} \end{bmatrix} \in \mathbb{C}^{N_\mathrm{R} \times N_\mathrm{T}}$
- Entries of channel matrix drawn i.i.d. from a continuous distribution

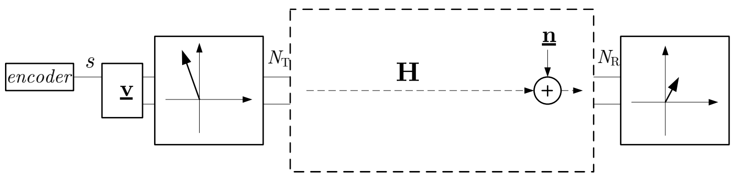
Point-to-point MIMO system

- Transmitter receiver relationship, single stream precoding

$$\underline{\mathbf{y}} = \mathbf{H}\underline{\mathbf{v}}s + \underline{\mathbf{n}}$$

with

- $\underline{\mathbf{y}} = [y_1, ..., y_{N_\mathrm{R}}]^\top \in \mathbb{C}^{N_\mathrm{R}}$, receive signal vector
- $\underline{\mathbf{v}} = [v_1, ..., v_{N_\mathrm{T}}]^\top \in \mathbb{C}^{N_\mathrm{T}}$, transmit precoding vector
- $s \in \mathbb{C}$, transmit symbol, $\mathbb{E}\{s\} = 0$, $\mathbb{E}\{|s|^2\} = E_s$, transmit energy
- $\underline{\mathbf{n}} = [n_1, ..., n_{N_\mathrm{R}}]^\top \in \mathbb{C}^{N_\mathrm{R}}$, noise vector, $n_i \sim \mathcal{CN}(0, N_0)$ with $\mathbb{E}\{\underline{\mathbf{n}}\underline{\mathbf{n}}^\mathrm{H}\} = N_0\mathbf{I}$

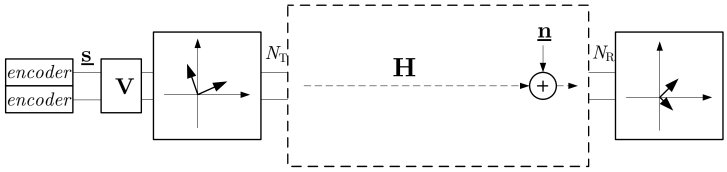


P2P MIMO with CSIT

- Channel knowledge $\mathbf{H}$ obtained through feedback or reciprocity in the case of time division duplex (TDD)
- Signal model

$$\underline{\mathbf{y}} = \mathbf{H}\mathbf{V}\underline{\mathbf{s}} + \underline{\mathbf{n}}$$

  - Rank of channel matrix: $\mathrm{rank}(\mathbf{H}) = \min(N_\mathrm{R}, N_\mathrm{T})$ w.p. 1
  - Channel decomposition $\mathbf{H} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^\mathrm{H}$, with $\mathbf{V} \in \mathbb{C}^{N_\mathrm{T}\times N_\mathrm{T}}$
  - Orthogonal linear precoding of $d = \min(N_\mathrm{T}, N_\mathrm{R})$ streams $\mathbf{V}\underline{\mathbf{s}}$ with $\underline{\mathbf{s}} = [s_1, ..., s_d, 0, ..., 0]^\top \in \mathbb{C}^{N_\mathrm{T}}$,$\mathbb{E}\{\underline{\mathbf{s}}\} = 0$, $\mathrm{tr}\{\mathbb{E}\{\underline{\mathbf{s}}\underline{\mathbf{s}}^\mathrm{H}\}\} = E_s$,



  - At the receiver: streams are still orthogonal, no inter-stream interference, stretching of the arrows according to eigenmodes of $\mathbf{H}$
  - Multiplexing gain, intuitively the number of *parallel pipes* available from the channel

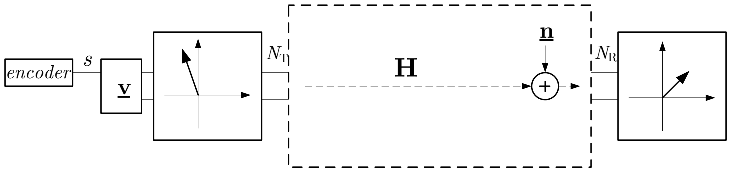
P2P MIMO with CSIT (II)

Maximum "Eigen"mode

- Goal is to realize array gain
- Signal model

$$\underline{\mathbf{y}} = \mathbf{H}\underline{\mathbf{v}}s + \underline{\mathbf{n}}$$

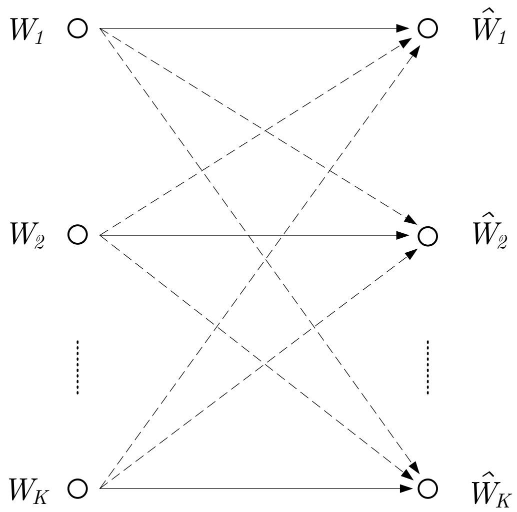
  - Channel can be decomposed in $\mathbf{H} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^\mathrm{H}$
  - Orthogonal linear precoding of one stream $\underline{\mathbf{x}} = \underline{\mathbf{v}}s$, with $\underline{\mathbf{v}}$ the singular-vector (column of $\mathbf{V}$) corresponding to the strongest singular-value of $\mathbf{H}$, increased power for one stream



  - At the receiver: stretching of the arrow according to strongest singular mode of $\mathbf{H}$, i.e. precoding matched to dominant eigenmode

Multi-user MIMO IFC

- K transmitter-receiver pairs (independent messages from transmitter *i* to receiver *i*)



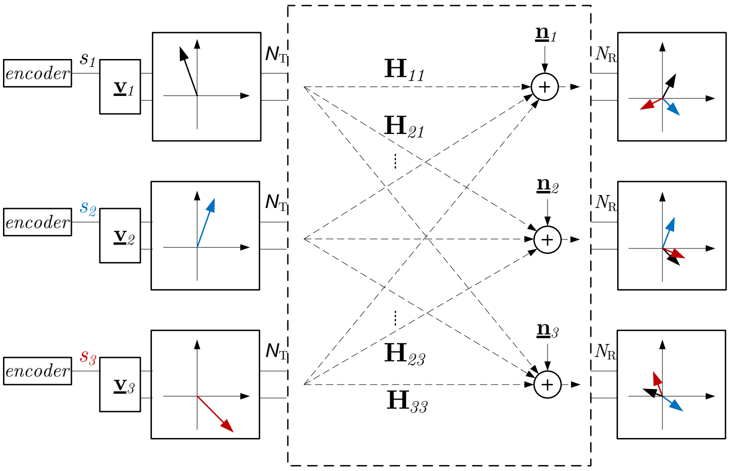
Multi-user MIMO IFC

- Channel matrix $\mathbf{H}_{ij}$ representing link between transmitter *j* and receiver *i*
- Channel matrix $\mathbf{H}_{ij} = \begin{bmatrix} h^{ij}_{1,1} & \cdots & h^{ij}_{1,N_\mathrm{T}} \\ \vdots & & \vdots \\ h^{ij}_{N_\mathrm{R},1} & \cdots & h^{ij}_{N_\mathrm{R},N_\mathrm{T}} \end{bmatrix} \in \mathbb{C}^{N_\mathrm{R} \times N_\mathrm{T}}$
- Assume that all transmitters and receivers have the same number of antennas, N_T and N_R, respectively for simplicity
- Entries of each channel matrix drawn i.i.d. from a continuous distribution and no dependency between individual channel matrices

Multi-user MIMO IFC

- Transmitter receiver relationship

$$\underline{\mathbf{y}}_i = \mathbf{H}_{ii}\underline{\mathbf{v}}_i s_i + \sum_{j=1, j\neq i}^{K} \mathbf{H}_{ij}\underline{\mathbf{v}}_j s_j + \underline{\mathbf{n}}_i$$



- Observe the relativity of the received signals, every receiver sees a different picture

Mutual information for the multi-user MIMO IFC

- No joint processing of signals either across all K transmitter or across all K receivers, *distributed nature* of the network
- Assume that
  - each receiver treats interference from any unintended source as additive noise (suboptimal)
  - transmitters use Gaussian codebook (possibly suboptimal)
  - linear precoding at transmitter side (here low rank precoding of one stream, can be generalized)
- Instantaneous mutual information for receiver i

$$I(s_i; \underline{\mathbf{y}}_i) = h(\underline{\mathbf{y}}_i) - h(\underline{\mathbf{y}}_i | s_i)$$

  - $h(\underline{\mathbf{y}}_i)$ differential entropy of $\underline{\mathbf{y}}_i$
  - $h(\underline{\mathbf{y}}_i | s_i)$ differential entropy of $\underline{\mathbf{y}}_i$ given s_i

Mutual information for the multi-user MIMO IFC (II)

$$\underline{\mathbf{y}}_i = \mathbf{H}_{ii}\underline{\mathbf{v}}_i s_i + \underbrace{\sum_{j=1,j\neq i}^{K} \mathbf{H}_{ij}\underline{\mathbf{v}}_j s_j + \underline{\mathbf{n}}_i}_{\underline{\tilde{\mathbf{n}}}_i}$$

- s_i and $\underline{\tilde{\mathbf{n}}}_i$ are independent, i.e. $h(\underline{\mathbf{y}}_i|s_i) = h(\underline{\tilde{\mathbf{n}}}_i)$, thus

$$I(s_i; \underline{\mathbf{y}}_i) = h(\underline{\mathbf{y}}_i) - h(\underline{\tilde{\mathbf{n}}}_i)$$

- The differential entropies are given as

$$h(\underline{\mathbf{y}}_i) = \log_2(\det(\pi e \mathbf{R}_{\underline{\mathbf{y}}_i\underline{\mathbf{y}}_i}))$$

$$h(\underline{\tilde{\mathbf{n}}}_i) = \log_2(\det(\pi e \mathbf{R}_{\underline{\tilde{\mathbf{n}}}_i\underline{\tilde{\mathbf{n}}}_i}))$$

with $\mathbf{R}_{\underline{\mathbf{y}}_i\underline{\mathbf{y}}_i} = E_s\mathbf{H}_{ii}\underline{\mathbf{v}}_i\underline{\mathbf{v}}_i^{\mathrm{H}}\mathbf{H}_{ii}^{\mathrm{H}} + \mathbf{R}_{\underline{\tilde{\mathbf{n}}}_i\underline{\tilde{\mathbf{n}}}_i}$ and
$\mathbf{R}_{\underline{\tilde{\mathbf{n}}}_i\underline{\tilde{\mathbf{n}}}_i} = \sum_{j=1,j\neq i}^{K} E_s\mathbf{H}_{ij}\underline{\mathbf{v}}_j\underline{\mathbf{v}}_j^{\mathrm{H}}\mathbf{H}_{ij}^{\mathrm{H}} + N_o\mathbf{I}_{N_\mathrm{R}}$

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Mutual information for the multi-user MIMO IFC (II)

- The mutual information therefore results in

$$I(s_i; \underline{\mathbf{y}}_i) = \log_2(\det(\mathbf{I}_{N_\mathrm{R}} + \mathbf{R}_{\underline{\tilde{\mathbf{n}}}_i\underline{\tilde{\mathbf{n}}}_i}^{-1} E_s\mathbf{H}_{ii}\underline{\mathbf{v}}_i\underline{\mathbf{v}}_i^{\mathrm{H}}\mathbf{H}_{ii}^{\mathrm{H}})) \qquad (1)$$

- This is the instantaneous achievable rate assuming Gaussian codebooks and optimal (multi-user) decoding at the receiver
- The choice of the precoders result in different achievable rates
- Crosscheck: if noise is white and we forget about the interference term, i.e. $\mathbf{R}_{\underline{\tilde{\mathbf{n}}}_i\underline{\tilde{\mathbf{n}}}_i}^{-1} = \frac{1}{N_0}\mathbf{I}_{N_\mathrm{R}}$ and
$I(s_i; \underline{\mathbf{y}}_i) = \log_2(\det(\mathbf{I}_{N_\mathrm{R}} + \frac{1}{N_0}E_s\mathbf{H}_{ii}\underline{\mathbf{v}}_i\underline{\mathbf{v}}_i^{\mathrm{H}}\mathbf{H}_{ii}^{\mathrm{H}}))$ see Lecture 3, Part II

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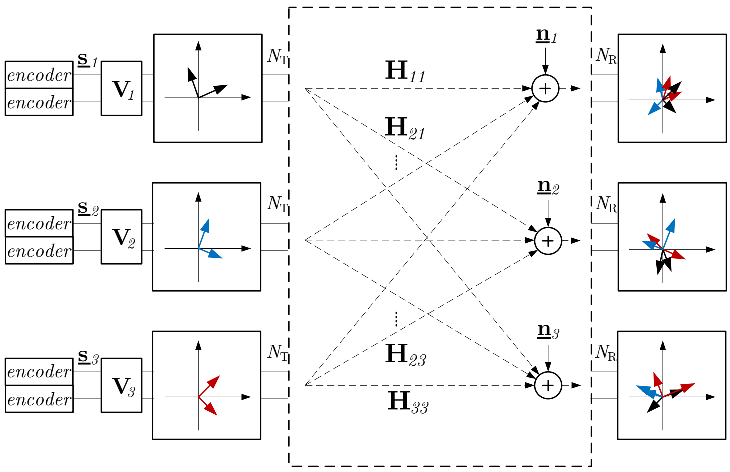
Towards optimal linear precoding for the IFC

- Based on our knowledge about MIMO P2P systems with CSIT we will gain intuition towards optimal linear precoding strategies
- Consider the 3-user 2x2 IFC as a motivating headliner
- We assume that through feedback we can acquire knowledge about all K^2 channel matrices $\mathbf{H}_{ij}$´s at all transmitters (global CSIT)
- Use naive selfish approaches
  - Precoding along the two eigenmodes of $\mathbf{H}_{ii}$, independently for each transmitter
  - Precoding along the dominant eigenmode of $\mathbf{H}_{ii}$, independently for each transmitter
  - Draw conclusions

Dual stream precoding for IFC

- Precoding along the two eigenmodes of $\mathbf{H}_{ii}$, individually for each transmitter
- Inspired by the capacity achieving strategy for P2P MIMO

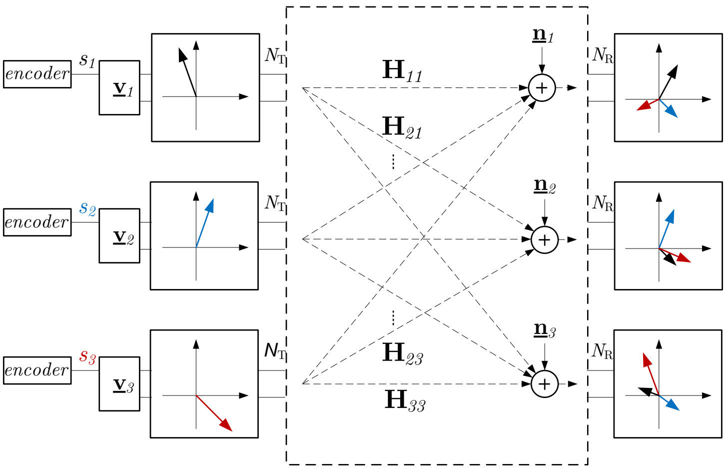


Dual stream precoding for IFC (II)

- Observations
  - individual channel modes of intended link are accessible, no inter-stream interference at the receivers, arrows orthogonal
  - intended signal of transmitter i spans the whole receive signal space of receiver i
  - interfering signal of transmitter j spans the whole receive signal space of receiver i
  - strong interference from unintended streams
- Lessons learned
  - give up one stream per transmitter

Single stream precoding for IFC

- Precoding along the strongest eigenmodes of $\mathbf{H}_{ii}$, independently for each transmitter
- Inspired by SNR maximizing precoding strategy for P2P MIMO



Single stream precoding for IFC (II)

- Observations
  - intended signal of transmitter i spans a subspace of receive signal space of receiver i
  - interfering signals unintended transmitters span the whole receive signal space of receiver i
  - residual interference from unintended streams
  - there does not exist a linear filter that can suppress the interference
- Lessons learned
  - we have to shift our focus on the interfering signals and try to align the subspaces that they span at each receiver simultaneously
  - with this strategy we restrain the interference leakage that unintended signals cause

Interference alignment (intuition)

- *Interference alignment refers to a construction of signals in such a manner that they cast overlapping shadows at the receivers where they constitute interference while they remain distinguishable at the receivers where they are desired [1]*
  - interference is not weak enough to treat it as noise
  - don't want to decode many strong interference signals
  - restrict subspace where interference is allowed to live in
  - remaining subspace used for interference free communication with intended transmitter
  - altruistic approach achieves a network multiplexing gain, intuitively parallel pipes through the IFC

Alignment constraints

- Problem can be formulated similarly: we look for $\underline{\mathbf{v}}_i$ (unit norm precoding-vectors) of dimensions $N_{\mathrm{T}} \times 1$ and $N_{\mathrm{R}} \times 1$ unit-norm vectors $\underline{\mathbf{u}}_i$ (interference suppression vectors) such that, for all $i \in (1, \ldots, K)$

$$\underline{\mathbf{u}}_i^{\mathrm{H}} \mathbf{H}_{ij} \underline{\mathbf{v}}_j = 0 \quad \forall j \neq i$$

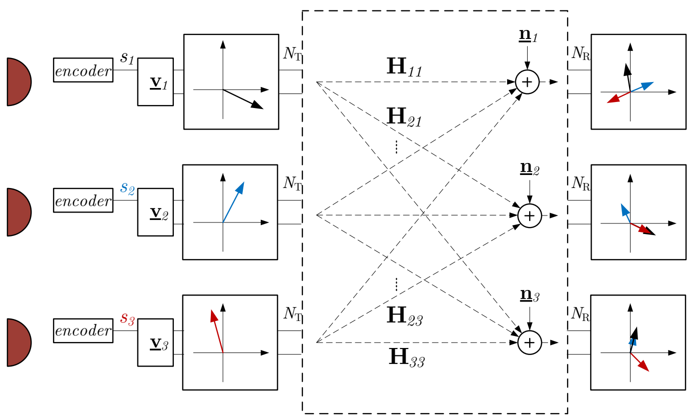
  i.e. condition for interference alignment, and

$$\mathrm{rank}(\underline{\mathbf{u}}_i^{\mathrm{H}} \mathbf{H}_{ii} \underline{\mathbf{v}}_i) = 1$$

  i.e. condition for non-zero signal power in the interference free subspace
- Interference suppression vectors $\underline{\mathbf{u}}_i$ span the orthogonal subspace of the interference

Interference alignment (in pictures)

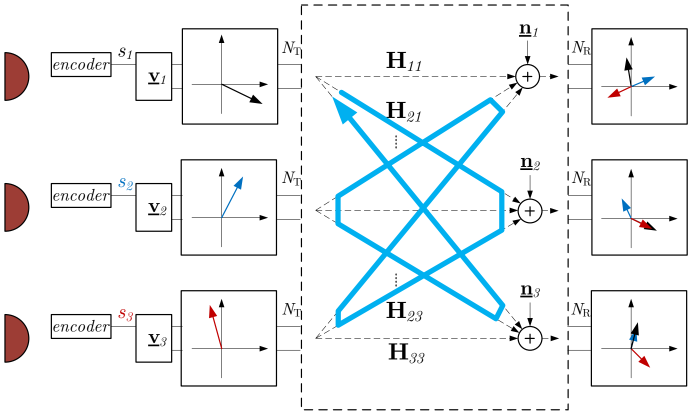
- Interference alignment is an *altruistic* approach, each transmitter primarily tries to minimize the interference to unintended receivers
- The result is that the K users can transmit half the spatial streams, *free from interference*, compared to the isolated P2P system



Alignment solution

- For the 3-user 2x2 MIMO IFC, there exists a closed form alignment solution
- Solve the following eigenvalue problem

$$\underline{\mathbf{v}}_1 = \lambda \mathbf{H}_{31}^{-1}\mathbf{H}_{32}\mathbf{H}_{12}^{-1}\mathbf{H}_{13}\mathbf{H}_{23}^{-1}\mathbf{H}_{21}\underline{\mathbf{v}}_1$$



- Pick $\underline{\mathbf{v}}_1$ as one of the two non-orthogonal eigenvectors of the above nonsymmetric eigenvalue problem

Alignment solution (II)

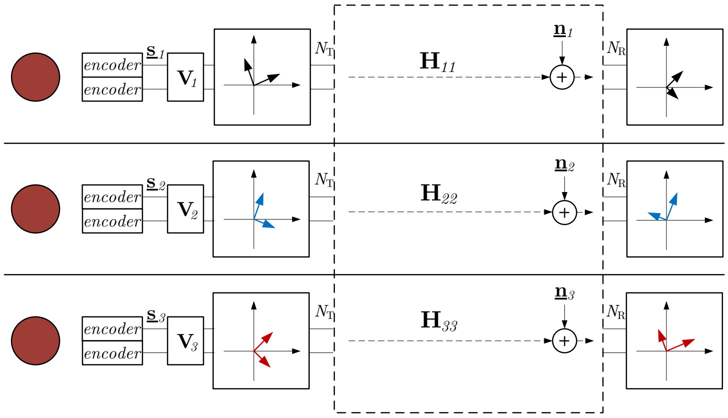
- The remaining precoders $\underline{\mathbf{v}}_2$, $\underline{\mathbf{v}}_3$ can be found recursively using the alignment conditions, e.g. collinearity of

$$\mathbf{H}_{31}\underline{\mathbf{v}}_1 = \mu \mathbf{H}_{32}\underline{\mathbf{v}}_2 \rightarrow \underline{\mathbf{v}}_2 = \frac{1}{\mu}\mathbf{H}_{32}^{-1}\mathbf{H}_{31}\underline{\mathbf{v}}_1$$

with μ a normalizing constant
- Reveals the coupled nature of the problem
- Interference decorrelators $\underline{\mathbf{u}}_1$, $\underline{\mathbf{u}}_2$, $\underline{\mathbf{u}}_3$ can be determined by finding an orthogonal basis of the interference subspace
- The alignment precoders depend on *all* channel matrices corresponding to *interfering links* and determined *independently* of the direct transceiver pair links

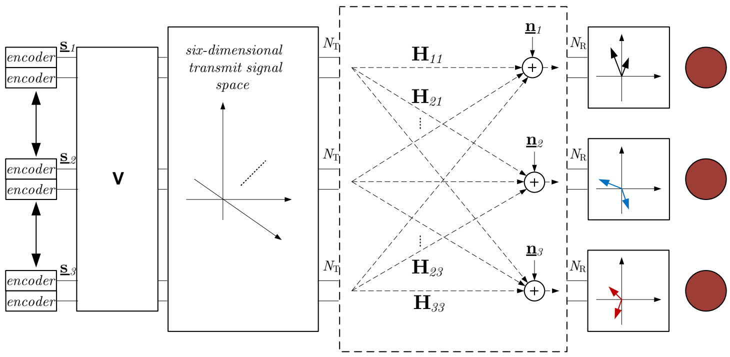
The full cake in the spatial domain

- We consider interference management in the spatial domain
- The full cake represents the number of spatial streams that a single transmitter-receiver pair, in the absence of interference, is able to communicate



Comparison to CoMP

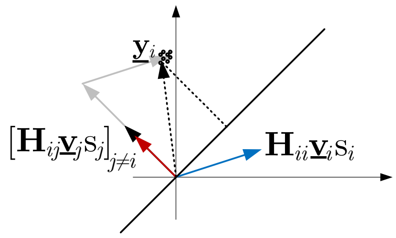
- Coordinated multipoint transmission (CoMP) or network MIMO allows for joint signal processing at transmitter side
- The information symbols are shared between transmitters
- Interference can be mitigated by joint precoding with a 6×6 precoding matrix V which e.g. block diagonalizes the overall 6×6 MIMO channel of the network
- Violates the basic assumption of the IFC



Receiver structure

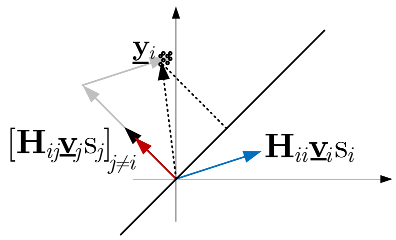
- Recall receive signal at receiver i, i.e.

$$\underline{\mathbf{y}}_i = \mathbf{H}_{ii}\underline{\mathbf{v}}_i s_i + \sum_{j=1, j\neq i}^{K} \mathbf{H}_{ij}\underline{\mathbf{v}}_j s_j + \underline{\mathbf{n}}_i$$



- Remove interference by projecting $\underline{\mathbf{y}}_i$ onto the subspace orthogonal to the one spanned by $[\mathbf{H}_{ij}\underline{\mathbf{v}}_j s_j]_{j\neq i}$ denoted as $V_I^{\perp}$
- The vector $\underline{\mathbf{u}}_i$ is the row of the orthonormal basis of $V_I^{\perp}$ (can be obtained trough QR decomposition of $\mathbf{H}_{ij}\underline{\mathbf{v}}_j$)

Linear decorrelator



- $\underline{\mathbf{u}}_i^{\mathrm{H}}\underline{\mathbf{y}}_i$ should be interpreted as the projection of $\underline{\mathbf{y}}_i$ onto $V_I^{\perp}$ but expressed in terms of the coordinates defined by the basis of $V_I^{\perp}$
- After the projection operation

$$\bar{y}_i = \underline{\mathbf{u}}_i^{\mathrm{H}}\underline{\mathbf{y}}_i = \underline{\mathbf{u}}_i^{\mathrm{H}}\mathbf{H}_{ii}\underline{\mathbf{v}}_i s_i + \sum_{j=1, j\neq i}^{K} \underline{\mathbf{u}}_i^{\mathrm{H}}\mathbf{H}_{ij}\underline{\mathbf{v}}_j s_j + \underline{\mathbf{u}}_i^{\mathrm{H}}\underline{\mathbf{n}}_i$$

which yields

$$\bar{y}_i = \underline{\mathbf{u}}_i^{\mathrm{H}}\mathbf{H}_{ii}\underline{\mathbf{v}}_i s_i + \bar{n}_i$$

with $\bar{n}_i = \underline{\mathbf{u}}_i^{\mathrm{H}}\underline{\mathbf{n}}_i$ is still white noise

Linear decorrelator (II)

- Therefore, alignment based precoding with linear interference decorrelators leads to stream wise detection with SNR for user i

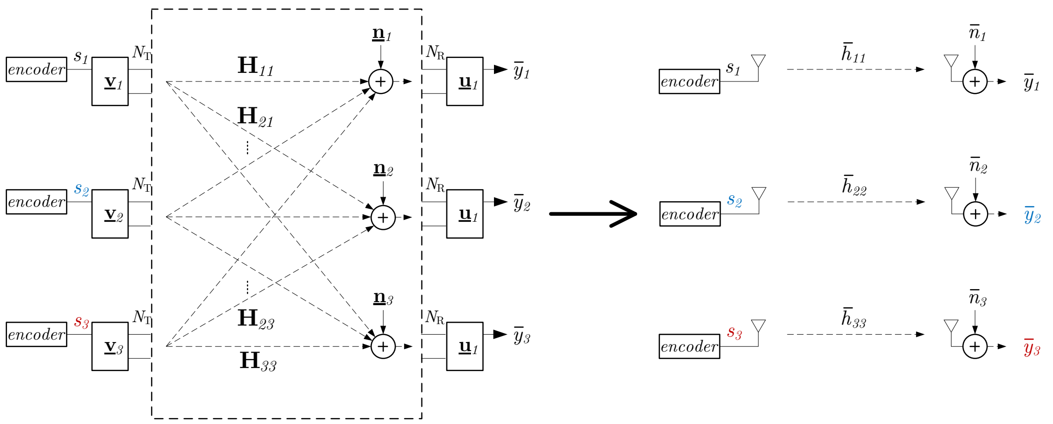
$$\frac{E_s|\underline{\mathbf{u}}_i^{\mathrm{H}}\mathbf{H}_{ii}\underline{\mathbf{v}}_i|^2}{N_o}$$

- Furthermore, if all channel gains $[h_{ij}^{m,n}]_{i,j,m,n}$ are i.i.d. Gaussian, i.e. $h_{ij}^{m,n} \sim \mathcal{CN}(0,1)$ then since unit-norm $\underline{\mathbf{v}}_i$'s and $\underline{\mathbf{u}}_i$'s are independent of $\mathbf{H}_{ii}$ it can be shown that the *effective scalar* channel gain

$$\bar{h}_{ii} = \underline{\mathbf{u}}_i^{\mathrm{H}}\mathbf{H}_{ii}\underline{\mathbf{v}}_i \sim \mathcal{CN}(0,1)$$

and $\bar{y}_i = \bar{h}_{ii}s_i + \bar{n}_i$ can be interpreted as a system with *single-antenna* terminals, interference completely mitigated

Take away from today...

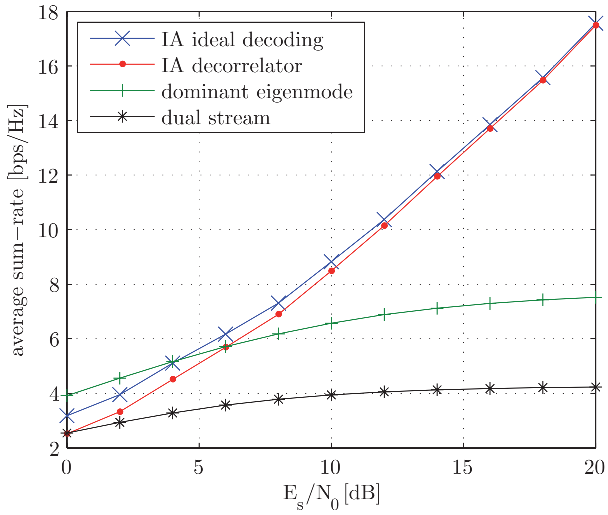


- The achievable instantaneous sum-rate of the 3-user 2x2 MIMO IFC using alignment-based precoding and interference decorrelators is

$$R_{\mathrm{sum}} = \sum_{i=1}^{3} R_i = \sum_{i=1}^{3} \log_2\left(1 + \frac{E_s|\bar{h}|^2}{N_0}\right) \qquad (2)$$

Performance of different precoders...

- The sum-rate averaged over different channel realizations is plotted versus SNR



- The achievable instantaneous sum-rate is evaluated using the mutual information formula (1) and (2) for interference decorrelator

The dream

"Each speaker is able to talk half the time and be heard *interference-free* by its desired audience"

picture source: http://de.academic.ru/dic.nsf/dewiki/438911

References

[1] Viveck R. Cadambe and Syed A. Jafar, "Interference Alignment and Degrees of Freedom of the K-User Interference Channel," IEEE Trans. Information Theory, vol. 54, no. 8, pp. 3425–3441, Aug. 2008
[2] Krishna Gomadam, Viveck R. Cadambe, and Syed A. Jafar, "Approaching the Capacity of Wireless Networks through Distributed Interference Alignment," IEEE Trans. Information Theory, 2008, submitted
[3] Cenk M. Yetis, Tiangao Gou, Syed A. Jafar, and Ahmet H. Kayran, "On Feasibility of Interference Alignment in MIMO Interference Networks," 2009, http://arxiv.org/abs/0911.4507
[4] David Tse and Pramod Viswanath "Fundamentals of Wireless Communication", Cambridge University Press, 2005