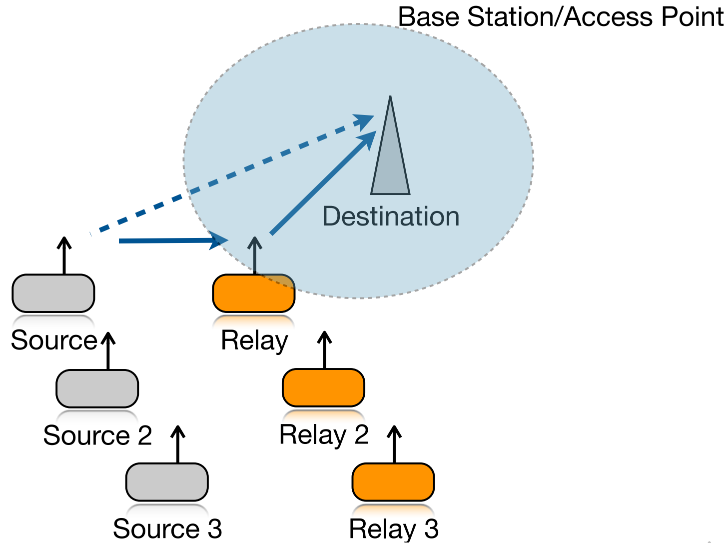
Cooperative Communications

Lecture 1

Thomas Zemen and Nicolai Czink

March 10, 2010

What the Lecture is about ...



When an Where

- Literature
  - Lecture notes are available as collection of slides
  - G. Kramer, I. Maric, and R.D. Yates: Cooperative Communications, Foundations and Trends in Networking. Hanover, MA: now Publishers Inc., vol. 1, no. 3-4, June 2007.
- Date and Time
  - When - Thursday, March 10th, 2010, 17:00-18:30
    - 10 minutes break
  - Where - SEM 389, 1st floor, room CG0118 (Institute of Communications and Radio-Frequency Engineering).
- Exam
  - Written exam at the end of the semester

Topics

- Communication theory (physical layer, link layer, network layer)
- Network models
- Physical layer channel models for mobile-users
- Cooperative strategies (Amplify-and-forward, compress-and-forward, decode-and-forward)
- Cooperative diversity
- Coded cooperation
- Virtual MIMO systems
- Interference alignment
- Cooperative vehicle-to-vehicle communications

Communication Network

- Wireless
  - Cellular networks - GSM, UMTS, UMTS LTE
  - Wireles local area network (LAN)
- Wireline
  - Internet
  - Plain old telephone systems (POTS)

A communication network is made out of

- devices, and
- channels.

Device and Channel Properties

	Devices (Nodes)	Channels (Edges)
Wireline	Constraints:	Independent channels
	– processing speed/energy	No interference
	– input/output (ports)	Limited bandwidth
	– delay	Slow changes
	Limited network knowledge	Packet erasures
Wireless	Constraints:	Broadcasting
	– transmit energy	Interference
	– processing speed/energy	Noise
	– half-duplex	Limited bandwidth
	– delay	Slow or rapid changes
	Limited network knowledge	
	Limited channel knowledge	

Table source: [1, Tab 1.1]

Network Purpose

Enable message exchange between devices (nodes)

- Traditional approach
  - Nodes use packet transmission with store-and-forward
  - Channels treated as point-to-point links
  - Data packets traverse paths: sequences of nodes
- Two other possibilities
  - Node coding: nodes process
    - Reliable message or packet bits (network coding)
    - Reliable or unreliable symbols (relaying/cooperation)
  - Broadcasting: nodes overhear wireless transmissions

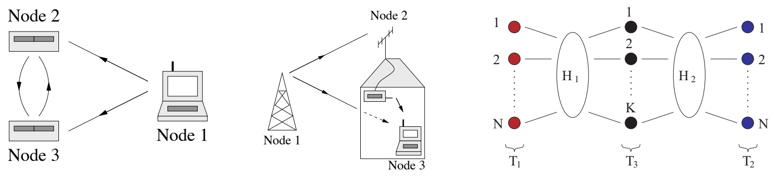
Cooperative Communications

Goals

- Increasing the communication rate
- Increasing the communication reliability

Cooperation Examples

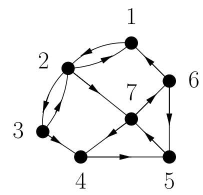
(a) Base stations (devices) cooperating via wired & wireless links
(b) Cellular network with remote antennas
(c) Multi user relaying



(a) (b) (c)

Figure source: [1, 2]

Abstract Networks



- A communication network has devices and channels
- One commonly represents a network as a directed graph $\mathcal{G} = (\mathcal{N}, \mathcal{E})$ where $\mathcal{N}$ is a set of nodes and $\mathcal{E}$ is a set of edges (an edge is an ordered pair of nodes).
  - Nodes represent communication devices
  - Edges represent communication channels (or links)

Figure source: [1]

Protocol Stack

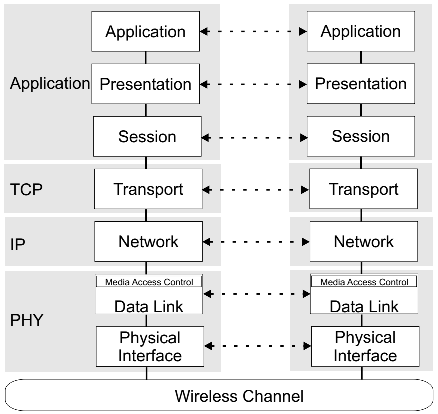
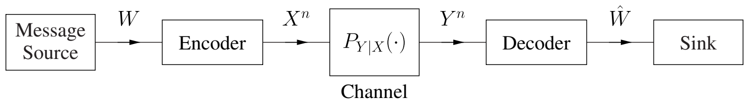


Figure source:[1, Fig 2.1]

A little bit of Communication Theory



- Channel input $X \in \mathcal{X}$, channel output $Y \in \mathcal{Y}$
- Discrete memoryless channel (DMC) described via conditional probability distribution $P_{Y|X}(\cdot)$ where $\mathcal{X}$ and $\mathcal{Y}$ are discete and finite.
- Encoder transmits string $X^n = \underline{X} = X_1, X_2, \ldots, X_n$ that is a function of W.
- Decoder sees $Y^n = Y_1, Y_2, \ldots, Y_n$ and puts out message estimate $\hat{W}$ that is a function of Y^n.

Figure source: [1, Fig. 2.2]

Capacity

- Source message W has B bits; rate $R = B/n$ bits per channel use for sufficiently large n.
- Capacity: maximum R at which one can transmit W reliably, hence $\Pr[\hat{W} \neq W]$ get close to zero for large code word length n,

$$R \leq C = \max_{P_X(\cdot)} I(X;Y) \quad \text{bits/use}$$

where

$$I(X;Y) = \sum_{a \in \mathcal{X}, b \in \mathcal{Y}, P_{XY(a,b)>0},} P_{XY}(a,b) \log_2 \frac{P_{XY}(a,b)}{P_X(a)P_Y(b)}$$

is the mutual information between X and Y.

Random Codes

- Random codebook design - not suitable for practical systems
  - Choose a probability distribution $P_X(\cdot)$
  - Generate 2^{nR} codewords $\underline{x}(w)$, $w = 1, 2, \ldots, 2^{nR}$, by choosing $x_i(w)$, $i = 1, 2, \ldots, n$, independently using $P_X(\cdot)$
- Encoder and Decoder
  - Encoder transmits $\underline{x}(w)$
  - Decoder sees $\underline{y}$ and puts out the $\hat{w}$ for which $\underline{x}(\hat{w})$ is "closest" to $\underline{y}$, where "closest" can be measured in terms of, e.g., maximum-likelihood or least squares.
- Error Probability
  - Over the ensemble of all codebooks constructed as above, one can make $\Pr[\hat{W} \neq W] \to 0$ as $n \to \infty$ if $R < I(X;Y)$
  - One can thus find a sequence of codebooks of increasing length n for which $R < I(X;Y)$ and $\Pr[\hat{W} \neq W] \to 0$
- Finally, optimize over $P_X(\cdot)$

Simple Channel - Real Valued I

- Additive white Gaussian noise (AWGN) channel

$$Y = \frac{h}{d^{\alpha/2}} X + Z$$

with block power constraint

$$\sum_{i=1}^{n} X_i^2 / n \leq P\,.$$

- X, Y, and Z are real random variables $X, Y, Z \in \mathbb{R}$
- $Z \sim \mathcal{N}(0, N)$ is Gaussian with variance N
- $h \in \mathbb{R}$ is the channel gain, d is the distance and α is the path loss exponent (model not valid for small d)

Simple Channel - Real Valued II

- Capacity formula generalized for continuous random variables

$$C = \max_{p_{X(\cdot)}} \int p_{XY}(a,b) \log_2 \frac{p_{XY}(a,b)}{p_X(a)p_Y(b)} \mathrm{d}a\,\mathrm{d}b \quad \text{bits/use}$$

- Maximum entropy theorem [3, p. 234]: best X is Gaussian with zero mean, $\mu = 0$, and variance $\sigma^2 = P$, and the capacity is

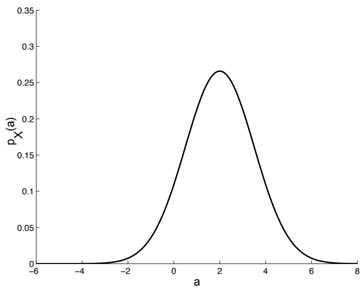
$$C = \frac{1}{2} \log_2(1 + \gamma) \quad \text{bits/use}\,,$$

where $\gamma = (\frac{P}{N}) \frac{|h|^2}{d^\alpha}$ is the signal to noise ratio (SNR).

Simple Channel - Real Valued III

- Normal (Gaussian) distribution:

$$X \sim \mathcal{N}(\mu, \sigma^2), \quad p_X(a) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(a-\mu)^2}{2\sigma^2}}$$



Gaussian pdf, $\mathcal{N}(2, 1.5^2)$ [http://users.isr.ist.utl.pt/~mir/pub/probability.pdf]

Simple Channel - Complex Valued I

- In this lecture we consider equivalent baseband notation. Modulation and demodulation to carrier frequency f_C is not explicitely stated, signals are complex valued.
- Complex valued AWGN channel

$$Y = \frac{h + a}{d^{\alpha/2}} X + Z$$

with block power constraint

$$\sum_{i=1}^{n} |X_i|^2 / n \leq P\,.$$

- X, Y, and Z are complex random variables $X, Y, Z \in \mathbb{C}$

Simple Channel - Complex Valued II

- $Z \sim \mathcal{CN}(0, N)$ is Gaussian, real and imaginary parts are indenpendt, each with variance $N/2$,

$$Z = Z_R + jZ_I\,.$$

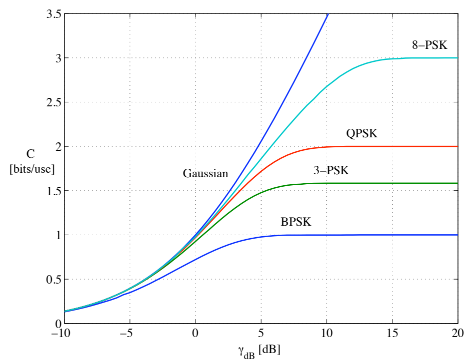
- Capacity

$$C = \log_2(1 + \gamma)$$

is achived with Gaussian X with independent real and imaginary parts, each with variance $P/2$.

Practical Modulation Alphabets

- Discrete X used for practical reasons, e.g. M-ary phase shift keying (PSK) with $X = \sqrt{P} e^{j2\pi m/M}, \quad m = 0, 1, \ldots, M-1\,.$



- SNR in decibels: $\gamma_{dB} = 10 \log_{10} \gamma$ dB

Figure source:[1, Fig 2.3]

Bandlimited Channels I

- Spectral efficiency includes the effect of bandwidth
- The spectral efficiency of a modulation set measures the number of bits per second per Herz that the set can support
- The carrier frequency f_C is much larger than the channel bandwidth W,

$$f_C - W/2 < |f| < f_C + W/2$$

- Equivalent baseband notation: Input and output signal are complex and bandlimited to $|f| < W/2$.
- Channel is used for a period of T second. Channel input and output described by $n = TW$ complex samples spaced $T_S = 1/W$ apart.

$$X(t) = \sum_{i=1}^{n} X_i \frac{\sin(\pi W t - \pi i)}{\pi W t - \pi i}$$

Bandlimited Channels II

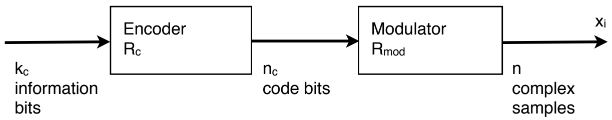
- Average energy per sample

$$E_S = P T_S = \sum_{i=1}^{n} \mathrm{E}[|X_i|^2]/n$$

- Channel: $Y_i = X_i + Z_i$, where $Z_i = Z_{R,i} + jZ_{I,i}$
  - $Z_{R,i}$ and $Z_{I,i}$ are independent, Gaussian random variables with variance $(NT_S)/2$ each.
  - Noise power N increases with bandwidth W: $N = N_0 W$, where $N_0/2$ denotes noise power per Hertz.

Bandlimited Channels III

- Modulation rate $R_{\text{mod}} = \log_2(M)$ bits, where $M = |\mathcal{X}|$ is the number of elements in the alphabet (QPSK: $R_{\text{mod}} = 2$)
- Encoder map k_c information bits to n_c coded bits. Code rate $R_c = k_c/n_c$.
- Overall coded modulation rate: $R = R_c R_{\text{mod}}$



- Energy per information bit: $E_b = PT_S/R = E_S/R$
- Noise energy or variance: $N_S = NT_S = N_0 W T_S = N_0$
- Note: $E_S/N_S = P/N = (E_b/N_0)R$

Bandlimited Channels IV

- Spectral efficiency: $\eta(E_b/N_0, P_b) = R_c^* R_{\text{mod}}$ bits/s/Hz
  R_c^* is the maximum code rate for which a bit-error probability of P_b can be achived given E_b/N_0.
- Typical values for P_b:
  - Wireless: 10^{-3}
  - Magnetic recording: 10^{-12}
  - Fiber optic: 10^{-14}
- For $P_b \leq 10^3$ we can also compute $\eta(E_b/N_0) = \eta(E_b/N_0, P_b)|_{P_b \to 0}$

Computing Spectral Efficiency I

- Passband AWGN channel

$$R \leq C = \log_2\left(1 + \frac{P}{N}\right) \quad \text{bits/s/Hz}$$

- Substituting $P/N = (E_b/N_0)R$ we get

$$R \leq \log_2\left(1 + \frac{E_b}{N_0}R\right)$$

- Spectral efficiency is unique solution of

$$\eta = \log_2\left(1 + \frac{E_b}{N_0}\eta\right)$$

- Capacity for finite alphabets have the form

$$R \leq C = \max_{P_X} I(X;Y) = f(P/N)$$

where $f(x)$ is some non-descreasing function in x.

Computing Spectral Efficiency II

- Again substituting $P/N = (E_b/N_0)R$ we get $\eta \leq f\left(\frac{E_b}{N_0}\eta\right)$

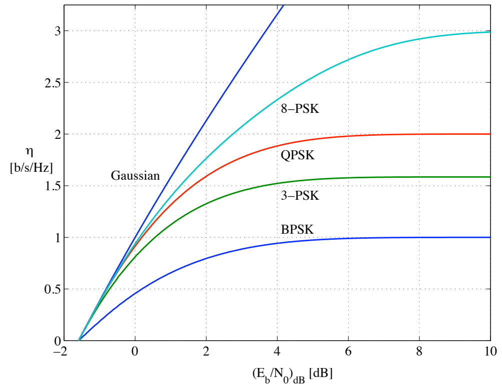


Figure source:[1, Fig 2.4]

References

- G. Kramer, I. Maric, and R. D. Yates, *Cooperative Communictions*, ser. Foundations and Trends in Networking. Hanover, MA, USA: now Publishers Inc., 2006.
- B. Rankov and A. Wittneben, "Spectral efficient protocols for half-duplex fading relay channels," *IEEE J. Sel. Areas Commun.*, vol. 25, no. 2, pp. 379–389, February 2007.
- T. M. Cover and J. A. Thomas, *Elements of Information Theory*. John Wiley & Sons, Inc., 1991.