Cooperative Communications

Lecture 2

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Outline I

Last Time, Lecture 1

- Communication networks - wireline and wireless
- Cooperative systems
- Basics of communication theory.

Today, Lecture 2

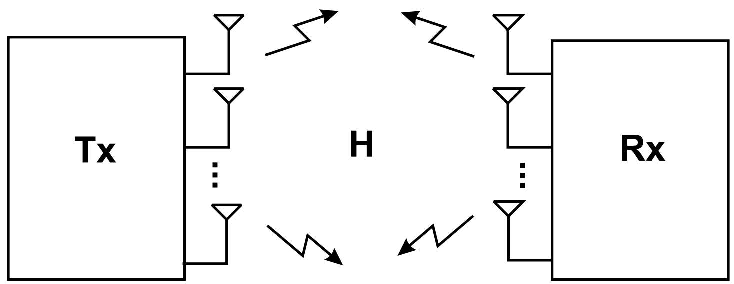
- MIMO systems as foundation for cooperative communications [1]
- Multipath propagation
- Equivalent baseband representation
- Large-scale and short-scale fading
- Path loss
- Characterization of small-scale fading

Outline II

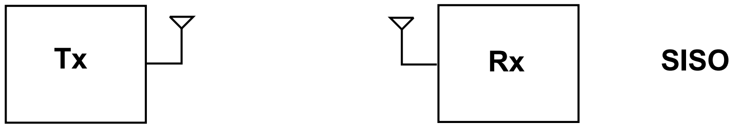
Lecture 2 cont.

- Wide sense stationarity (WSS) and uncorrelated scattering (US)
- SISO, SIMO, MISO, and MIMO signal model.
- Double directional channel model
- Classical iid channel model
- Singular value decomposition and Frobenius norm
- Spatial fading correlations

Multiple-Input Multiple-Output (MIMO) Communications



Single-Input Single-Output (SISO), SIMO, MISO



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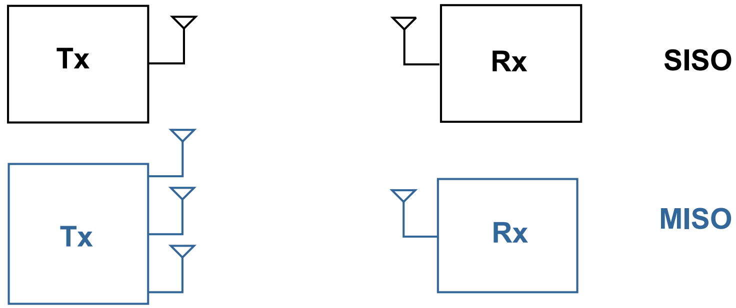
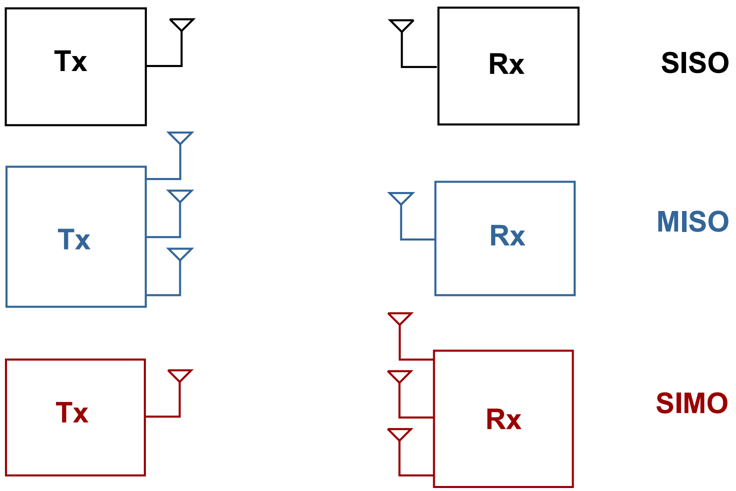


Single-Input Single-Output (SISO), SIMO, MISO



Advantages of Multiple Antennas

- MIMO systems - multiple antennas are connected to a single device
- Cooperative communication systems - each device may have a single antenna only. Cooperation allows to achive (partial) MIMO gains.

Multiple antenna gains:

- **Energy efficiency** (array gain): Signal to thermal noise ratio is improved. Increased coverage.
- **Error rate reduction** (diversity gain): Mitigates fading through spatial diversity. Improved quality.
- **Spectral efficiency** (multiplexing gain): Increased bits/channel access (bpca) rate.
- **Interference reduction**: Improve the reuse factor in multi-user scenarios.

Diversity gain and spatial multiplexing may be mutually conflicting goals both in MIMO and in cooperative communications systems.

Obstacles to MIMO Implementations

- **Hardware costs**: Multiple antennas mean multiple RF chains.
- **Battery**: More involved signal processing requires more computing power and energy.
- Portable consumer devices are especially sensitive to cost arguments.

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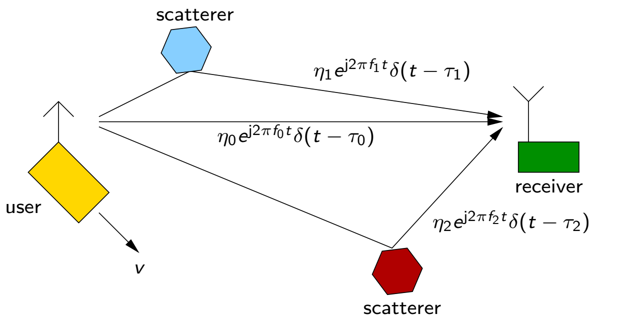
Are cooperative communcation systems the better solution? Maybe we will know a partial answer at the end of the lecture.

Multipath Propagation: SISO Case



- v velocity
- ℓ path
- η_ℓ attenuation
- τ_ℓ time delay
- f_ℓ Doppler shift
- L' number of paths
- ϕ_ℓ angle of arrival
- f_C carrier frequency
- c_0 speed of light

Time-variant channel impulse response

$$h_p(t,\tau) = \sum_{\ell=0}^{L'-1} \eta_\ell e^{j2\pi f_\ell t}\delta(\tau-\tau_\ell), \qquad f_\ell = \frac{v\cos\phi_\ell f_C}{c_0}$$

Equivalent Baseband Description I

- Transmitted signal

$$s(t) = \mathrm{Re}[s_b(t)e^{j2\pi f_c t}],$$

  with carrier frequency f_c, baseband signal $s_b(t)$.

- Received signal

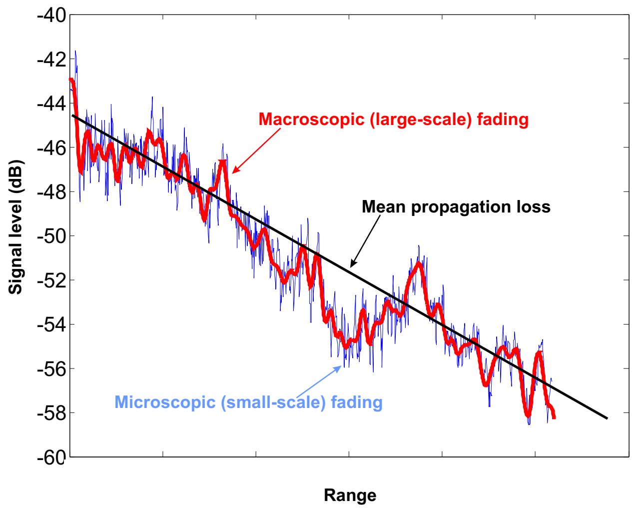
$$x(t) = \mathrm{Re}[\underbrace{\left\{\sum_{\ell=0}^{L-1}\eta_\ell e^{j2\pi f_\ell t}e^{-j2\pi f_c\tau_\ell}s_b(t-\tau_\ell)\right\}}_{r_b(t)} e^{j2\pi f_c t}]$$

  $r_b(t)$ baseband received signal

- Equivalent baseband physical channel description

$$h_p(t,\tau) = \sum_{\ell=0}^{L-1}\eta_\ell e^{j2\pi f_\ell t}\delta(t-\tau_\ell)$$

Large-scale and short-scale fading



Path loss I

Free space propagation

$$P_r = P_t \left(\frac{\lambda_c}{4\pi d} \right)^2 G_t G_r \,, \quad \lambda_c = \frac{c_0}{f_c}$$

$P_t\,,\,P_r$ transmitted power, received power

$G_t\,,\,G_r$ antenna gains

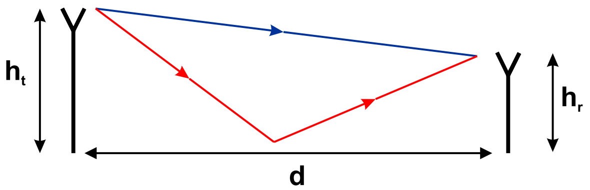
λ_c wavelength

c_0 speed of light

f_c carrier frequency

d distance between receiver and transmitter

Path loss II



Cellular environment with surface reflection

$$P_r = P_t \frac{(h_t h_r)^2}{d^4} G_t G_r \,, \quad d^2 \gg h_t h_r$$

$h_t,\ h_r$ height of antennas

Path loss exponent varies from 2.5 to 6 depending on terrain and foliage.

Large-Scale Fading

Long term signal power fluctuations due to

- buildings,
- terrain.

Characterized by a log-normal distribution with probability density function (pdf)

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

x signal power level in dB

μ mean received signal power level

σ standard deviation (typical value 8 dB)

Small-Scale Fading

Rapid fluctuations of the received signal in space, time and frequency.

- Large number of independent scattered components
- Central limit theorem
- **Rayleigh fading** - channel h is complex Gaussian with zero mean, envelope $r = |h|$ is Rayleigh distributed

$$f(r) = \frac{2r}{\Omega} e^{-\frac{r^2}{\Omega}} u(r),$$

Ω average receive power

$u(r)$ unit step function, $u(r) = \begin{cases} 1\,, & r \geq 0, r \in \mathbb{R} \\ 0\,, & r < 0, r \in \mathbb{R} \end{cases}$

- **Ricean fading** - additional (deterministic) line of sight (LOS) component

Characterization of Small-Scale Fading

- **Time selective fading**: coherence time - time separation between individual channel fades
- **Frequency selective fading**: coherence bandwidth - frequency separation between individual channel fades
- **Space selective fading**: coherence distance - separation of antenna elements for independent fading

Time-Selective Fading - Doppler Spread

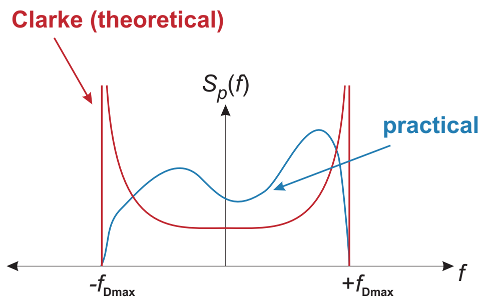
- Doppler shift ν is caused by scatterer or user movement

$$\nu = \frac{v}{\lambda_c} \cos(\theta)\,,$$

where θ is the angle between velocity vector and the propagation direction.

- Superposition of many paths leads to Doppler spread.
- For UMTS with carrier frequency 2GHz and users moving with $v = 100$km/h, Doppler is $\nu = 185$Hz.

Doppler spectrum



Clarke's model

Uniformly distributed scatterers around the mobile [2]

$$\Psi_D(\nu) = \begin{cases} \frac{1}{\pi \nu_D \sqrt{1 - (\nu/\nu_D)^2}} & \text{for } |\nu| < \nu_D\,, \\ 0 & \text{otherwise.} \end{cases}$$

Coherence time $T_C \propto 1/\nu_D$

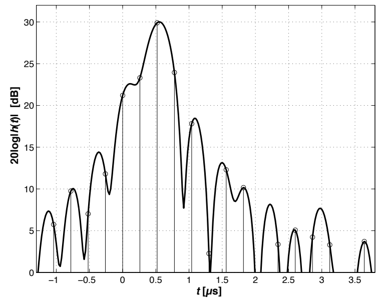
Power-Delay Profile

- Exponentially decaying power delay profile $\eta^2(\tau)$ with (root mean square) delay spread T_D.

$$\eta^2(\tau) = \frac{1}{T_\mathrm{D}} \mathrm{e}^{-\frac{\tau}{T_\mathrm{D}}}$$

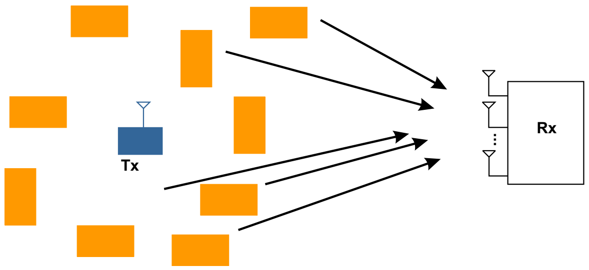
- Coherence bandwidth $B_c \propto 1/T_\mathrm{D}$.
- Typical values for urban environment $0.26\mu\mathrm{s} \leq T_\mathrm{D} \leq 1\mu\mathrm{s}$.

Frequency Selective Fading - Delay Spread



- Impulse response magnitude $|h(t)|$ (exponential PDP, non-line-of-sight environment)
- Delay spread leads to frequency-selective fading

Space-Selective-Fading - Angle Spread

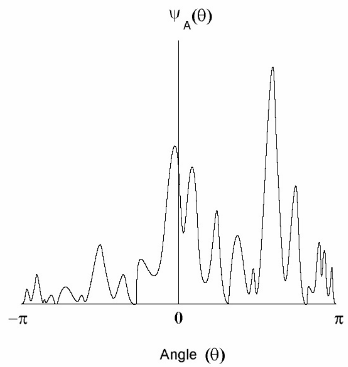


Angle spread due to

- local scatterers near the mobile
- local scatterers near the base station
- remote scatterers

Range: 2 (flat rural) to 30 (hilly terain) degrees

Angle spectrum

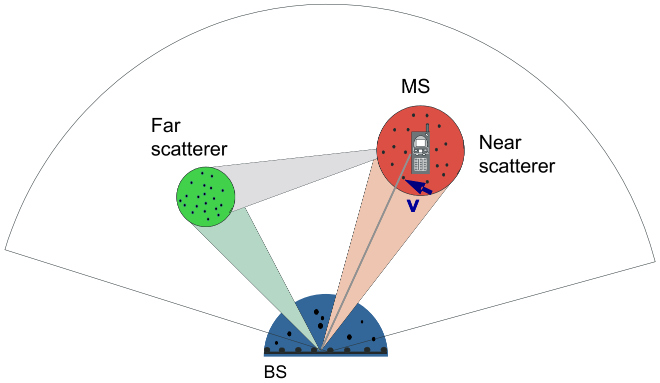


Root mean square (RMS) angle spread

$$\theta_{RMS} = \sqrt{\frac{\int_{-\pi}^{\pi} (\theta - \bar{\theta})^2 \Psi_A(\theta) \mathrm{d}\theta}{\int_{-\pi}^{\pi} \Psi_A(\theta) \mathrm{d}\theta}}, \quad \bar{\theta} = \frac{\int_{-\pi}^{\pi} \theta \Psi_A(\theta) \mathrm{d}\theta}{\int_{-\pi}^{\pi} \Psi_A(\theta) \mathrm{d}\theta}$$

Coherence distance $D_c \propto 1/\theta_{RMS}$

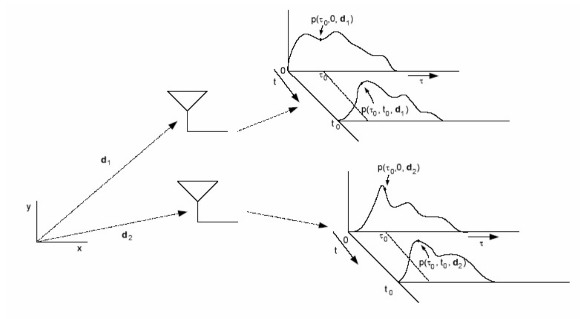
Macro-Cell Scatterer Model



Scattering

- Local to mobile station (MS) - angle spread, Doppler spread
- Local to base station (BS) - angle spread
- Far scatterer - delay spread, angle spread

Time and Space-Variant Impulse Response



- Impulse response $h_p(\tau, t, \boldsymbol{d})$ at the receive antenna at location $\boldsymbol{d}$ for an impulse launched at $t - \tau$.
- Assume zero-mean impulse response $\mathrm{E}\{h_p(\tau, t, \boldsymbol{d})\} = 0$ (no line of sight (LOS) component).
- $h_p(\tau, t, \boldsymbol{d})$ **includes** the antenna characteristics.

Wide Sense Stationarity (WSS) and Uncorrelated Scattering (US) I

Wide sense stationarity (WSS) (see [3])

Second order statistics of the channel are stationary

$$\mathrm{E}\{h_p(\tau, t) h_p^*(\tau', t')\} = R_{hh}(\tau, \tau', \Delta t),$$

Correlation function $R_{hh}(\tau, \tau', \Delta t)$ depends only on the time-difference $\Delta t = t - t'$.

WSS + Uncorrelated Scattering (US)

Attenuation and phase shift at delay τ is uncorrelated with delay τ'

$$\mathrm{E}\{h_p(\tau, t) h_p^*(\tau', t')\} = R_{hh}(\tau, \Delta t)\delta(\tau - \tau'),$$

Wide Sense Stationarity (WSS) and Uncorrelated Scattering (US) II

Delay Doppler spreading function

$$S_h(\nu, \tau) = \int_{-\infty}^{\infty} h_p(\tau, t) \mathrm{e}^{-\mathrm{j}2\pi\nu t} \mathrm{d}t$$

For WSSUS channels, paths with different delay or Doppler are uncorrelated

$$\mathrm{E}\{S_h(\nu, \tau) S_h^*(\nu', \tau')\} = C(\nu, \tau)\delta(\nu - \nu')\delta(\tau - \tau')$$

Homogeneous (HO) Channel

- $h_p(\tau, t, \boldsymbol{d})$ is WSS in space

$$\mathrm{E}\{h_p(\tau, t, \boldsymbol{d}) h_p^*(\tau, t, \boldsymbol{d} + \boldsymbol{\Delta d})\} = R_{\boldsymbol{d}}(\tau, t, \boldsymbol{\Delta d}),$$

where $R_{\boldsymbol{d}}(\tau, t, \boldsymbol{\Delta d})$ is the lagged-space correlation function.

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where $R_{\boldsymbol{d}}(\tau, t, \boldsymbol{\Delta d})$ is the lagged-space correlation function.

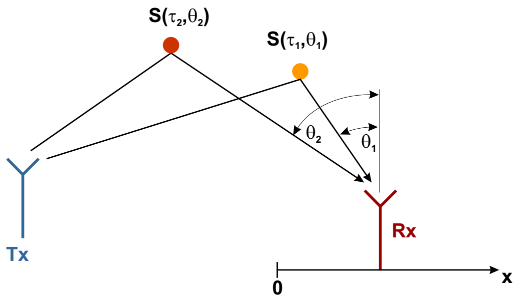
- This assumption is usually not fulfilled in cooperative communications!

Scattering Function

- **One dimensional case**: $\boldsymbol{d}$ lies on the x-axis. Channel description in the delay(τ)-time(t)-angle(θ) domain is given by $S(\tau, t, \theta)$ where

$$h_p(\tau, t, x) = \int_{-\pi}^{\pi} S(\tau, t, \theta) \mathrm{e}^{-\mathrm{j}2\pi \sin(\theta) \frac{x}{\lambda}} \mathrm{d}\theta$$

- **Time invariant case**: $S(\tau_i, \theta_i)$ is the scattering amplitude of a scatterer located at τ_i and θ_i.



MIMO Channel and Signal Models

SISO channel

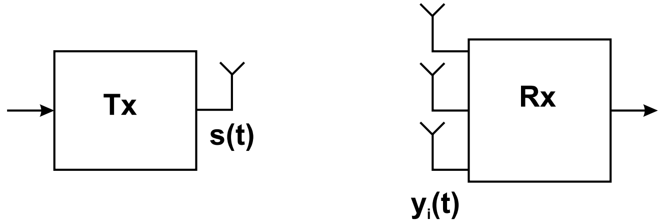
- Time-variant impulse response: $h(\tau, t) = h_p(\tau, t) * g(\tau)$
  - $h_p(\tau, t)$ *physical* propagation channel
  - $g(\tau)$ combined effect of *transmit pulse-shaping and receive matched-filtering*.
- Received signal (without noise)

$$y(t) = \int_0^{\tau_{max}} h(\tau, t) s(t - \tau) \mathrm{d}\tau = h(\tau, t) * s(t)$$

- For linear modulation, $s(t)$ is a train of discrete amplitude pulses at symbol spacing T_S

$$s(t) = \sum_{m=-\infty}^{\infty} (a_m + \mathrm{j}b_m)\delta(t_m T_S)$$

SIMO Channel



- SIMO channel represented by $M_R \times 1$ vector $\boldsymbol{h}(\tau, t)$

$$\boldsymbol{h}(\tau, t) = [h_1(\tau, t), h_2(\tau, t), \ldots, h_{M_R}(\tau, t)]^{\mathsf{T}}$$

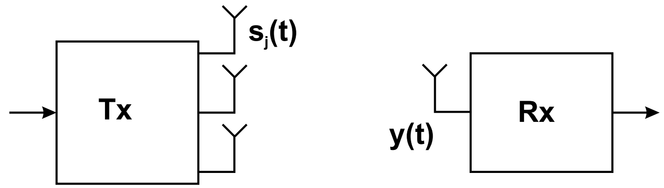
- For input signal $s(t)$, output at i-th antenna is given by

$$y_i(t) = h_i(\tau, t) * s(t), \quad i \in \{1, 2, \ldots, M_R\}$$

- In vector notation using $\boldsymbol{y}(t) = [y_1(t), y_2(t), \ldots, y_{M_R}(t)]^{\mathsf{T}}$

$$\boldsymbol{y}(t) = \boldsymbol{h}(\tau, t) * s(t)$$

MISO Channel



- MISO channel represented by $1 \times M_T$ vector $\boldsymbol{h}(\tau, t)$

$$\boldsymbol{h}(\tau, t) = [h_1(\tau, t), h_2(\tau, t), \ldots, h_{M_T}(\tau, t)]$$

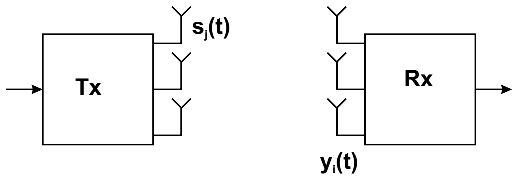
- For input signals $s_j(t)$, output is given by

$$y(t) = \sum_{j=1}^{M_T} h_j(\tau, t) * s_j(t)$$

- In vector notation using $\boldsymbol{s}(t) = [s_1(t), s_2(t), \ldots, s_{M_T}(t)]^{\mathsf{T}}$

$$y(t) = \boldsymbol{h}(\tau, t) * \boldsymbol{s}(t)$$

MIMO Channel



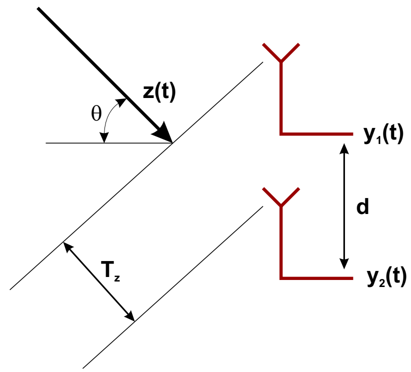
- $h_{i,j}(\tau, t)$ is impulse response between j-th ($j = 1, 2, \ldots, M_T$) transmit antenna and i-th ($i = 1, 2, \ldots, M_R$) receive antenna.
- $M_R \times M_T$ MIMO channel matrix $\boldsymbol{H}(\tau, t)$

$$\boldsymbol{H}(\tau, t) = \begin{bmatrix} h_{1,1}(\tau, t) & h_{1,2}(\tau, t) & \ldots & h_{1,M_T}(\tau, t) \\ h_{2,1}(\tau, t) & h_{2,2}(\tau, t) & \ldots & h_{2,M_T}(\tau, t) \\ \vdots & \vdots & \ddots & \vdots \\ h_{M_R,1}(\tau, t) & h_{M_R,2}(\tau, t) & \ldots & h_{M_R,M_T}(\tau, t) \end{bmatrix}$$

- In vector notation

$$\boldsymbol{y}(t) = \boldsymbol{H}(\tau, t) * \boldsymbol{s}(t)$$

Narrowband Array Assumption I



- Single Planar Wavefront
- Wavefront $z(t) = \beta(t) e^{j2\pi f_c t}$ arrives at an array having two antennas with inter-element spacing d, at angle θ.

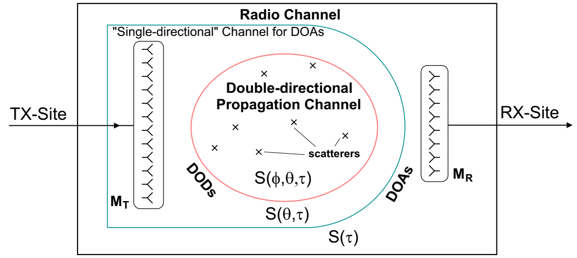
Narrowband Array Assumption II

- Wavefront $z(t)$ has bandwidth B
- Narrowband assumption: $B \ll 1/T_z \Rightarrow \beta(t - T_z) \approx \beta(t)$

$$y_1(t) = z(t) \quad y_2(t) = z(t)\mathrm{e}^{\mathrm{j}2\pi \sin(\theta)\frac{d}{\lambda}}$$

- Array response vector $\boldsymbol{a}(\theta) = [1, \mathrm{e}^{\mathrm{j}2\pi \sin(\theta)\frac{d}{\lambda}}]^{\mathrm{T}}$, equal to the array manifold (for omnidirectional antennas).
- Not fulfilled in cooperative communications!

Double Directional Channel Model



$$\boldsymbol{S}(\theta, \tau) = \int_{-\pi}^{\pi} S(\phi, \theta, \tau)\boldsymbol{b}^{\mathrm{T}}(\psi)\mathrm{d}\phi$$

$\boldsymbol{b}(\phi) \in \mathbb{C}^{M_T}$ array manifold of transmit array

$$\boldsymbol{H}(\tau) = \int_{-\pi}^{\pi}\int_{0}^{\tau_{max}} \boldsymbol{a}(\theta)\boldsymbol{S}(\theta, \tau')g(\tau - \tau')\mathrm{d}\tau'\mathrm{d}\theta$$

$\boldsymbol{a}(\theta) \in \mathbb{C}^{M_R}$ array manifold of receive array

Figure source: Steinbauer et.al [4]

Classical IID channel model

For small delay spread $T_{\mathrm{D}} \ll T_{\mathrm{S}}$ we can write

$$\boldsymbol{H}(\tau) = \left(\int_{-\pi}^{\pi} \boldsymbol{a}(\theta)\boldsymbol{S}(\theta, \tau)\mathrm{d}\theta\right) g(\tau) = \boldsymbol{H}g(\tau)$$

- If we assume **rich scattering** and **sufficient antenna spacing:**
  - Central limit theorem: Elements of $\boldsymbol{H}$ are zero-mean circularly symmetric complex Gaussian random variables with unit variance: $\boldsymbol{H} = \boldsymbol{H}_w$
  - Some properties of $\boldsymbol{H}_w$

$$[\boldsymbol{H}_w]_{i,\ell} \sim \mathcal{CN}(0,1),$$

$$\mathrm{E}\{[\boldsymbol{H}_w]_{i,\ell}\} = 0$$

$$\mathrm{E}\{|[\boldsymbol{H}_w]_{i,\ell}|^2\} = 1 \leftarrow \text{not in coop. comm.!}$$

$$\mathrm{E}\{[\boldsymbol{H}_w]_{i,\ell}\boldsymbol{H}_w]^*_{i',\ell'}\} = 0 \text{ for } i \neq i' \text{ or } \ell \neq \ell'$$

Frequency Selective Channels

MIMO channel in the frequency domain

$$\tilde{\boldsymbol{H}}(f) = \int_{-\infty}^{\infty} \boldsymbol{H}(\tau)\mathrm{e}^{-\mathrm{j}2\pi f\tau}\mathrm{d}\tau$$

More details later in the context of orthogonal frequency division multiplexing (OFDM).

Singular Value Decomposition of $\boldsymbol{H}$

- Any channel $\boldsymbol{H}$ can be decomposed as

$$\boldsymbol{H} = \boldsymbol{U}\boldsymbol{\Sigma}\boldsymbol{V}^{\mathrm{H}}$$

  - $\boldsymbol{U}^{\mathrm{H}}\boldsymbol{U} = \boldsymbol{V}^{\mathrm{H}}\boldsymbol{V} = \boldsymbol{I}_r$
  - $\boldsymbol{\Sigma} = \mathrm{diag}\{\sigma_i\}_{i=1}^r$, $\sigma_i > 0$, r is the rank of the channel $\boldsymbol{H}$

- $\boldsymbol{H}\boldsymbol{H}^{\mathrm{H}}$ is Hermitian with eigendecomposition

$$\boldsymbol{H}\boldsymbol{H}^{\mathrm{H}} = \boldsymbol{Q}\boldsymbol{\Lambda}\boldsymbol{Q}^{\mathrm{H}}$$

  - $\boldsymbol{Q}\boldsymbol{Q}^{\mathrm{H}} = \boldsymbol{Q}^{\mathrm{H}}\boldsymbol{Q} = \boldsymbol{I}_{M_R}$
  - $\boldsymbol{\Lambda} = \mathrm{diag}\{\lambda_i\}_{i=1}^{M_R}$

- $\lambda_i = \sigma_i^2$ is a random variable for random $\boldsymbol{H}$

Square Frobenius Norm of $\boldsymbol{H}$

- Square Frobenius Norm of $\boldsymbol{H}$ can be interpreted as the total power gain of the MIMO channel

$$||\boldsymbol{H}||_F^2 = \mathrm{Tr}(\boldsymbol{H}\boldsymbol{H}^{\mathrm{H}}) = \sum_{i=1}^{M_R}\sum_{j=1}^{M_T}|[\boldsymbol{H}]_{i,j}|^2 = \sum_{i=1}^{M_R}\lambda_i$$

- Power distribution:
  $||\boldsymbol{H}||_F^2$ is a chi-square distributed random variable with $2M_TM_R$ degrees of freedom,

$$f(x) = \frac{x^{M_TM_R-1}}{(M_RM_R-1)!}\mathrm{e}^{-x}u(x)$$

Spatial Fading Correlation I

Insufficient antenna spacing or lack of scattering causes individual antennas to be correlated.

- The channel may be modeled as

$$\mathrm{vec}(\boldsymbol{H}) = \boldsymbol{C}^{1/2}\mathrm{vec}(\boldsymbol{H})_w$$

- $\boldsymbol{C}$ is the $M_TM_R \times M_TM_R$ covariance matrix given by

$$\boldsymbol{C} = \mathrm{E}\{\mathrm{vec}(\boldsymbol{H})\mathrm{vec}(\boldsymbol{H})^{\mathrm{H}}\}$$

- $\boldsymbol{C}$ is Hermitian positive semi-definite

$$\boldsymbol{C} = \boldsymbol{Q}\boldsymbol{\Lambda}\boldsymbol{Q}^{\mathrm{H}} \quad \boldsymbol{C}^{1/2} = \boldsymbol{Q}\boldsymbol{\Lambda}^{1/2}\boldsymbol{Q}^{\mathrm{H}}$$

Spatial Fading Correlation II

Simpler more restrictive correlation model (Kronecker model)

$$\boldsymbol{H} = \boldsymbol{R}_r^{1/2}\boldsymbol{H}_w\boldsymbol{R}_t^{1/2}$$

- $\boldsymbol{R}_r$ is the $M_R \times M_R$ receive correlation matrix.
- $\boldsymbol{R}_t$ is the $M_T \times M_T$ transmit correlation matrix.
- $\boldsymbol{C}$, $\boldsymbol{R}_r$ and $\boldsymbol{R}_t$ are related through the Kronecker product ($\otimes$)

$$\boldsymbol{C} = \boldsymbol{R}_t^{\mathrm{T}} \otimes \boldsymbol{R}_r$$

Ricean Fading

Presence of line of sight (LOS) components

- Channel is modeled as sum of LOS and scattered components

$$\boldsymbol{H} = \sqrt{\frac{K}{1+K}}\overline{\boldsymbol{H}} + \sqrt{\frac{1}{1+K}}\tilde{\boldsymbol{H}}$$

- $\sqrt{\frac{K}{1+K}}\overline{\boldsymbol{H}} = \mathrm{E}\{\boldsymbol{H}\}$ is the LOS component and $\sqrt{\frac{1}{1+K}}\tilde{\boldsymbol{H}}$ is the fading component of the channel.
- $K \geq 0$ is the **Ricean K-factor**.

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