Cooperative Communications

Lecture 3

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Outline I

Today, Lecture 3 Part I

- Wide sense stationarity (WSS) and uncorrelated scattering (US)
- SISO, SIMO, MISO, and MIMO signal model.
- Double directional channel model
- Classical iid channel model

Today, Lecture 3 Part II

- MIMO Channel Capacity
- Channel unkown at transmitter
- Channel known at transmitter
- Diversity Multiplexing Tradeoff
- Alamouti Scheme
- OFDM

Discrete-Time Transmission on Flat-Fading MIMO Channels

For now, the channel gains are assumed constant:

$$\boldsymbol{y}_n = \boldsymbol{H}\boldsymbol{s}_n + \boldsymbol{n}_n$$

Properties of transmitted symbols:

- $\mathrm{E}\{\boldsymbol{s_n}\} = \boldsymbol{0}$,
- $\boldsymbol{R}_{ss} = \mathrm{E}\{\boldsymbol{s}_n\boldsymbol{s}_n^{\mathrm{H}}\}$,
- $\mathrm{tr}\{\boldsymbol{R}_{ss}\} = E_s$: transmit energy

Important points:

- All antennas transmit at the same time on the same resource (frequency...).
- Their signals interfere.
- This is not a problem if we decode them jointly (the receive antennas "cooperate").

Capacity of a Deterministic MIMO Channel

Method introduced by Telatar in 1995 ([1]):

- Mutual information

$$I(\boldsymbol{s};\boldsymbol{y}) = \log_2(\det(\boldsymbol{I}_{M_R} + \frac{1}{N_0}\boldsymbol{H}\boldsymbol{R}_{ss}\boldsymbol{H}^{\mathrm{H}})) \quad \text{bps/Hz}$$

- Channel capacity

$$C = \max_{\mathrm{tr}(\boldsymbol{R}_{ss})=E_s} \log_2(\det(\boldsymbol{I}_{M_R} + \frac{1}{N_0}\boldsymbol{H}\boldsymbol{R}_{ss}\boldsymbol{H}^{\mathrm{H}})) \quad \text{bps/Hz}.$$

Channel Unknown at Transmitter I

If the channel is unknown at the transmitter it is reasonable to choose

$$\boldsymbol{R}_{ss} = \frac{E_s}{M_T}\boldsymbol{I}_{M_T}$$

- This choice maximizes the average mutual information over the class of Gaussian i.i.d. channels $\boldsymbol{H}$.
- The mutual information achieved in this case is

$$I = \log_2(\det(\boldsymbol{I}_{M_R} + \frac{E_S}{M_T N_0}\boldsymbol{H}\boldsymbol{H}^{\mathrm{H}}))$$

.

Channel Unknown at Transmitter II

The mutual information

$$I = \log_2\left(\det\left(\boldsymbol{I}_{M_R} + \frac{E_S}{M_T N_0}\boldsymbol{H}\boldsymbol{H}^{\mathrm{H}}\right)\right)$$

can be written alternatively as

$$I = \log_2\left(\det\left(\boldsymbol{I}_{M_R} + \frac{E_S}{M_T N_0}\boldsymbol{U}\boldsymbol{\Lambda}\boldsymbol{U}^{\mathrm{H}}\right)\right)$$

Using the identity $\det(\boldsymbol{I}_m + \boldsymbol{A}\boldsymbol{B}) = \det(\boldsymbol{I}_n + \boldsymbol{B}\boldsymbol{A})$ for matrices $\boldsymbol{A}$ $(m \times n)$ and $\boldsymbol{B}$ $(n \times m)$ we can write

$$I = \sum_{i=1}^{r} \log_2\left(1 + \frac{E_S}{M_T N_0}\lambda_i\right)$$

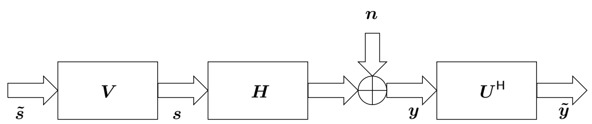
where r is the rank of the channel and λ_i are the positive eigenvalues of $\boldsymbol{H}\boldsymbol{H}^{\mathrm{H}}$.

Channel Unknown at Transmitter III

Summary:

- Multiple antennas effectively open r scalar data pipes (modes) between transmitter and receiver.
- $\boldsymbol{R}_{ss} = \frac{E_S}{M_T}\boldsymbol{I}_{M_T}$ results in equal energy allocation across the spatial modes (with wastage if $r < M_T$).
- In the absence of channel knowledge the individual channel modes are not accessible.

Channel Known at Transmitter I



- Channel knowledge $\boldsymbol{H}$ obtained through feedback or reciprocity in the case of time division duplexing (TDD)
- Individual channel modes can be accessed by linear processing at the receiver
- Input vector $\tilde{\mathbf{s}} \in \mathbb{C}^r$ with $\mathrm{E}\{\tilde{\mathbf{s}}\tilde{\mathbf{s}}^{\mathrm{H}}\} = M_T$

Channel Known at Transmitter II

Signal model

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{s} + \boldsymbol{n}$$

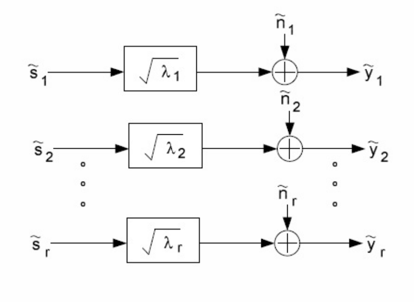
- Channel can be decomposed in $\boldsymbol{H} = \boldsymbol{U}\boldsymbol{\Sigma}\boldsymbol{V}^{\mathrm{H}}$
- Precoding at transmitter: $\boldsymbol{s} = \boldsymbol{V}\tilde{\boldsymbol{s}}$
- At the receiver: $\tilde{\boldsymbol{y}} = \boldsymbol{U}^{\mathrm{H}}\boldsymbol{y}$

$$\tilde{\boldsymbol{y}} = \boldsymbol{U}^{\mathrm{H}}\boldsymbol{H}\boldsymbol{V}\tilde{\boldsymbol{s}} + \boldsymbol{U}^{\mathrm{H}}\boldsymbol{n}$$

- Inserting the singular value decomposition of $\boldsymbol{H}$ yields

$$\tilde{\boldsymbol{y}} = \boldsymbol{\Sigma}\tilde{\boldsymbol{s}} + \tilde{\boldsymbol{n}}$$

Channel Known at Transmitter III



The matrix channel $\boldsymbol{H}$ is decomposed into r parallel sub-channels

$$\tilde{y}_i = \sqrt{\lambda_i}\tilde{s}_i + \tilde{n}_i$$

for $i \in \{1, \ldots, r\}$.

Channel Known at Transmitter IV

Mutual Information is sum of individual SISO channels

$$I = \sum_{i=1}^{r} \log_2 \left(1 + \frac{\gamma_i}{N_0}\lambda_i\right)$$

where

- $\gamma_i = \mathrm{E}\{|\tilde{s}_i|^2\}$ is transmit energy in i-th sub channel and
- $\sum_{i=1}^{r} \gamma_i = E_s$.

Capacity achieved through optimal distribution of energy on the individual sub-channels

$$C = \max_{\sum_{i=1}^{r}\gamma_i = E_s} \sum_{i=1}^{r} \log_2 \left(1 + \frac{\gamma_i}{N_0}\lambda_i\right)$$

Optimal Energy Allocation

Maximization through Lagrangian multipliers

$$\gamma_i^{opt} = \left(\mu - \frac{N_0}{\lambda_i}\right)^+, \quad i = 1, \ldots, r$$

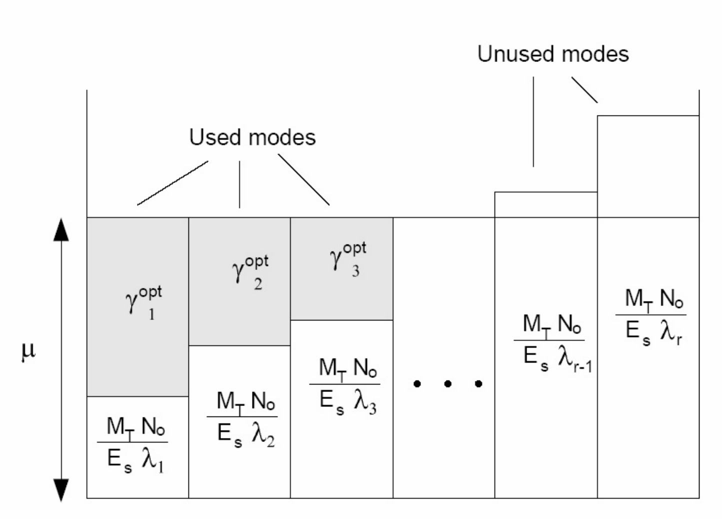
$$\sum_{i=1}^{r} \gamma_i^{opt} = E_s$$

where μ is a constant and $(x)^+$ is defined as

$$(x)^+ = \begin{cases} x & \text{if} \quad x \geq 0 \\ 0 & \text{if} \quad x < 0 \end{cases}$$

In order to ensure positive γ_i we apply the waterpouring algorithm to find μ.

Waterpouring Algorithm



Optimal Transmit Covariance Matrix

Recall that

$$\boldsymbol{s} = \boldsymbol{V}\tilde{\boldsymbol{s}}$$

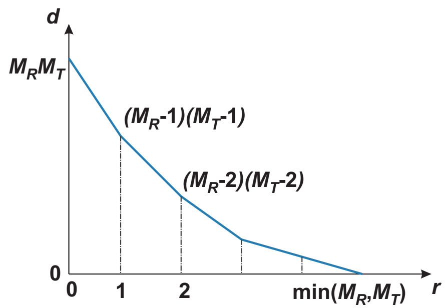
Optimal transmit covariance matrix is given by

$$\boldsymbol{R}_{ss}^{opt} = \boldsymbol{V}\boldsymbol{R}_{\tilde{s}\tilde{s}}^{opt}\boldsymbol{V}^{\mathrm{H}}$$

where

$$\boldsymbol{R}_{\tilde{s}\tilde{s}}^{opt} = \mathrm{diag}([\gamma_1^{opt}, \ldots, \gamma_r^{opt}])$$

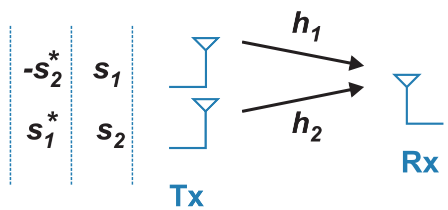
Diversity-Multiplexing Tradeoff



- Flexible tradeoff between diversity d and multiplexing gain r can be achieved.
- For the $\boldsymbol{H}_w$ (independent identical distributed) MIMO channel $d(r)$ is piecewise linear:

$$d(r) = (M_R - r)(M_T - r)$$

Space-Time Coding - Alamouti-Scheme I



- Symbol period 1: $y_1 = \sqrt{\frac{E_S}{2}}(h_1 s_1 + h_2 s_2) + n_1$,
- Symbol period 2: $y_2 = \sqrt{\frac{E_S}{2}}(-h_1 s_2^* + h_2 s_1^*) + n_2$,
- Decode by: $\hat{s}_1 = \frac{y_1 h_1^* + y_2^* h_2}{|h_1|^2 + |h_2|^2}$, $\hat{s}_2 = \frac{y_1 h_2^* - y_2^* h_1}{|h_1|^2 + |h_2|^2}$

$\Rightarrow$ *Do the maths!!*

Space-Time Coding - Alamouti-Scheme II

Alamouti scheme has diversity order $2M_R$.

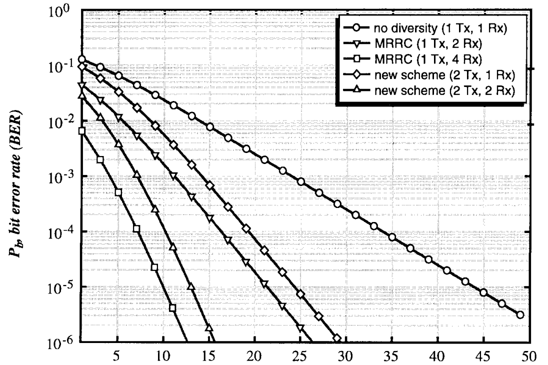
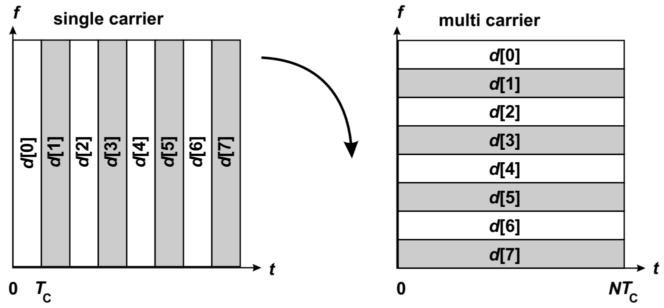


Fig. 4. The BER performance comparison of coherent BPSK with MRRC and two-branch transmit diversity in Rayleigh fading.

Figure source:[2, Fig. 4]

OFDM Fundamentals I

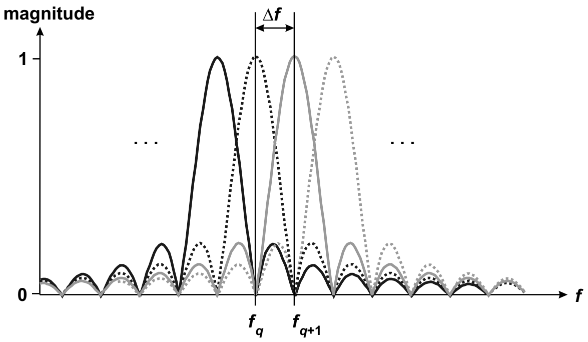
Single carrier versus multi carrier



$1/T_C$	Chip rate
N	Number of subcarriers
$d[0] \ldots d[7]$	Data symbols

OFDM Fundamentals II

Orthogonal subcarriers

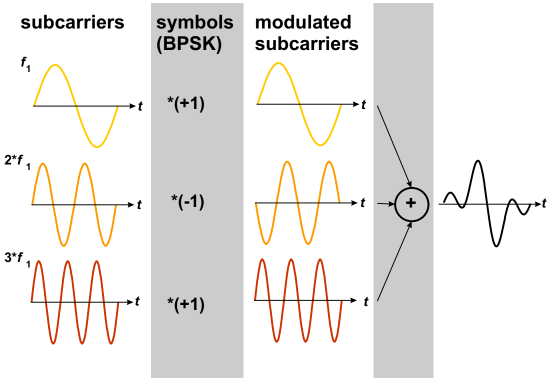


$$f_q = q/(NT_C)$$

$q \in \{0, \ldots, N-1\}$ Subcarrier index

OFDM Fundamentals III

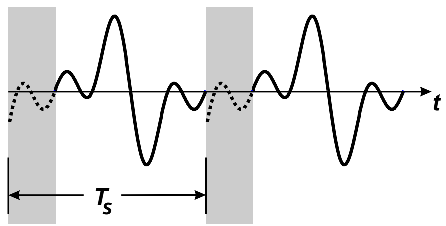
Processing steps



Efficiently implementable by means of an inverse discrete Fourier transform.

OFDM Fundamentals IV

Cyclic prefix insertion

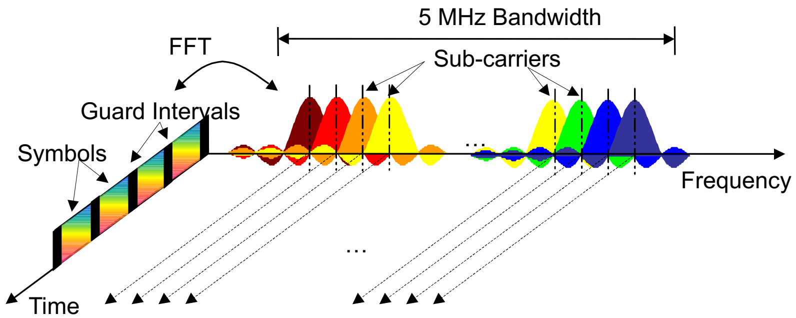


T_S OFDM symbol duration.

- A copy of the signal tail (length T_G) is inserted at the beginning of each OFDM symbol.
- Absorbs multipath components, and turns a convolution into a cyclic convolution.

OFDM Fundamentals V

OFDM time frequency representation



OFDM System Design

- No inter-symbol interference: Guard interval larger than the delay spread T_D
$$T_G > T_D$$
- Spectral efficiency: Symbol duration much larger than delay spread
$$T_S \gg T_D$$
- Inter-carrier interference: Symbol rate much higher than Doppler shift f_D
$$1/T_S \gg f_D$$
$[T_S = (N + G)T_C]$

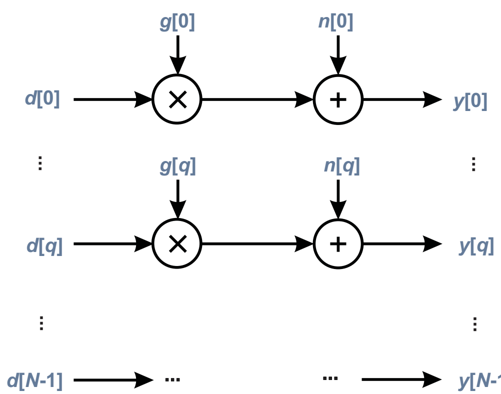
$T_G = GT_C$ cyclic prefix length

G length of cyclic prefix in samples (chips)

$T_S = (N + G)T_C$ OFDM symbol length

Receiver Side Processing

- Drop cyclic prefix and perform DFT
- Channel partitioned in N parallel frequency flat channels
- Simple equalization - complexity grows with $N \log(N)$



q subcarrier index

d data symbol

g subcarrier channel coefficient

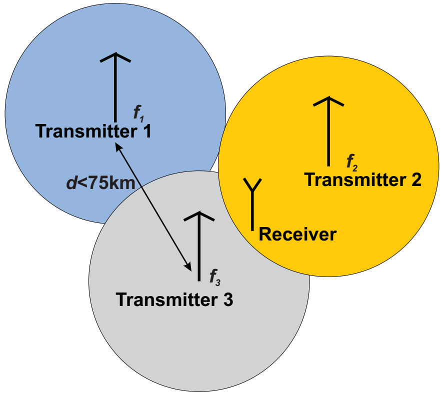
n additive noise

y received symbol

Digital Video Broadcasting (DVB-T)

Single frequency broadcasting networks

- All transmitters use the same frequency $f_1 = f_2 = f_3$
- Large distances $d \leq 75$km between individual (high power) transmitters cause long delay spreads $T_\mathrm{D} \leq 128\,\mu$s



$$
\begin{aligned}
B &= 8\,\mathrm{MHz} && \text{bandwidth}\\
N &= 8192 && \text{subcarriers}\\
G &= 1024 && \text{cyclic prefix length}\\
\Delta f &= 988\,\mathrm{Hz} && \text{subcarrier bandwidth}
\end{aligned}
$$

Wireless LAN (802.11a)

Indoor application

- Multipath propagation, delay spread $T_\mathrm{D} < 800\,\mathrm{ns}$

$$
\begin{aligned}
B &= 20\,\mathrm{MHz} && \text{bandwidth}\\
N &= 64 && \text{subcarriers}\\
G &= 15 && \text{cyclic prefix length}\\
\Delta f &= 312\,\mathrm{kHz} && \text{subcarrier bandwidth}
\end{aligned}
$$

- Physical layer data rate 54 Mb/s
- 802.11n: Extensions to MIMO systems currently under development [3]

References I

I. Telatar, "Capacity of multi-antenna gaussian channels," AT&T Bell Labs, Tech. Rep., Jun. 1995.

S. Alamouti, "A simple transmitter diversity sceme for wireless communications," *IEEE J. Sel. Areas Commun.*, vol. 16, no. 8, pp. 1451–1458, Oct. 1998.

S. Nanda, R. Walton, J. Ketchum, M. Wallace, and S. Howard, "A high-performance MIMO OFDM wireless LAN," *IEEE Commun. Mag.*, vol. 43, no. 2, pp. 101–109, Feb. 2005.