Cooperative Communications

Lecture 4

Thomas Zemen, Nicolai Czink

March 31, 2011

Outline I

Last Time, Lecture 3

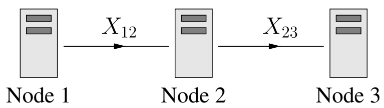
- MIMO Channel Capacity
- Channel unkown at transmitter
- Channel known at transmitter
- Diversity multiplexing tradeoff
- Alamouti scheme
- OFDM

Today, Lecture 4

- Vehicular channel properties
- Network models (wireline and wireless)
- Wireline cooperation methods
- Wireless cooperation methods

Network Models

- Network represented by a graph
  - set of $\mathcal{N}$ nodes
  - set of $\mathcal{E}$ edges, that are pairs of nodes
  - directed edge (u, v) goes from node u to node v
  - Edge (u, v) has capacity C_{uv}
  - Edge variables have alphabet of size $2^{C_{uv}}$

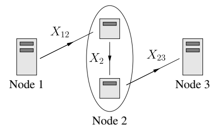


$$\mathcal{N} = \{1, 2, 3\}, \mathcal{E} = \{(1, 2), (2, 3)\}$$

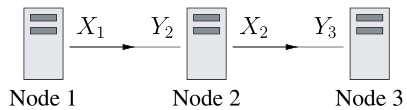
Figure source: [1, Fig. 3.1]

Node Constraints

- Node 2 has limited processing power C_2



- Half duplex constraint - a port can either transmit or receive

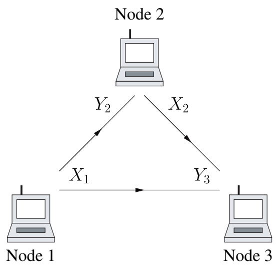


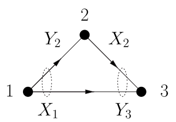
$$Y_2 = \begin{cases} X_1 & \text{if } X_2 = 0, \\ 0 & \text{if } X_2 \neq 0. \end{cases}$$

The symbol 0 might represent a "silence" symbol.

Figure source: [1, Fig. 3.2 and Fig. 3.3]

Wireless Relay Channel





$$Y_2 = \frac{h_{12}}{d_{12}^{\alpha/2}} X_1 + Z_2$$
$$Y_3 = \frac{h_{13}}{d_{13}^{\alpha/2}} X_1 + \frac{h_{23}}{d_{23}^{\alpha/2}} X_2 + Z_3$$

- h_{uv} are fading gains
- d_{uv} are distances
- α is the attenuation exponent

Figure source: [1, Fig. 3.5]

Fast and Slow Fading

- Marginal distributions:
  - Assume the $H_{uv,i}$, $i = 1, 2, \dots, n$, have the same marginal distribution H_{uv} during a communication session (stationarity)
  - No fading: H_{uv} is a known constant
  - Rayleigh fading (see Lecture 2 and 3): $H_{uv} \sim \mathcal{CN}\{0, 1\}$ is complex Gaussian, with zero mean and unit variance.
- Temporal correlation
  - Fast fading: $h_{uv,i}$ are independent realizations of H_{uv}
  - Slow fading: $H_{uv,i} = H_{uv}$, hence H_{uv} is drawn once for all $i = 1, 2, \dots, n$.
  - Correlated fading e.g. according to Clarke's model will be treated later.
- A channel is fast fading if each packet encounters many channel realizations. A channel is slow fading if a packet encounters one channel realizations.

Wireless Device Models

- Power and energy constraints:
  - Block constraint: $\sum_{i=1}^{n} |X_i|^2/n \leq P$ or $\sum_{i=1}^{n} \mathrm{E}\left\{|X_i|^2\right\}/n \leq P$
  - Symbol-wise constraints: $|X_i|^2 \leq P$ or $\mathrm{E}\left\{|X_i|^2\right\} \leq P$ for all i
- Half-duplex constraint

$$Y_2 = \begin{cases} \frac{h_{12}}{d_{12}^{\alpha/2}} X_1 + Z_2 & \text{if } X_2 = 0 \\ 0 & \text{if } X_2 \neq 0 \end{cases}$$

- Limited channel knowledge
  - (no) channel state information at the transmitter (CSIT) and (no) channel state information at the receiver (CSIR)

Discrete Memoryless Network Models I

- Node u has one input variable X_u and one output variable Y_u.
- Network clock: node u transmits $X_{u,i}$ between clock tick $i-1$ and tick i, and receives $Y_{u,i}$ at tick i. The clock ticks n times.
- Causality: $X_{u,i}$ function of its own messages and its past channel outputs $Y_u^{i-1} = Y_{u,1}, Y_{u,2}, \dots, Y_{u,i-1}$.
- M sources, source m puts out message W_m with B_m bits and rate $R_m = B_m/n$.
- A sink accepts an estimate $W_m(u)$ at node u.

Discrete Memoryless Network Models II

- Capacity region: closure of the set of rate tuples $(R_1, R_2, \ldots, R_M)$ for which
$$\Pr\left[\bigcup_{m=1}^{M}\bigcup_{u\in\mathcal{D}_m}\left\{\hat{W}_m(u)\neq W_m\right\}\right]$$
can be made close to zero, where $\mathcal{D}_m$ is the set of nodes that decode W_m.
- The capacity region is not known for any memoryless network except for the discrete memoryless channel (DMC) and the multiple access channel (MAC).

Basic Networks

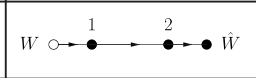
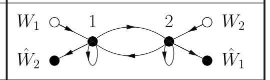
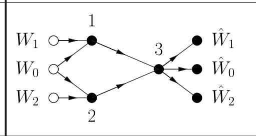
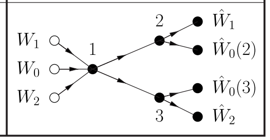
Network	Device Nodes	Channel Edges	Sources	Sinks	Graph
Point–to–Point Channel (DMC & AWGN Channel)	2	1	1	1	
Two–way Channel (2WC)	2	4	2	2	
Multiaccess Channel (MAC)	3	2	3	3	
Broadcast Channel (BC)	3	2	3	4	

Figure source: [1, Fig. 3.7]

Cooperative Networks

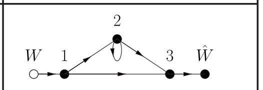
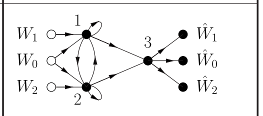
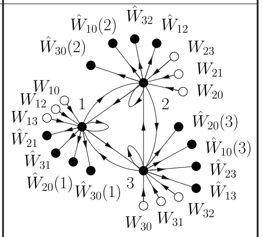
Network	Device Nodes	Channel Edges	Sources	Sinks	Graph
Relay Channel (RC)	3	4	1	1	
MAC with Generalized Feedback (MAC–GF)	3	6	3	3	
Three–way Channel (3WC)	3	9	9	12	

Figure source: [1, Fig. 3.8]

Networks with Four Device Nodes

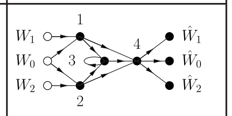
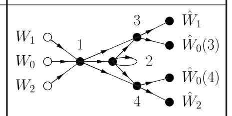
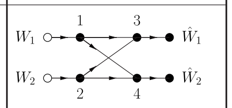
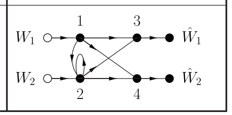
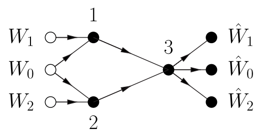
Network	Device Nodes	Channel Edges	Sources	Sinks	Graph
MAC with a Dedicated Relay (MAC–DR)	4	6	3	3	
BC with a Dedicated Relay (BC–DR)	4	6	3	4	
Interference Channel (IC)	4	4	2	2	
Cognitive Radio Channel	4	6	2	2	

Figure source: [1, Fig. 3.9]

AWGN MAC



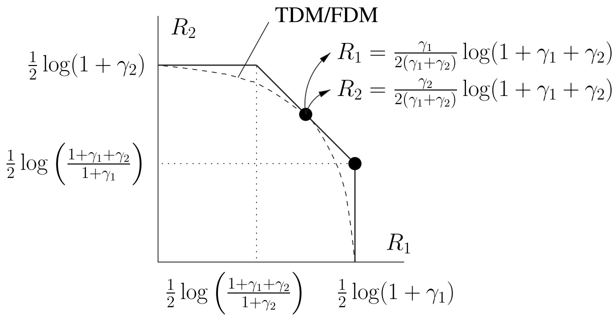
$$Y = \frac{h_1}{d_1^{\alpha/2}} X_1 + \frac{h_2}{d_2^{\alpha/2}} X_2 + Z, \quad \sum_{i=1}^{n} \mathrm{E}\left\{|X_{u,i}|^2\right\}/n \leq P_u, \ u = 1, 2$$

Rate region defined by

$$R_1 \leq \frac{1}{2}\log_2(1+\gamma_1)$$
$$R_2 \leq \frac{1}{2}\log_2(1+\gamma_2)$$
$$R_1 + R_2 \leq \frac{1}{2}\log_2(1+\gamma_1+\gamma_2)$$

where $\gamma_u = \left(\frac{P_u}{N}\right)\frac{|h_u|^2}{d_u^{\alpha}}$, $u = 1, 2$.

Capacity Region



Frequency division mutliplex (FDM):

- Node 1 and 2 use the fraction α and $1-\alpha$ of the bandwidth.
- Noise power reduces to αN and $(1-\alpha)N$
- $R_1 = \alpha/2\log_2(1+\gamma_1/\alpha)$, $R_2 = (1-\alpha)/2\log_2\left(1+\gamma_2/(1-\alpha)\right)$

Figure source: [1, Fig. 3.10]

Routing I

Routing assigns flows to every path so that no coding , i.e. combining of bits symbols or packets is done.

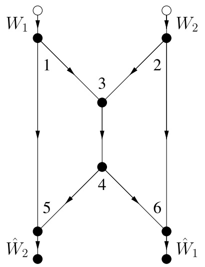


Figure source: [1, Fig. 4.1]

- Directed network: From node 1 to node 6 exists exactly one path $(1, 3, 4, 6)$ exists.
- Routing achieves the rate pair $(R_1, R_2) = (1-\beta, \beta)$ for $0 \leq \beta \leq 1$.

Routing II

Undirected network

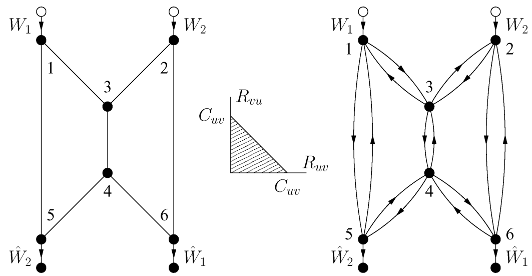


Figure source: [1, Fig. 4.2]

Each edge is modeled as two-way-channel (2WC) defined by

$$(Y_{uv}, Y_{vu}) = \begin{cases} (X_{uv}, Z_{vu}) & \text{if} \quad X_{uv} \neq 0, X_{vu} = 0, \\ (Z_{uv}, X_{vu}) & \text{if} \quad X_{uv} = 0, X_{vu} \neq 0, \\ (Z_{uv}, Z_{vu}) & \text{if} \quad X_{uv} = 0, X_{vu} = 0. \end{cases}$$

Routing III

Optimal routing for undirected butterfly network

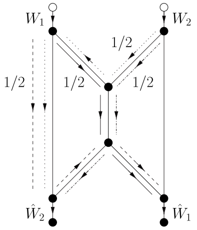


Figure source: [1, Fig. 4.3]

- We assume $C_{uv} = 1$ for all edges (u, v).
- Four paths from node 1 to node 6 exist: $(1, 3, 4, 6)$, $(1, 3, 2, 6)$, $(1, 5, 4, 6)$, $(1, 5, 4, 3, 2, 6)$
- Rate pair $(R_1, R_2) = (1, 1)$ can be achieved.

Network Coding

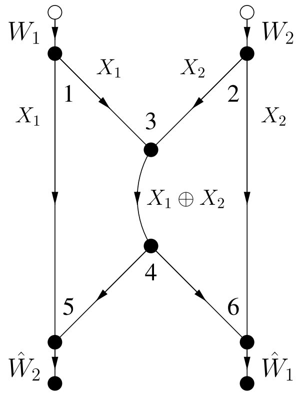


Figure source: [1, Fig. 4.4]

- Routing: Smaller rate region for directed than for undirected networks
- Network coding: allow combination of packets.
  - Node 3 combines packets by XORing them bitwise, $X_1 \oplus X_2$.
  - Called linear network coding if combining operation if done over a finite field.

Network coding achieves $(R_1, R_2) = (1, 1)$ for a directed network.

Wireless Strategies

Cooperative coding combines symbols atht the physical (and higher) layer to produce new symbols.

Cooperative coding types:

- amplify-and-forward (AF)
- classic multi-hop
- compress-and-forward (CF)
- decode-and-forward (DF)
- multipath decode-and-forward(MDF)
- ...

First we will use idealized wirless models

- full duplex radio
- CSIR, no CSIT

Basic Model I

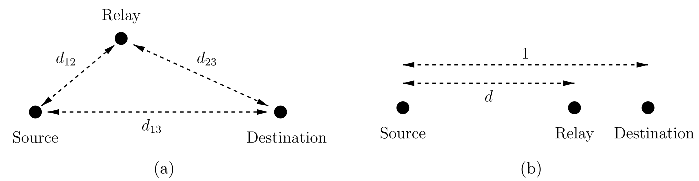


Figure source: [1, Fig. 4.8]

(a) Nodes u and v at distance d_{uv}

(b) Linear geometry

- source and destination at distance $d_{13} = 1$.
- Relay at distance $d_{12} = |d|$ to the source and $d_{23} = |1 - d|$ to the destination
- Long-range attenuation is included in power constraints

Basic Model II

Signal model:

$$Y_2 = \frac{H_{12}}{|d|^{\alpha/2}} X_1 + Z_2$$

$$Y_3 = H_{13} X_1 + \frac{H_{23}}{|1-d|^{\alpha/2}} X_2 + Z_3$$

with $Z_i \sim \mathcal{CN}(0, N)$.
We will consider three kinds of fading:

1. no fading - H_{uv} is constant
2. fast uniform phase fading - H_{uv} are independet and uniform over $\left\{ e^{j\phi} : \phi \in [0, 2\pi) \right\}$.
3. fast Rayleigh fading - H_{uv} are indendent and Gaussian with zero mean and unit variance

Amplify-and-Forward I

The relay amplifies the received signal

$$X_{2,i} = a Y_{2,i-1} = a \left(\frac{H_{12,i-1}}{|d|^{\alpha/2}} X_{1,i-1} + Z_{2,i-1} \right)$$

where a is chosen to satify the relay's power constraint.
Destination output

$$Y_{3,i} = H_{13,i} X_{1,i} + \frac{H_{23,i}}{|1-d|^{\alpha/2}} X_{2,i} + Z_{3,i}, \quad (1)$$

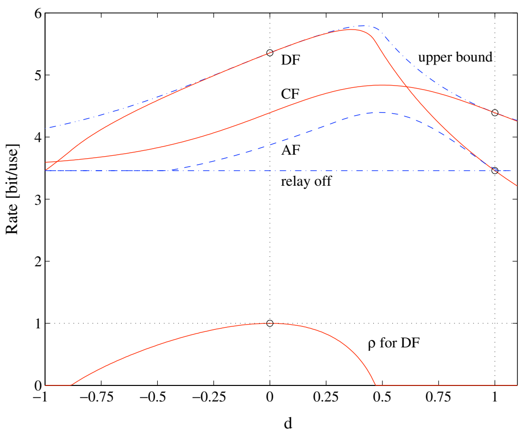
$$= H_{13,i} X_{1,i} + a \frac{H_{12,i-1} H_{23,i}}{|d|^{\alpha/2} |1-d|^{\alpha/2}} X_{1,i-1} + a \frac{H_{23,i}}{|1-d|^{\alpha/2}} Z_{2,i-1} + Z_{3,i} \quad (2)$$

To fulfil the power constraint

$$|a|^2 \leq \frac{P_2}{N + P_1 \, \mathrm{E}\left[|H_{12}|^2\right] / |d|^{\alpha}}$$

Amplify-and-Forward II

Without fading (2) is an AWGN channel with unit memory intersymbol interference $\rightarrow$ waterfilling optimization of the spectrum of X_1^n [2, Sec. VII.B], [3, Sec. 5.3.2].



- $P_1/N = P_2/N = 10$
- $H_{uv} = 1$
- $\alpha = 2$
- relay off for $d \leq 0.5$

Figure source: [1, Fig. 4.9]

Classic Multi-Hop

- Source transmitts message W to the relay in one-time slot
- Relay fowards W to the destination in second-time slot
- Time fraction τ assigned to first hop and $\bar{\tau} = 1 - \tau$ to second hop
- For constant H_{12} and H_{23}

$$R = \min \left[\tau \log_2 \left(1 + \frac{P_1 |H_{12}|^2}{\tau |d|^{\alpha} N} \right), \bar{\tau} \log_2 \left(1 + \frac{P_2 |H_{23}|^2}{\bar{\tau} |1-d|^{\alpha} N} \right) \right]$$

Classic Multi-Hop performs worse than using no relay for any d.
Multi-hop works well for half-duplex relays if $\alpha > 2$.

References I

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- G. Kramer, M. Gastpar, and P. Gupta, "Cooperative strategies and capacity theorems for relay networks," *IEEE Trans. Inf. Theory*, vol. 51, no. 9, pp. 3037–3063, Sept. 2005.
- J. N. Laneman, "Cooperative diversity in wireless networks: Algorithms and architectures," Ph.D. dissertation, Massachusetts Institute of Technology, 2002.