Cooperative Communications

Lecture 5

Thomas Zemen, Nicolai Czink

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Outline I

Last Time, Lecture 4 Part II

- Theory and Bounds
  - Wireline cooperation methods
  - Wireless cooperation methods

Today, Lecture 5

- Towards practical implementations
  - Gaussian (half-duplex) relay channel
  - Degraded relay channel
  - Decode-and-forward (DF)
  - Low density parity check (LDPC) codes

Wireless Strategies I

Cooperative strategies

- Decode-and-Forwards (DF)
- Estimate-and Fowards (EF) (Compress-and-Forward (CF))

Recap of the relay channel (with simplified notation)

- Source (S) sends data to
- destination (D), and is aided by the
- relay (R) which has no data of its own to transmit.

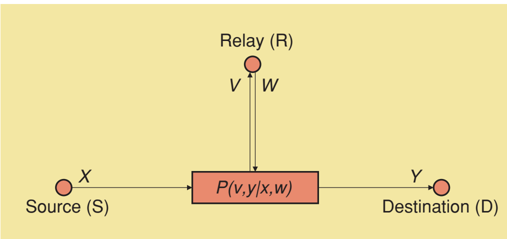


Figure source: [1, Fig. 1]

Wireless Strategies II

Half duplex relays

- a half duplex relay cannot transmit and receive simultaneously in the same frequency band
- full duplex operation is impractical
  - requires accurate interference cancellation between transmitted and received signal
  - power difference of this two signals is several orders of magnitude ($\approx 100\,$dB)
- Half-duplex operation
  - separating transmitted and received signal in time or frequency
  - using orthogonal signals (e.g. orhtogonal spreading codes, multicarrier system)

Relay Channel Model I

Time-division hald-duplex communication takes place over two time slots of normalized duration t and $t' = (1-t)$.

- First slot
  - S transmits information that is received by both R and D
  - Broadcast (BC) mode
- Second slot
  - Both S and R transmit to D
  - Multiple-access (MAC) mode

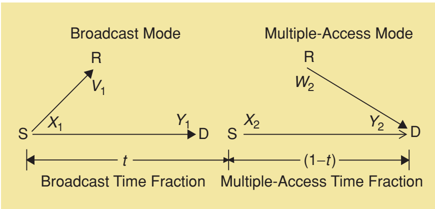


Figure source: [1, Fig. 2]

Relay Channel Model II

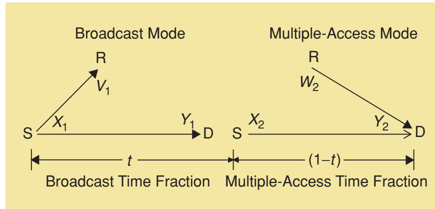
Gaussian relay channel model:

$$V_1 = h_{SR}X_1 + N_{R_1}, \quad Y_1 = h_{SD}X_1 + N_{D_1}$$

$$Y_2 = h_{SD}X_2 + h_{RD}W_2 + N_{D_2}$$

Variable naming convention:

- X ... signal transmitted by the source
- V ... signal received by the relay
- W ... signal transmitte by the relay
- Y ... signal received by the destination
- subscript $\cdot_1$ ... BC mode
- subscipt $\cdot_2$ ... MAC mode
- SR channel ... source-relay channel



Relay Channel Model III

- N_{R_1} ... noise realization at the relay receiver in BC mode
- h_{SR} ... SR channel realization
- $\gamma_{SR} = |h_{SR}|^2$ ... SR channel gain
- All noise variables are zero mean and unit variance Gaussian $\sim \mathcal{N}(0,1)$
- Are variables are considered to be real valued (extension to the complex valued case is straight forward)
- Instantaneous channel state information (CSI) is assumed to be perfectly known at transmitter, relay, and receiver!
- $\rightarrow$ design applicable for slow fading channels - channel remains constant for the whole codeword
- Perfect synchronization is assumed.

Relay Channel Model IV

- Transmission power constraint

$$\mathcal{P}: \quad tP_{S_1} + t'(P_{S_2} + P_{R_2}) \leq P$$

  where
  - $P_{S_1} = \mathrm{E}[X_1^2]$ ... source transmission power in BC mode
  - P ... total system transmission power
  - $SNR = P$ ... because noice variance is normalized to unity.
- Fair comparison of relaying with direct transmission: Sum of source and relay transmission power for the relay link must be equal to the source transmission power in the direct link.

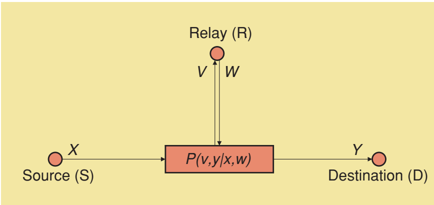
Degraded Relay Channel

Relay channel (full-duplex): $(\mathcal{X} \times \mathcal{W}, p(Y, V|X, W), \mathcal{Y} \times \mathcal{V})$

- $\mathcal{X}, \mathcal{Y}, \mathcal{V}, \mathcal{W}$ denote the alphabet of X, Y, V, W
- Relay channel is called physically degraded [2, Sec. 15.7], if

$$p(Y, V|X, W) = p(V|X, W)p(Y|V, W)$$

This means the output of the relay channel at the destination D does not depend on the source signal X. That is why it is called physically degraded. There is simply no connection between source and destination.



Relay Channel Geometry

- Relay position denoted by d

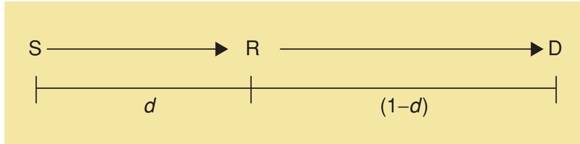


Figure source: [1, Fig. 3]

- Distance SD normalized to unity.
- $\gamma_{SD} = 1 \ldots$ SD channel gain
- $\gamma_{SR} = 1/d^{\alpha} \ldots$ SR channel gain
- $\gamma_{RD} = 1/(1-d)^{\alpha} \ldots$ RD channel gain
- $\alpha \ldots$ channel attenuation exponent

Decode-and-Forward (DF) Relay Coding I

Basic idea

- The source transmission if first decoded by the relay.
- The relay helps then the destination by transmitting additional information about the codeword.
- Different code families can be utilized for relaying
  - convoluational codes
  - turbo codes
  - low density parity check (LDPC) codes [3]
- A variant of DF is coded cooperation that allows to realize diversity benefits $\rightarrow$ we will discuss this variant in detail in some later lecture.

Decode-and-Forward (DF) Relay Coding II

Rate of the DF protocol in bits/channel use:

$$\begin{aligned} R_{DF} = \max_{0 \leq t \leq 1} \max_{p(x1), p(x_2, w_2)} \min\{ & tI(X_1; V_1) + t'I(X_2; Y_2|W_2) \\ & , tI(X_1; Y_1) + t'I(X_2, W_2; Y_2)\} \end{aligned} \quad (1)$$

where the channel probabilities are of the form

$$p(x_1, v_1, y_1) = p(x_1)p(v_1, y_1|x_1)$$

and

$$p(x_2, w_2, y_2) = p(x_2, w_2)p(y_2|x_2, w_2) .$$

Decode-and-Forward (DF) Relay Coding III

For the Gaussian relay channel the expression simplifies to

$$R_{DF_G} = \max_{\mathcal{P}} \max_{0\le t,r\le 1} \min\left\{tC\left(P_{SR}\right) + t'C\left((1-r^2)P_{SD_2}\right),\right.$$
$$\left. tC\left(P_{SD_1}\right) + t'C\left(P_{SD_2} + P_{RD} + 2r\sqrt{P_{SD_2}P_{RD}}\right)\right\}$$

where

- $r \ldots$ correlation between source and relay signals in MAC mode
- $C(x) = \frac{1}{2}\log(1+x) \ldots$ capacity of a Gaussian link
- Received powers
  - $P_{SR} = P_{S_1}\gamma_{SR} \ldots$ relay from source in BC mode
  - $P_{SD_1} = P_{S_1}\gamma_{SD} \ldots$ destination from source in BC mode
  - $P_{RD} = P_{R_2}\gamma_{RD} \ldots$ destination from relay in MAC mode
  - $P_{SD_2} = P_{S_2}\gamma_{SD} \ldots$ destintion from source in MAC mode

Information Theoretic DF Coding

Aim: achive the rate given in (1), see slide 12.

- First divide information at source into two independent parts (ω, ν).
- In total N (code) symbols are transmitted
  - $tN \in \mathbb{Z}$ symbols in BC mode
  - $(1-t)N$ in MAC mode

Encoding and decoding in BC mode

The source endcodes ω to produce a tN symbol length codeword $c_{SR_1} \in \mathcal{C}_{SR_1}$ with rate $R_{SR_1} = I(X_1; V_1)$

- The codeword c_{SR_1} with added noise is received by R and D
- Relay can decode c_{SR_1} reliably since R_{SR_1} is an achievable rate for the SR link.
- Destination cannot decode because the capacity of the SD link is less than the SR link. Received signal is stored for decoding after the MAC mode.

DF relaying outperforms direct communications only if the SR link is better than the SD link.

Encoding and decoding in MAC mode I

- Destination has $tNI(X_1; Y_1)$ bits of information in the undecodeable noisy codeword c_{SR_1} from BC mode.
- Additionally $tN\left(I(X_1; V_1) - I(X_1; Y_1)\right)$ bits are needed to reliably decode c_{SR_1}.
- Extra bits are send jointly by S and R using the codeword $c_{RD_2} \in \mathcal{C}_{RD_2}$ of rate

$$R_{RD_2} = \frac{t}{t'}\left(I(X_1; V_1) - I(X_1; Y_1)\right)$$

Encoding and decoding in MAC mode II

- Second part of information ν is also sent in MAC mode using a codeword $c_{SD_2} \in \mathcal{C}_{SD_2}$ utilizing the remaining capacity of the MAC channel.
- New information sent by S only, rate is given by rate region of MAC (S,R to D)

$$R_{SD_2} = min\left\{ I(X_2, W_2; Y_2) - \frac{t}{t'}\left(I(X_1; V_1) - I(X_1; Y_1)\right), \right. \\ \left. I(X_2; Y_2|W_2) \right\}$$

- The average rate in BC and MAC mode: $tR_{SR_1} + t'R_{SD_2}$ bits/channel use
- Destination first decodes codewords c_{RD_2} and c_{SD_2} in MAC mode.

Encoding and decoding in MAC mode III

- Source rate R_{SD_2} and relay rate R_{RD_2} correspond to a point on the multiple-access capacity region that can be achieved by successive decoding (also known as onion peeling, stripping or superposition coding) of a pair of single-user codes.
- After decoding c_{RD_2} and c_{SD_2} desination can decode c_{SR_1} using the additional information carried by c_{RD_2} as side information.
- c_{SR_1} is treated as a codeword $c_{SD_1} \in \mathcal{C}_{SD_1}$ of rate $R_{SD_1} = I(X_1; Y_1) \leq R_{SR_1}$ using the additional side information from c_{RD_2} (additional parity information).

Decoding Regions for DF in BC mode

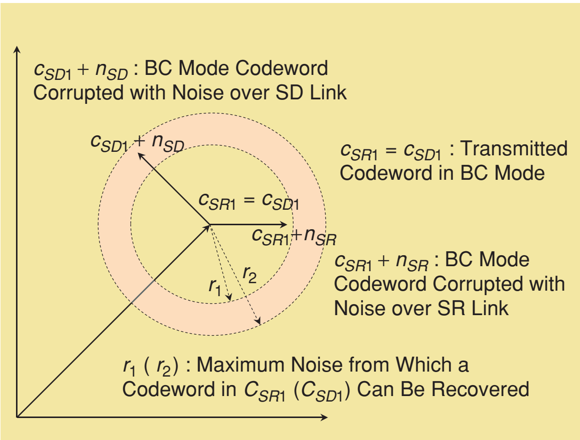


Figure source: [1, Fig. 4]

Introduction to LDPC Codes I

Binary LDPC code:

- Linear block code with $n \times m$ sparse parity-check matrix
- Matrix can be represented by a bipartite graph with
  - n variable nodes corresponding to rows (bits in the codeword)
  - m check nodes corresponding to columns (parity check equations)
  - an edge between a variable and a check node represents a one in a certain row and column

Introduction to LDPC Codes II

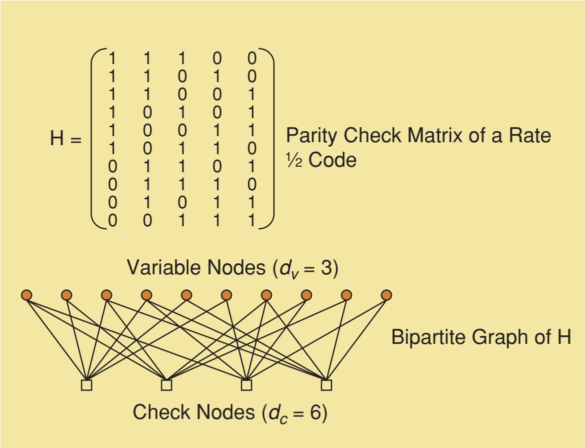


Figure source: [1, Fig. 5]

LDPC Code Design for DF Relaying I

Important for code construction:

- Source and relay signal should have an optimal correlation r in MAC mode.
- Any correlation of inputs can be achieved by weighed addition of completely correlated signals ($r = 1$) and completely uncorrelated signals ($r = 0$)
- $\rightarrow$ code design for two extreme cases of $r = 0, 1$ only.
- LDPC code optimization using density evolution using Gaussian approximation, see [3] for more details.

LDPC Code Design for DF Relaying II

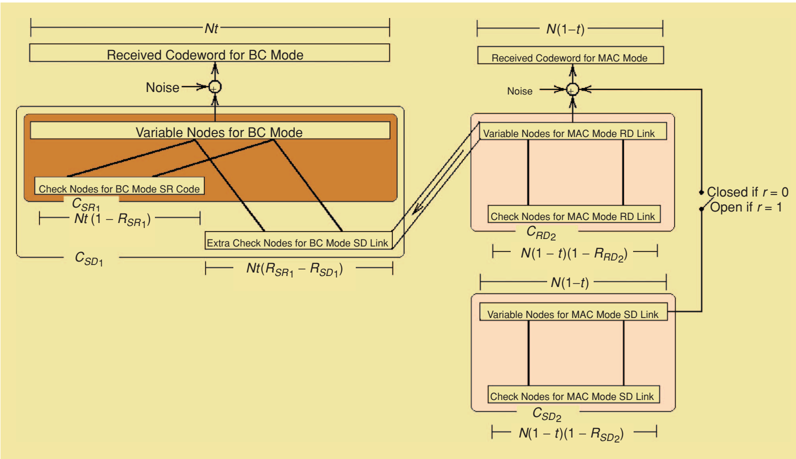


Figure source: [1, Fig. 6]

Numerical Results I

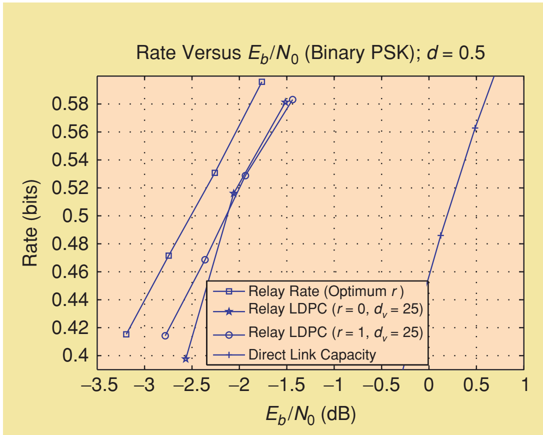


Figure source: [1, Fig. 7 (a)]

Limiting performance of LDPC coding scheme compared to binary signaling in the direct link.

Numerical Results II

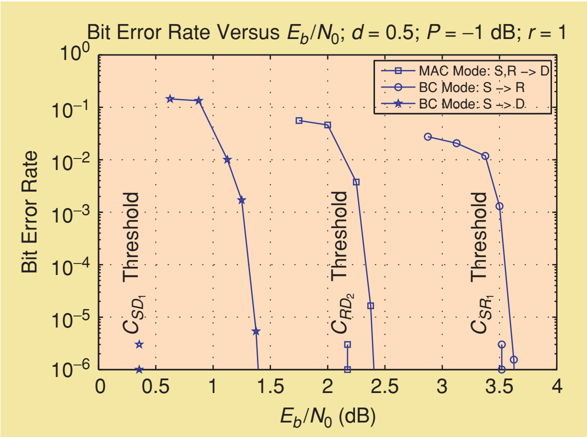


Figure source: [1, Fig. 7 (b)]

BER vs. E_b/N_0 for each of the three constituent codes.

References I

- A. Chakrabarti, E. Erkip, A. Sabharwal, and B. Aazhang, "Code design for cooperative communications," *IEEE Signal Process. Mag.*, pp. 16–26, September 2007.
- T. M. Cover and J. A. Thomas, *Elements of Information Theory*. John Wiley & Sons, Inc., 1991.
- A. Chakrabarti, A. de Baynast, A. Sabharwal, and B. Aazhang, "Low density parity check codes for the relay channel," *IEEE J. Sel. Areas Commun.*, vol. 25, no. 2, pp. 280–291, February 2007.