Cooperative Communications

Lecture 6

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Outline

Last Time, Lecture 5

- Towards practical implementations
  - Gaussian (half-duplex) relay channel
  - Degraded relay channel
  - Decode-and-forward (DF)
  - Low density parity check (LDPC) codes

Today, Lecture 6

- Coded Cooperation
  - Introduction for the block fading case and links to decode-and-forward
  - Rate-compatible puncture convolutional (RCPC) codes
  - Cooperation scheme
  - Time-variant fading channel
  - Union bound and pairwise error probability (PEP)
  - Spatial diversity vs. temporal diversity
  - Results and *cooperative regions*

- Introduction for the block fading case and links to decode-and-forward

Relay Channel Model Reprise I

Time-division hald-duplex communication takes place over two time slots of normalized duration t and $t' = (1-t)$.

- First slot
  - S transmits information that is received by both R and D
  - Broadcast (BC) mode
- Second slot
  - Both S and R transmit to D
  - Multiple-access (MAC) mode

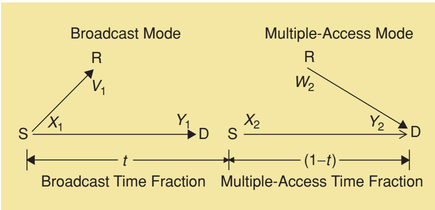


Figure source: [1, Fig. 2]

Relay Channel Model Reprise II

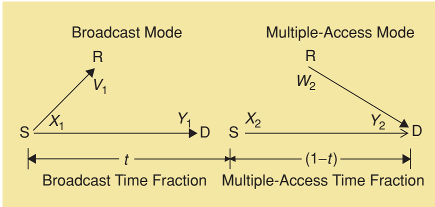
Gaussian relay channel model (cf. Lecture 7):

$$V_1 = h_{SR} X_1 + N_{R_1}, \quad Y_1 = h_{SD} X_1 + N_{D_1}$$

$$Y_2 = h_{SD} X_2 + h_{RD} W_2 + N_{D_2}$$

Variable naming convention:

- X ... signal transmitted by the source
- V ... signal received by the relay
- W ... signal transmitte by the relay
- Y ... signal received by the destination
- subscript $\cdot_1$ ... BC mode
- subscipt $\cdot_2$ ... MAC mode
- SR channel ... source-relay channel



Decode-and-Forward (DF) Relay Coding

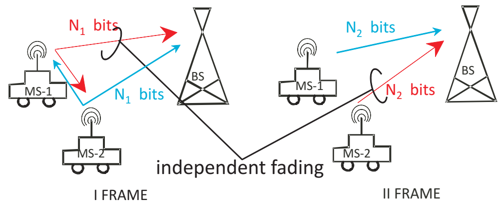
Basic idea

- The source transmission is first decoded by the relay.
- The relay helps then the destination by transmitting additional information about the codeword.
- Different code families can be utilized for relaying
  - convolutional codes [1][3](and other non-iterative linear codes)
  - turbo codes
  - low density parity check (LDPC) codes [2]
- Coded cooperation, a variant of DF, achieves spatial diversity at no bandwidth/power/rate costs: no repetition!
  More efficient than basic Decode-&-Forward

Coded Cooperation I

New situation

- Two (not too distant) users want to communicate to the same base station
- No dedicated relay is present
- Can this two users help each other to obtain better diversity?



Coded Cooperation II

Basic idea

- Assume a baseline wireless system using a rate-R channel code
- Coded cooperation uses the same overall rate by just re-arranging the coded symbols of the two users
- K information bits per block and N coded bits per block, $R = K/N$
- Divide each codeword into two segments of length N_1 and N_2, $N_1 + N_2 = N$.

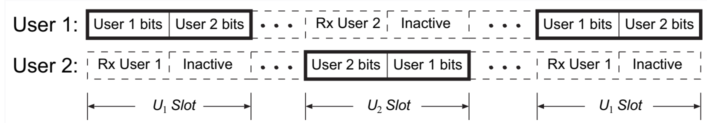


Figure source: [2, Fig. 1]

Coded Cooperation III

- First segment:
  - sub-codeword of rate $R_1 = K/N_1$ is broadcasted by the user and received to varying degree by base station and partner.
  - Each user will receive a noisy version of the coded message from its partner
- Second segment:
  - Correct decoding can be checked by cyclic redundancy check (CRC)
  - In case of correct decoding the N_2 additional parity bits for the partner are computed and transmitted
  - otherwise N_2 additional partiy bits for users own data are transmitted

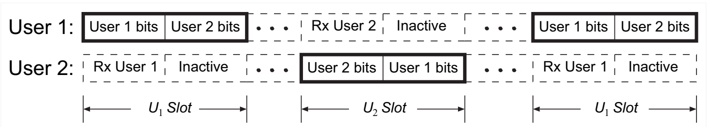
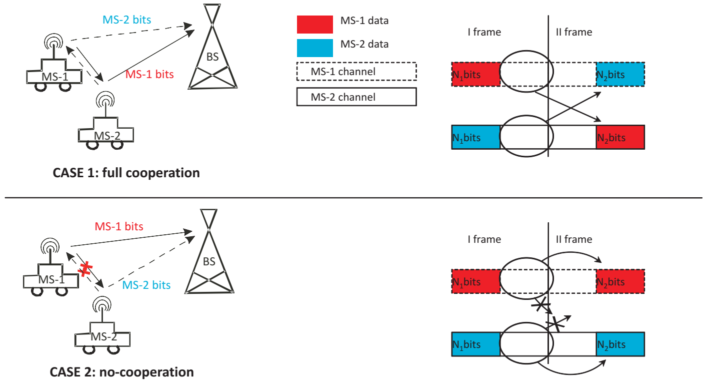


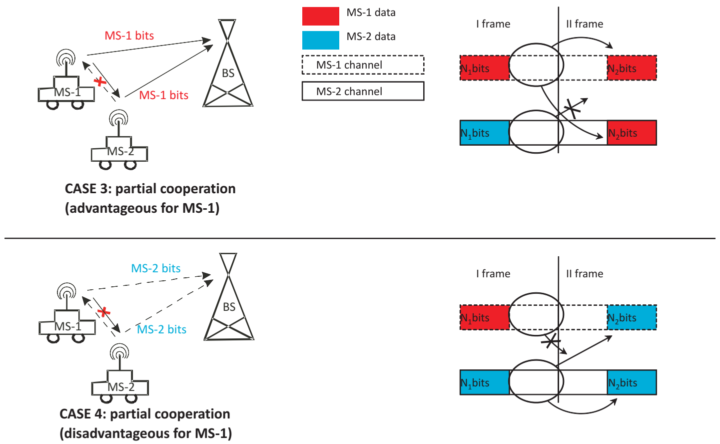
Figure source: [2, Fig. 1]

Coded Cooperation IV

- 4 configurations of the punctured bits transmission in the II frame (depending on the success of the inter-user detection)



Coded Cooperation V



Coded Cooperation VI

Performance for block fading channels (slow fading)

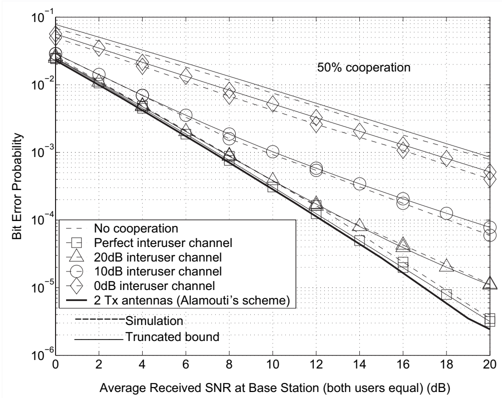


Figure source: [2, Fig. 3]

- Rate-compatible puncture convolutional (RCPC) codes

RCPC codes

$$\mathbf{x} = C(\mathbf{u}) = \begin{bmatrix} \mathbf{x}_{\mathrm{c}} \\ \mathbf{x}_{\mathrm{p}} \end{bmatrix}$$

The rate-compatibility restriction on the puncturing tables ensures that all code bits of the higher rate $R_1 = \frac{K}{N_1}$ code are used by the lower rate $R = \frac{K}{N}$ code. [1]

[1] A RCPC code is almost as good as the best known general convolutional code of the respective rate.
Source bits: $\mathbf{u} = [u_0, \ldots, u_K]^{\mathrm{T}} \in \{\pm 1\}$;
coded bits: $\mathbf{x} = [x_0, \ldots, x_{N-1}]^{\mathrm{T}} \in \{\pm 1\}$;
rate-R_1code: $\mathbf{x}_{\mathrm{c}} = \left[x_{\mathrm{c};0}, \ldots, x_{\mathrm{c};N_1-1}\right]^{\mathrm{T}}$;
punctured bits: $\mathbf{x}_{\mathrm{p}} = \left[x_{\mathrm{p};0}, \ldots, x_{\mathrm{p};N_2-1}\right]^{\mathrm{T}}$.

Transmitter side

- MS-i interleaves its codeword $\mathbf{x} \rightarrow \mathbf{x}_{\mathrm{I}}$. The interleaved coded bits $\mathbf{x}_{\mathrm{I}} = \begin{bmatrix} x_{\mathrm{I};0}, & \ldots, & x_{\mathrm{I};N-1} \end{bmatrix}^{\mathrm{T}}$ are mapped onto $M - J$ QPSK symbols[2]

$$s_i[k] = \sqrt{\frac{E_{\mathrm{S}}}{2}}(x_{\mathrm{I};2k} + jx_{\mathrm{I};2k+1})$$

- J QPSK random pilot symbols are placed uniformly along the frame

[2] E_{S}: transmitted energy per QPSK symbol;
$k \in \{0, \ldots, M - J - 1\}$.

Receiver side

- Received signal at the BS from MS-i: $y_i[m]$[3]

$$y_i[m] = h_i[m]s_i[m] + z_i[m]$$

- Matched filter and zero-forcing (channel known at the receiver)

$$\tilde{y}_i[m] = \frac{h_i^*[m]\, y_i[m]}{|h_i[m]|^2} \sim \mathcal{CN}\left(s_i[m], \frac{\sigma_z^2}{|h_i[m]|^2}\right)$$

[3] $m \in \{0, \ldots, M-1\}$;
Rayleigh fading impulse response $h_i[m] \sim \mathcal{CN}(0, \Omega)$;
AWGN $z_i[m] \sim \mathcal{CN}\left(0, \sigma_z^2\right)$.

Soft Input Viterbi decoder I

At the input of the Viterbi decoder, the single de-interleaved code bit $\tilde{y}_n$ (de-mapped from $s_i[m]$)[4]

$$\tilde{y}_n = x_n + \tilde{z}_n$$

[4] $\tilde{z}_n \sim \mathcal{N}\left(0, \frac{\sigma_z^2}{E_S\,|h_i[m]|^2}\right)$; $n \in \left\{ 0, \quad \dots, \quad N-1 \right\}$.

Soft Input Viterbi decoder II

The soft inputs for the Viterbi decoder, given the equalized bits $\widetilde{\mathbf{y}}_n = \left[\tilde{y}_0, \quad \dots, \quad \tilde{y}_{N-1} \right]^{\mathrm{T}}$, are the Log-Likelihood ratios (LLR) $\mathbf{w} = \left[w_0, \quad \dots, \quad w_{N-1} \right]^{\mathrm{T}}$[5]

$$w_n = \log \frac{P(\tilde{y}_n \mid x_n = +1)}{P(\tilde{y}_n \mid x_n = -1)} = \log \left[\frac{\exp\left(-\frac{(\tilde{y}_n - 1)^2}{2\tilde{\sigma}_z^2}\right)}{\exp\left(-\frac{(\tilde{y}_n + 1)^2}{2\tilde{\sigma}_z^2}\right)} \right] = \frac{2\tilde{y}_n}{\tilde{\sigma}_z^2}$$

[5] $\tilde{\sigma}_z^2 = \frac{\sigma_z^2}{E_S\,|h_i[m]|^2}$ is the normalized noise variance

Soft Input Viterbi decoder III

The output of the Viterbi decoder is the maximum likelihood estimate $\tilde{\mathbf{u}} = \left[\tilde{u}_0, \quad \dots, \quad \tilde{u}_K \right]^{\mathrm{T}}$ of the encoded bits[6]

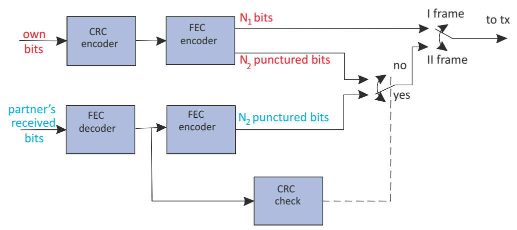
$$\tilde{\mathbf{u}} = \arg \max_{\widehat{\mathbf{u}} = \mathcal{C}^{-1}(\widehat{\mathbf{x}})} P(\widetilde{\mathbf{y}}_n \mid \widehat{\mathbf{x}}) = \arg \max_{\widehat{\mathbf{u}} = \mathcal{C}^{-1}(\widehat{\mathbf{x}})} \mathbf{w}^{\mathrm{T}}\,\widehat{\mathbf{x}}$$

[6] $\widehat{\mathbf{x}} = \left[\hat{x}_0, \quad \dots, \quad \hat{x}_{N-1} \right]^{\mathrm{T}} \in \{\pm 1\}$ is a valid RCPC codeword generated from a possible source block $\widehat{\mathbf{u}}$

- Cooperation scheme

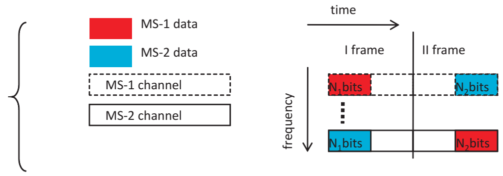
Coded Cooperation: Encoding

- The RCPC code is punctured
- An error detection code (e.g. CRC) avoids error propagation: the partner's punctured bits are transmitted only if the detection is successful



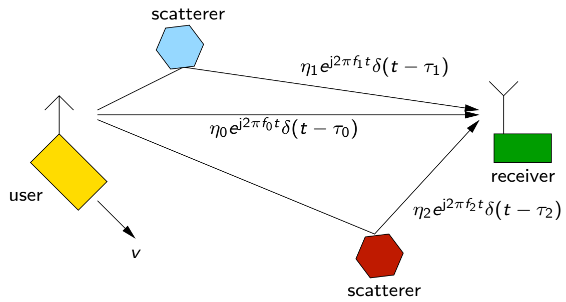
Coded Cooperation: Half Duplex Constraint

- Assumption: FDMA



- Time-variant fading channel

Multipath Propagation



v velocity
p path
η_p attenuation
τ_p time delay
f_p Doppler shift
P number of paths (scatterers)
ϕ_p angle of departure
f_c carrier frequency
c_0 speed of light

Time-variant channel impulse response

$$h(t,\tau) = \sum_{p=0}^{P-1} \eta_p e^{j2\pi f_p t}\delta(\tau-\tau_p)\,, \qquad f_p = \frac{v\cos\phi_p f_c}{c_0}$$

Channel Model: Uplink

The **time-bandwidth product** [max Doppler shift x codeword duration] measures the temporal variability

$$\mathrm{TBP}_i = \frac{f_\mathrm{c} v_i}{c_0} \cdot 2M \cdot T_\mathrm{s}$$

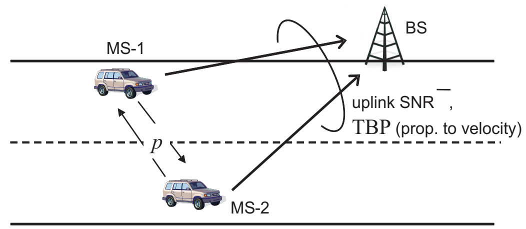
- Frequency-flat Rayleigh fading uplink channels $h_i\,[m]$
- Clarke's autocorrelation function ($\uparrow \mathrm{TBP} \Leftrightarrow \uparrow \mathrm{decorrelation}$)

$$R_i\,[k] = \mathbb{E}\left\{h_i\,[m]\,h_i^*\,[m+k]\right\} = \bar{\gamma}_i J_0\left(\pi k \cdot \mathrm{TBP}_i/M\right),$$

where $\bar{\gamma}_i$ is the **average SNR**

Channel Model: Inter-MS link

- Independent inter-MS channels
- **Inter-MS block error rate** p[7]



[7] The probability of successful coded cooperation is $(1-p)^2$..

- Union bound and pairwise error probability (PEP)

Union bound

The average BER at the output ofthe Viterbi decoder at the receiver-side is upperbounded

$$P_\mathrm{b} \leq \sum_{\Theta=1}^{4} \mathrm{Pr}(\Theta) \left[\frac{1}{k} \sum_{d \geqq d_\mathrm{free}} \sum_{\mathbf{c} \in \mathcal{E}(d)} w(\mathbf{c}) P\left(\mathbf{c}\,|\Theta\right)\right]$$

The average PEP $P\left(\mathbf{c}\right)$ depends on the fading statistics, while the values of the other parameters are fixed by the code.[8]

[8] $\mathrm{Pr}(\Theta)$: probability of case Θ; k : number of input bits for each branch of the convolutional code trellis; d_free: minimum Hamming distance between the codewords; $\mathcal{E}\left(d\right)$: set of error events $\mathbf{c}$ at a certain Hamming distance d from the all-zero codeword; $w(\mathbf{c})$: Hamming weight of the input sequence corresponding to $\mathbf{c}$; $P\left(\mathbf{c}\right)$: average pairwise error probability (PEP).

Pairwise Error Probability I

The *average* PEP is the probability of receiving the codeword $\mathbf{y} = \mathbf{x} + \mathbf{c}$ instead of the transmitted $\mathbf{x}$. Also defined, due to the linearity of the convolutional codes, as the probability of receiving $\mathbf{c}$ (*error event*) instead of the all-zero codeword.

The PEP is averaged with respect to the probability density function of the *effective* SNR γ_{eff}[9]

$$P(\mathbf{c}) = \int_0^\infty Q\left(\sqrt{2\gamma_{\text{eff}}}\right) P(\gamma_{\text{eff}})\, d\gamma_{\text{eff}}$$

[9] $\gamma_{\text{eff}} = \sum_{k\in\mathcal{T}_{\mathbf{c}}} \gamma(k) = \sum_{k\in\mathcal{T}_{\mathbf{c}}} \frac{\mathrm{E_s}\,|h_i(k)|^2}{\sigma_z^2} = \left\|\widetilde{\mathbf{h}}\right\|^2$;
$\mathcal{T}_{\mathbf{c}} = \left\{\tau_{\mathbf{c},1}, \ldots, \tau_{\mathbf{c},d}\right\}$: set of time instants associated with $\mathbf{c}$;
$\gamma_{\mathbf{c}} = \left[\ \gamma(\tau_{\mathbf{c},1}), \quad \ldots, \quad \gamma(\tau_{\mathbf{c},d})\ \right]^{\mathrm{T}}$: the corresponding *instantaneous* SNR;
$\widetilde{\mathbf{h}} = \frac{\sqrt{E_{\mathrm{s}}}}{\sigma_z}\left[\ h(\tau_{\mathbf{c},1}), \quad \ldots, \quad h(\tau_{\mathbf{c},d})\ \right]^{\mathrm{T}}$.

Pairwise Error Probability II

γ_{eff} is the sum of d *correlated* exponential random variables, whose autocorrelation is $\mathbf{R}_{\mathbf{c}} = \mathbb{E}\left[\widetilde{\mathbf{h}}\widetilde{\mathbf{h}}^{\mathrm{H}}\right]$.[10] Through an *eigenvalue decomposition* (EVD), the effective SNR becomes the sum of r *uncorrelated* exponential random variables $\left\{b_k^2\right\}_{k=1}^{r}$[11]

$$\gamma_{\text{eff}} = \left\|\widetilde{\mathbf{h}}\right\|^2 = \sum_{k=1}^{r} b_k^2 = \|\mathbf{b}\|^2$$

where $\mathbf{b} = \mathbf{U}^{\mathrm{H}}\widetilde{\mathbf{h}} = [b_1 \ldots b_r]^{\mathrm{T}}$is the linear projection of the channel onto the r-dimensional column-space of $\mathbf{R}_{\mathbf{c}}$.

[10] For its definition and for the stationarity of $\mathbf{R}_{\mathbf{c}}$the fading process, the elements on the diagonal of $\mathbf{R}_{\mathbf{c}}$are equal to the average SNR $\bar{\gamma} = \Omega\frac{E_{\mathrm{s}}}{\sigma_z^2}$,where $\Omega = \mathbb{E}\left\{|h\,[m]|^2\right\}$

[11] $r = \mathit{rank}\left\{\mathbf{R}_{\mathbf{c}}\right\}$;
$\mathbf{U}$: $d \times r$ matrix containing the eigenvectors of $\mathbf{R}_{\mathbf{c}}$.

Pairwise Error Probability III

The d error bits are split into two groups, d_1 and d_2, coming from the MS's and the partner's independent uplink channels $i \in \{1, 2\}$, respectively at time instants $\mathcal{T}_{\mathbf{c};i} = \{\tau_{\mathbf{c},1;i}, \ldots, \tau_{\mathbf{c},d_i;i}\}$. The autocorrelation of
$\widetilde{\mathbf{h}} = \frac{\sqrt{E_{\mathrm{s}}}}{\sigma_z}\left[h_1(\tau_{\mathbf{c},1;1}), \ldots, h_1(\tau_{\mathbf{c},d_1;1}), h_2(\tau_{\mathbf{c},1;2}), \ldots, h_2(\tau_{\mathbf{c},d_2;2})\right]^{\mathrm{T}}$becomes[12]

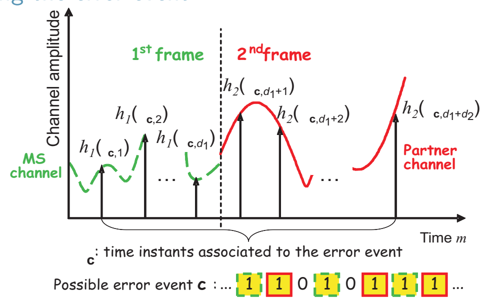
$$\mathbf{R}_{\mathbf{c}} = \begin{bmatrix} [\mathbf{R}_{\mathbf{c},1}]_{d_1\times d_1} & \mathbf{0} \\ \mathbf{0} & [\mathbf{R}_{\mathbf{c},2}]_{d_2\times d_2} \end{bmatrix}_{d\times d} =$$

$$= \begin{bmatrix} \mathbf{U}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{U}_2 \end{bmatrix} \begin{bmatrix} \mathbf{\Lambda}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{\Lambda}_2 \end{bmatrix} \begin{bmatrix} \mathbf{U}_1^{\mathrm{H}} & \mathbf{0} \\ \mathbf{0} & \mathbf{U}^{\mathrm{H}}{}_2 \end{bmatrix}$$

[12] Thereby, the eigenvalues of $\mathbf{R}_{\mathbf{c}}$ are $\boldsymbol{\lambda} = [\boldsymbol{\lambda}_1^{\mathrm{T}}, \boldsymbol{\lambda}_2^{\mathrm{T}}]^{\mathrm{T}}$, where $\boldsymbol{\lambda}_i = [\lambda_{1,1}, \ldots, \lambda_{1,r_i}]^{\mathrm{T}}$ are the eigenvalues of the respective submatrices $\mathbf{R}_{\mathbf{c},i}$ with rank r_i.

Pairwise Error Probability IV

Fading affecting the error event



Pairwise Error Probability V

Moment Generating Function (MGF) method

The pdf of the effective SNR exhibits the MGF

$$\mathbf{M}_{\gamma_{\text{eff}}}(s) = \int_0^\infty e^{s\gamma_{\text{eff}}} P(\gamma_{\text{eff}})\, d\gamma_{\text{eff}} = \prod_{i=1}^{r_1} \frac{1}{1-\lambda_{1,i}s} \prod_{j=1}^{r_2} \frac{1}{1-\lambda_{2,j}s}$$

Since $Q(x) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} e^{-\frac{x^2}{2\sin^2\vartheta}}\, d\vartheta$, the PEP becomes

$$P(\mathbf{c}) = \int_0^\infty Q\left(\sqrt{2\gamma_{\text{eff}}}\right) P(\gamma_{\text{eff}})\, d\gamma_{\text{eff}} = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \left[\int_0^\infty e^{-\frac{\gamma_{\text{eff}}}{\sin^2\vartheta}} P(\gamma_{\text{eff}})\, d\gamma_{\text{eff}}\right] d\vartheta$$

$$= \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \mathbf{M}_{\gamma_{\text{eff}}}\left(-\frac{1}{\sin^2\vartheta}\right) d\vartheta = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \prod_{i=1}^{r_1}\left(1+\frac{\lambda_{1,i}}{\sin^2\vartheta}\right)^{-1} \prod_{j=1}^{r_2}\left(1+\frac{\lambda_{2,j}}{\sin^2\vartheta}\right)^{-}$$

Pairwise Error Probability VI

Upperbound on the PEP [3, (12)-(13)]

$$P(\mathbf{c}) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \prod_{i=1}^{r_1}\left(1+\frac{\lambda_{1,i}}{\sin^2\vartheta}\right)^{-1} \prod_{j=1}^{r_2}\left(1+\frac{\lambda_{2,j}}{\sin^2\vartheta}\right)^{-1} d\vartheta =$$

$$\leq \frac{1}{2}\prod_{i=1}^{r_1}\frac{1}{1+\lambda_{1,i}} \prod_{j=1}^{r_2}\frac{1}{1+\lambda_{2,j}}$$

by taking $\sin^2\vartheta = 1$

Pairwise Error Probability VII

Interleaver

The $d = d_1 + d_2$ non-zero bits of the error event $\mathbf{c}$ can appear within the two time frames in several possible configurations, each corresponding to a different, but equivalent, shift of $\mathbf{c}$ at theinput of the Viterbi decoder. Thus, $P(\mathbf{c}) \leq P(\mathbf{c_p})$, where $\mathbf{c_p}$ is the most probable among the equivalent translations of $\mathbf{c}$.

Pairwise Error Probability VIII

Autocorrelation matrices for the 4 cases[13] [3, (10)(15)-(17)]

- coop: $\mathbf{R_c}(\Theta = 1) = \begin{bmatrix} [\mathbf{R_{c,1}}]_{d_1\times d_1} & \mathbf{0} \\ \mathbf{0} & [\mathbf{R_{c,2}}]_{d_2\times d_2} \end{bmatrix}$
- no-coop: $\mathbf{R_c}(\Theta = 2) = \begin{bmatrix} [\mathbf{R_{c,1}}]_{d_1\times d_1} & [\mathbf{R_{c,1}}]_{d_1\times d_2} \\ \left[[\mathbf{R_{c,1}}]_{d_1\times d_2}\right]^{\mathrm{H}} & [\mathbf{R_{c,1}}]_{d_2\times d_2} \end{bmatrix}$
- disadvantageous partial coop: $\mathbf{R_c}(\Theta = 3) = \left[\mathbf{R_{c,1}}\right]_{d_1\times d_1}$
- advantageous partial coop: $\mathbf{R_c}(\Theta = 4) = \begin{bmatrix} \mathbf{R_c}(\Theta = 2) & \mathbf{0}_{d\times d_2} \\ \mathbf{0}_{d_2\times d} & [\mathbf{R_{c,2}}]_{d_2\times d_2} \end{bmatrix}$

[13] denoting the MS' uplink channel as channel-1 and the partner's uplink channel as channel-2. $[\mathbf{R_{c,1}}]_{d_1\times d_1}$: autocorrelation of the channel-1 gains $\widetilde{\mathbf{h}}_{1,d_1}$ associated to the first frame d_1 error bits of $\mathbf{c}$; $[\mathbf{R_{c,1}}]_{d_2\times d_2}$: autocorrelation of the channel-1 gains $\widetilde{\mathbf{h}}_{1,d_2}$ associated to the second frame d_2 error bits; $[\mathbf{R_{c,2}}]_{d_2\times d_2}$ autocorrelation of the channel-2 gains $\widetilde{\mathbf{h}}_{2,d_2}$ associated to the second frame d_2 error bits; $[\mathbf{R_{c,1}}]_{d_1\times d_2}$: cross-correlation between the gains $\widetilde{\mathbf{h}}_{1,d_1}$ and the gains $\widetilde{\mathbf{h}}_{1,d_2}$.

- Spatial diversity vs. temporal diversity

Diversity order I

Normalizing $[\mathbf{R}_{\mathbf{c},1}]_{d_1\times d_1}$ and $[\mathbf{R}_{\mathbf{c},2}]_{d_2\times d_2}$ by the corresponding uplink average SNR, $\bar{\gamma}_1$ and $\bar{\gamma}_2$ respectively, the eigenvalues become $\widehat{\boldsymbol{\lambda}} = [\hat{\boldsymbol{\lambda}}_1^{\mathrm{T}}, \hat{\boldsymbol{\lambda}}_2^{\mathrm{T}}]^{\mathrm{T}}$, where $\widehat{\boldsymbol{\lambda}}_{i\in\{1,2\}} = \left[\frac{\lambda_{i,1}}{\bar{\gamma}_i}, \dots, \frac{\lambda_{i,r_i}}{\bar{\gamma}_i}\right]^{\mathrm{T}} = \left[\hat{\lambda}_{i,1}, \dots, \hat{\lambda}_{i,r_1}\right]^{\mathrm{T}}$.
Scaling the average SNR to infinity:

$$\lim_{\bar{\gamma}\to\infty} P(\mathbf{c}) \leq \lim_{\bar{\gamma}\to\infty} \frac{1}{2}\prod_{i=1}^{r=r_1+r_2} \frac{1}{1+\hat{\lambda}_i\bar{\gamma}} = \frac{1}{2}\prod_{i=1}^{r}\frac{1}{\hat{\lambda}_i\bar{\gamma}} = \mathrm{const}\cdot\left(\frac{1}{\bar{\gamma}}\right)^r$$

Diversity order II

$$\lim_{\bar{\gamma}\to\infty} P_{\mathrm{b}} \leq \mathrm{const}\cdot\left(\frac{1}{\bar{\gamma}}\right)^{\Psi}$$

where Ψ is the diversity order achieved in a convolutional coded transmission over a correlated Rayleigh fading channel.
Being $\mathcal{E}_{\mathrm{all}}$ the set of all the error events $\mathbf{c}$ of the convolutional code, the *diversity order* is:

$$\Psi = \min_{\mathbf{c}\in\mathcal{E}_{\mathrm{all}}} r$$

If $\mathbf{R}_{\mathbf{c}}$ is non-singular for every $\mathbf{c}$, the diversity order is equal to d_{free}.

Finite SNR diversity

For practical purposes it is more interesting to define a *finite SNR diversity order* $\Psi_{\mathrm{eff}}(\bar{\gamma})$, achievable for SNR$\simeq\bar{\gamma}$. It is possible to derive empirically an *effective* rank r_{eff}, that is the number of eigenvalues $\lambda_i = \bar{\gamma}\hat{\lambda}_i \gg 1$, with $i \in \{1, \dots, r_{\mathrm{eff}}\}$:

$$P(\mathbf{c}) \leq \frac{1}{2}\prod_{j=r_{\mathrm{eff}}+1}^{r}\frac{1}{1+\hat{\lambda}_j\bar{\gamma}}\prod_{i=1}^{r_{\mathrm{eff}}}\frac{1}{\hat{\lambda}_i\bar{\gamma}}$$

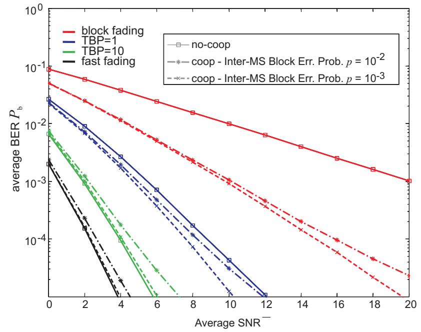
$P(\mathbf{c})$ decreases *at least* with $\left(\frac{1}{\bar{\gamma}}\right)^{r_{\mathrm{eff}}}$. The finite SNR diversity order is lower-bounded:

$$\Psi_{\mathrm{eff}}(\bar{\gamma}) \geq \min_{\mathbf{c}\in\mathcal{E}_{\mathrm{all}}} r_{\mathrm{eff}}(\mathbf{c})$$

- Results and *cooperative regions*

Results: from BF to FF

- From BF ($\mathrm{TBP}_i = \frac{f_c v_i}{c_0} \cdot 2M \cdot T_\mathrm{s} = 0$) to FF ($\mathrm{TBP}_i \Rightarrow \infty$)



- At finite SNR, the slope (diversity) decreases with increasing p

Results: Realistic Time-Variability

- Rate $R = 1/3$
- carrier frequency $f_\mathrm{C} = 5.2$ GHz
- symbol duration $T_\mathrm{S} = 10\,\mu$s
- frame length $M = 192$ symbols

coded cooperation advantageous up to $\mathrm{TBP} \simeq 3 \iff v \simeq 160$ km/h in the *finite SNR* regime (perfect inter-MS detection)

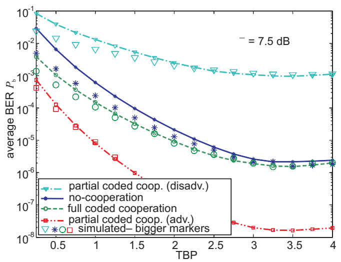


Figure source: [3, Fig. 9]

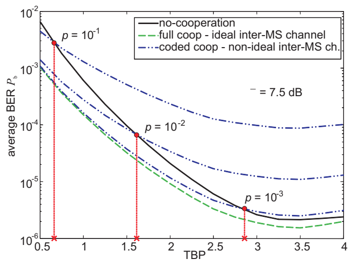


Figure source: [3, Fig. 10]

Results: Cooperative Regions

- The *cooperative region*: collection of mobility (TBP) and channel ($\bar{\gamma}$, p) settings for which coded cooperation is beneficial

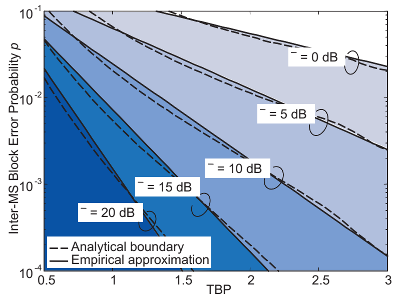


Figure source: [3, Fig. 11]

- Increasing p, the cooperative region decreases
- Increasing $\bar{\gamma}$, the cooperative region decreases
- At high velocities, the performance is dominated by the worst case of cooperation

Results: Fading Asymmetries

- Mobility asymmetry (β) affects the performance
- In general, long-term statistics unbalances are drawbacks for cooperative diversity!

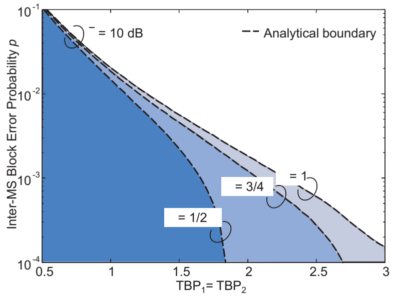


Figure source: [3, Fig. 12]

Conclusions

- For MSs moving at high speeds ($\gtrsim$ 40 km/h), the inter-MS channel quality plays a significant role in the definition of the *cooperative region*
  This is contrary to what has been shown for quasi-static channels!
- Benefits of coded cooperation in high mobility scenarios arise for those applications where energy efficiency (for low SNR, e.g. $\lesssim 10\,\mathrm{dB}$) is a key issue

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