Cooperative Communications

Lecture 8

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Outline I

Today, Lecture 8

- Distributed Synchronization
  - Brief history
  - Wireless networks
  - Packet-coupling vs. pulse-coupling

Introduction

- Local clocks at different nodes separated by (large) distances
- Synchronization: achieving and maintaining coordination among these local clock via exchange of local time information [1]
- Synchronization schemes classification by method used for
  - encoding,
  - exchange, and
  - processing

  of information.
- Wireless communication in decentralized cooperative communication networks and sensor networks heavily rely on synchronization.

Brief History

- 19th century - synchronization of distant clock to a reference time (unidirectional or master-slave synchronization)
  - implemented by telegraphy and later by wireless transmission
  - applications enabled
    - railroad transport
    - geodesy (meas. of longitude)
    - localization
- 20th century - spontaneous synchronization in nature as role model
  - Fireflies
  - activity of muscle fibres
  - clapping in a concert hall

Wireless Networks I

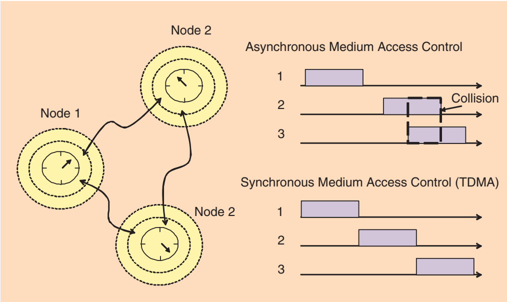

Figure source: [1, Fig. 1]

Examples:

- Distributed estimation and data fusion in sensor networks
- Multiple access schemes avoiding collisions
- Cooperative transmission
  - space time coding
  - distributed beamforming

Wireless Networks II

Classic synchronization methods:

- Central access point broadcast beacon timing signal (GSM, UMTS, IEEE 802.15.4 ZigBee)
- Satellite-synchronization (oudoor)

Distributed Synchronization

Mutual synchronization in distributed wireless networks

- Problems:
  - Random delays between transmission and reception of a timing signal
    - wave propagation
    - processing latency
  - Hardware and clock inaccuracies
- Specific aspects of wireless networks
  - Energy efficiency
  - Scalabillity
  - Application specifity

Basics I

Definitions

- $t_i(n) \ldots n$th tick $(n = 0, 1, 2, \ldots)$ of the ith clock $(i = 1, 2, \ldots, N)$ where N is the number of nodes
- Local periods $T_i = t_i(n) - t_i(n-1)$

Uncoupled nodes

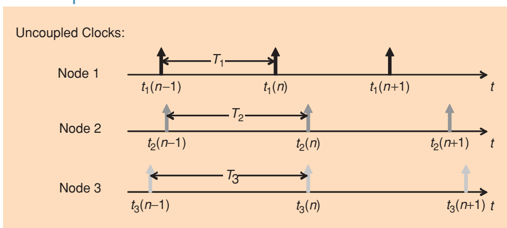

Figure source: [1, Fig. 2 (a)]

No local timing information is exchanged, local periods T_i and phases $t_i(n)$ differ.

Basics II

Frequency synchronization

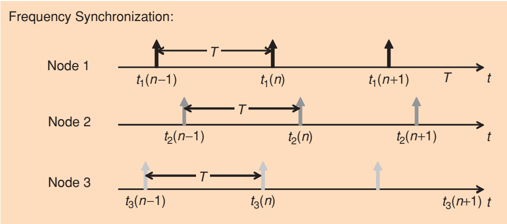

Figure source: [1, Fig. 2 (b)]

$$T_i = T$$

Basics III

Full synchronization

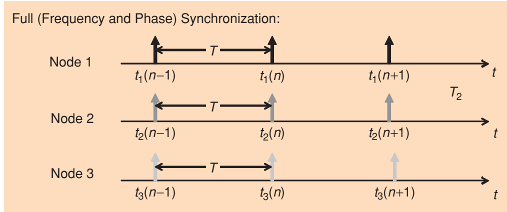

Figure source: [1, Fig. 2 (c)]

$$T_i = T$$

and

$$t_i(n) = t_j(n), \quad i \neq j$$

Packet Coupling vs. Pulse Coupling I

Packet Coupling

- Periodic exchange of packets carrying time stamps containing the local time $t_i(n)$ at the sender (point-to-point or broadcast)
- Source of errors through random delays q_{ij}
  - packet construction
  - queuing at the MAC
  - propagation
  - processing at receiver side
- Node i receives timing packet from node j at time $t_i(n) + q_{ij}$
- Accuracy in the order of milliseconds to microseconds can be achieved
- Large number of packets must be exchanged - limited scalability

Packet Coupling vs. Pulse Coupling II

Pulse Coupling

- Local time information is encoded in the transmission times of given waveforms $g(t)$
- Each nodes radiates a periodic train of waveforms

$$\sum_n g(t - t_i(n))$$

according to its local clock.

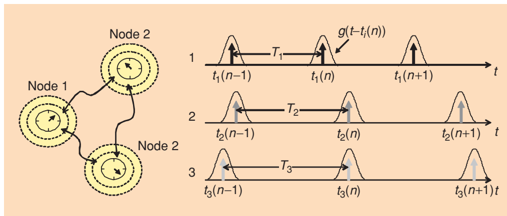

Figure source: [1, Fig. 3]

Packet Coupling vs. Pulse Coupling III

- Update of local clocks by processing received signal
- Better scalability since signalling is independent of N
- Half-duplex constraint requires special attention (cannot receive while sending)

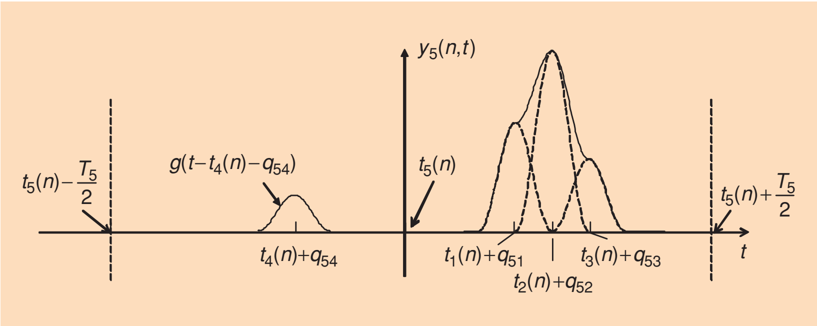

Figure source: [1, Fig. 4]

Uncoupled Clocks I

- Analog clock oscillator

$$s_i(t) = \cos \phi_i(t)$$

where $\phi_i(t)$ is the total instantaneous phase evolving as

$$\phi_i(t) = \phi_i(0) + \frac{2\pi}{T_i} + \zeta_i(t)$$

$T_i = T_{\mathrm{nom}} + \Delta T_i$ ... free running oscillator period
T_{nom} ... nominal period
ΔT_i ... random offset (frequency offset, skew)
$\zeta_i(t)$ ... phase noise (modeled as random process)

Uncoupled Clocks II

- Discrete-time clock

$$t_i(n) = t_i(0) + nT_i + \nu(n)$$

modeled as sequence $t_i(n)$ of significant time instants of an analog clock (e.g. upward zero crossing points, $\phi_i(t_i(n)) = n \cdot 2\pi$) with $n \in \{0, 1, 2, \ldots\}$.

$\nu(n)$ additive phase noise term

Coupled Clocks

- Analog clocks
  - Frequency synchronicity: for t sufficient large, there exists a common period of oscillation T for all nodes

$$\phi_i(t) = \phi_i(t+T), i = 1, 2, \ldots, N\,.$$

  - Full (frequency and phase) synchronicity: for t sufficiently large

$$\phi_i(t) = \phi_j(t) \quad \forall \quad i \neq j$$

- Digital clocks
  - Frequency synchronicity: for n sufficiently large

$$t_i(n+1) - t_i(n) = T\,, i = 1, \ldots, N\,.$$

  - Full (frequency and phase) synchronicity: for n sufficiently large

$$t_i(n) = t_j(n) \quad \forall \quad i \neq j\,.$$

Diffusion Protocols

Basic mechanism

- each node transmits (diffuses) its local time (either phase $\phi_j(t)$ or clock tick $t_j(n)$) to its neighboring nodes
- discrete timing $t_j(n)$ information encoded either as
  - time stamp in a packet (packet coupling)
  - transmission time of a given waveform (pulse coupling) $g(t - t_j(n))$

Connectivity Graph I

- Topology of connections between clocks ic crucial for achieving a synchronized state.
- Connectivity graph $\mathcal{G}$ for $N = 5$

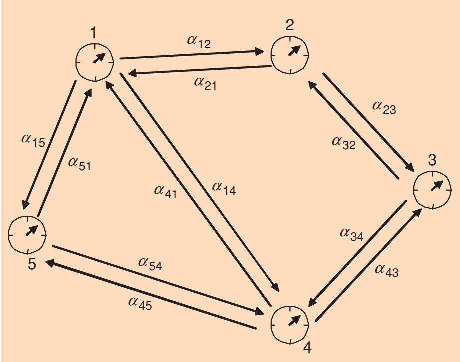

Figure source: [1, Fig. 5]

Connectivity Graph II

- Edge weight α_{ij} represents relative signal strength received by i from j with respect to the other neighbors of i,

$$\sum_j \alpha_{ij} = 1\,.$$

- Typical choice

$$\alpha_{ij} = \frac{P_{ij}}{\sum_{j \in \mathcal{I}_i} P_{ij}}$$

where

P_{ij} . . . power received by node i from node j
$\mathcal{I}_i$ . . . set of neighbors of i, $\mathcal{I}_i = \{j : P_{ij} > P_0\}$
P_0 . . . power threshold

Connectivity Graph III

- Diffusion synchronization protocols described by linear dynamic systems
- System matrix $\boldsymbol{L}$ is linearly related to connectivity graph $\mathcal{G}$
- Laplacian matrix $\boldsymbol{L}$ is key algebraic quantity describing $\mathcal{G}$,

$$\boldsymbol{L} = \boldsymbol{I} - \boldsymbol{A}$$

where $\boldsymbol{A}$ is the adjacency matrix of the graph

$$[\boldsymbol{A}]_{ij} = \alpha_{ij} \quad \text{for} \quad i \neq j$$

$$[\boldsymbol{A}]_{ii} = 0$$

Connectivity Graph IV

- Performance of mutual synchronization depends on the network topoplogy through the eigenstructure of the Laplacian matrix $\boldsymbol{L}$
- The directed and weighted graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \boldsymbol{A})$ with N vertices $\mathcal{V}$ and edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$
- Laplacian matrix $\boldsymbol{L} = \boldsymbol{I} - \boldsymbol{A}$ has the following properties:
  - existence of a zero eigenvalue, $\lambda_1(\boldsymbol{L}) = 0$
  - all the eigenvalues $\lambda_k(\boldsymbol{L})$ satisfy $|\lambda_k(\boldsymbol{L}) - 1| \leq 1$
  - if weights are symmetric, $\alpha_{ij} = \alpha_{ji}$, matrix $\boldsymbol{L}$ is symmetric and positive semidefinite so that $\lambda_k(\boldsymbol{L})$ are real and satisfy $0 \leq \lambda_k(\boldsymbol{L}) \leq 2$.

Connectivity Graph V

- Conditions for mutual synchronization
  - Continuously coupled first order PLL: $\Re\{\lambda_k(\boldsymbol{L})\} > 0$ for $k \neq 1$.
  - Pulse-coupled: $|\lambda_k(\boldsymbol{L})| > 0$ for $k \neq 1$
  - Exponential rate of convergence of synchronization depends on the "smallest" eigenvalue (see two cases above)
  - If graph is strongly connected, i.e. there exists a path between any pair of nodes, multiplicity of $\lambda_1(\boldsymbol{L}) = 0$ is one.

Continuously Coupled Analog Clocks I

- Applications
  - Cooperative Beamforming
  - FDMA in ad-hoc networks
- Historically the first model studied
- Each node transmits a signal proportional to its local oscillator $s_i(t)$ and updates the instantaneous phase $\phi_i(t)$ based on the received signal from the other nodes (full duplex is assumed).
- Basic mechanism $\rightarrow$ phase locking
- Each node measures through its phase detector (PD) the convex combination of phase differences

$$\Delta\phi_i(t) = \sum_{j=1, j\neq i}^{N} \alpha_{ij} f(\phi_j(t) - \phi_i(t)) \qquad (1)$$

where $f(\cdot)$ is a nonlinear function of the PD.

Continuously Coupled Analog Clocks II

- Convex combination in (1) ensures that $\Delta\phi_i(t)$ takes values between the minimum and maximum of phase differences $f(\phi_j(t) - \phi_i(t))$.
- $\Delta\phi_i(t)$ is fed to a loop filter $\epsilon(s)$ whose output drives a voltage controlled oscillator (VCO)

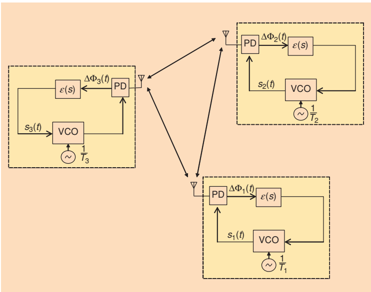

Figure source: [1, Fig. 6]

Continuously Coupled Analog Clocks III

- VCO update the local phase as

$$\dot{\phi}_i(t) = \frac{2\pi}{T_i} + \epsilon_0 \sum_{j=1, j\neq i}^{N} \alpha_{ij} f(\phi_j(t) - \phi_i(t))$$

  where a simple loop filter $\epsilon(s) = \epsilon_0$ was assumed.
- Note that we assumed the following simplifications:
  - no phase noise
  - instantaneous coupling among clocks (nor propagation delay)
  - time invariant network topology

Kuramoto's Model

- First model of coupled analog oscillators was proposed by Kuramoto
  - all-to-all connectivity (not directely applicable to wireless networks)
  - sinusoidal phase detector $f(x) = \sin(x)$
  - simple loop filter $\epsilon(s) = \epsilon_0$ (first order PLL)
- Critical value $\epsilon_0 > \epsilon_0^*$ for loop gain $\rightarrow$ state of partial frequency synchronization
  - part of the oscillators is in phase
  - and part is out of synchrony
  - full synchronization is eventually achieved for $\epsilon_0 \rightarrow \infty$.
- For $\epsilon_0 < \epsilon_0^*$ the clocks remain in an incoherent state

Continously Coupled Linear PLL

- Linear phase detectors $f(x) = x$
- Arbitrary connections α_{ij}
- Loop filter
  - first order PLL $\epsilon(s) = \epsilon_0$
  - second order PLL $\epsilon(s) = \frac{\epsilon_0}{1-\frac{s}{\mu}}$
- Vector linear time-invariant differential equation

$$\dot{\boldsymbol{\phi}}(t) = \boldsymbol{\omega} - \epsilon_0 \boldsymbol{L} \boldsymbol{\phi}(t)$$

  where $\boldsymbol{\phi}(t) = [\phi_1(t), \ldots, \phi_N(t)]^{\mathrm{T}}$ and $\boldsymbol{\omega} = [2\pi/T_1, \ldots, 2\pi/T_N]^{\mathrm{T}}$.
- Steady state frequency synchronization

$$\frac{1}{T} = \sum_{i=1}^{N} v_i \frac{1}{T_i}$$

  where $\boldsymbol{v} = [v_1, \ldots, v_N]^{\mathrm{T}}$ is the normalized eigenvalue of $\boldsymbol{L}$ corresponding to the zero eigenvalue $\boldsymbol{L}^{\mathrm{T}} \boldsymbol{v} = \boldsymbol{0}$.
- Generally phase synchronization is not attained!

Pulse-Coupled Discrete Time Clocks I

- Two approaches
  - integrate-and-fire oscillators
  - distributed discrete time PLLs
- Integrate-and-fire
  - for analysis no frequency mismatch is assumed first $T_i = T_{\mathrm{nom}}$

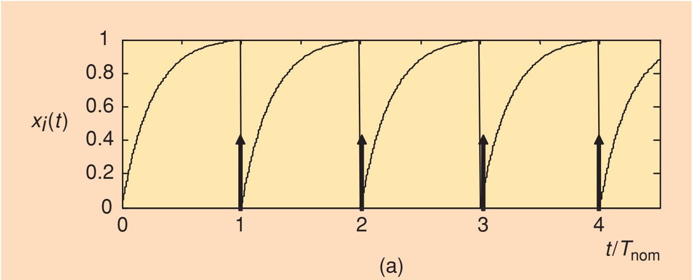

Figure source: [1, Fig. 7 (a)]

Pulse-Coupled Discrete Time Clocks II

- Oscillator described by state variable $x_i(t) = g(\phi_i(t))$ where $g(\cdot)$ is a periodic function with period 2π, smooth, concave and monotonically increasing from zero to one.
- Clock ticks $t_i(n)$ correspond to time instants when variable reaches $x_i(t_i(n)) = 1$ and returns to zero.
- Upon detection of the pulse sent by any node j at time $t_j(n)$ the ith clock adds ϵ to its state variable moving its firing instant closer to that of clock j,

$$x_i(t_j(n)^+) = \begin{cases} x_i(t_j(n)^-) + \epsilon & \text{if } x_i(t_j(n)^-) + \epsilon < 1 \\ 0 & \text{otherwise} \end{cases}$$

Pulse-Coupled Discrete Time Clocks III

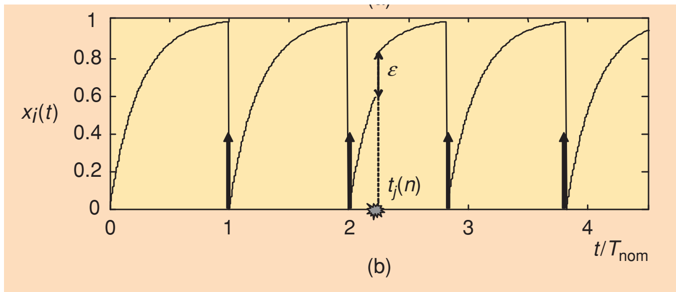

Figure source: [1, Fig. 7 (b)]

- Lyapunov stability theory shows that Laplacian $\boldsymbol{L}$ determins stability similarly to the case of analog oscillators.

References I

O. Simeone, U. S. andd Yeheskel Bar-Ness, and S. H. Strogatz, "Distributed synchronization in wireless networks," *IEEE Signal Process. Mag.*, pp. 81–97, September 2008.