DOUBLE KRYLOV SUBSPACE APPROXIMATION FOR LOW COMPLEXITY ITERATIVE MULTI-USER DECODING AND TIME-VARIANT CHANNEL ESTIMATION

Charlotte Dumard and Thomas Zemen

ftw. Forschungszentrum Telekommunikation Wien
Tech Gate, Donau-City Str. 1/3, A-1220 Vienna, Austria
E-mail: {dumard, zemen}@ftw.at

ABSTRACT

Iterative multi-user detection and time-variant channel estimation in a multi-carrier (MC) code division multiple access (CDMA) uplink requires high computational complexity. This is mainly due to the linear minimum mean square error (MMSE) filters for data detection and time-variant channel estimation. We develop an algorithm based on the Krylov subspace method to solve a linear system with low complexity, trading accuracy for efficiency. This approach enables drastic reduction of computational complexity for time-variant channel estimation as well as storage reduction and parallelization of the computations for multi-user detection. The performance of the low-complexity iterative Krylov subspace receiver is validated by simulations.

1. INTRODUCTION

An iterative multi-user detector with time-variant channel estimator for a multi-carrier (MC) code division multiple access (CDMA) uplink is described in [1]. Both, data detection and time-variant channel estimation are based on linear minimum mean square error (MMSE) filters. The matrix inversion that is necessary to calculate the linear MMSE filters largely determines the computational complexity of the receiver. The Krylov subspace method is an efficient way to solve linear equation systems by trading accuracy for efficiency. We apply the Krylov subspace method in order to implement efficiently the two linear MMSE filters for data detection and time-variant channel estimation.

In iterative receivers, the soft information gained about the transmitted data symbols is used to enhance the channel estimation and data detection in consecutive iterations. For data detection we apply parallel interference cancellation (PIC) and individual linear MMSE filtering [1, 2]. For time-variant channel estimation we exploit the fact that the maximum variation in time of the wireless channel is upper bounded by the maximum (one sided) normalized Doppler bandwidth

$$\nu_{\mathrm{Dmax}} = \frac{v_{\max} f_C}{c_0} T_{\mathrm{S}}$$

, where $v_{\max}$ is the maximum supported velocity, T_{S} is the symbol duration, and c_0 denotes the speed of light. MC-CDMA is based on orthogonal frequency division multiplexing (OFDM). Thus each time-variant frequency-flat subcarrier is fully described through a sequence of complex scalars at the OFDM symbol rate $1/T_{\mathrm{S}}$. This sequence is bandlimited by ν_{Dmax}. We make use of Slepian's basic result that time-limited parts (snapshots) of band-limited sequences span a low dimensional subspace of the signals space [3]. The basis functions of this subspace are the discrete prolate spheroidal sequences. Using these results from the theory of time-concentrated and bandlimited sequences we represent a time-variant subcarrier through a Slepian basis expansion of low dimensionality [4].

Our contribution is the application of the Krylov subspace method to data detection on one hand, and time variant channel estimation on the other hand, in order to reduce the complexity of the linear MMSE filters. In the case of data detection with preceding PIC, we show that the Krylov subspace method allows to reduce the storage requirements and enables parallelization. In the case of channel estimation the Krylov subspace method allows to reduce the computational complexity of the linear MMSE estimator drastically.

The rest of the paper is organized as follows: We define the notation in Section 2 and introduce the Krylov subspace method in Section 3. The signal model for the multi-user uplink is presented in Section 4. Sections 4.1 and 4.2 outline respectively the iterative multi-user detection and the time-variant channel estimation, and show the use of the Krylov subspace method in both cases. Simulation results are given in Section 5 and conclusions are drawn in Section 6.

2. NOTATION

We denote a column vector by $\boldsymbol{a}$ and its i-th element with $a[i]$. Equivalently, we denote a matrix by $\boldsymbol{A}$, its i, ℓ-th element by $[\boldsymbol{A}]_{i,\ell}$. Its transpose is given by $\boldsymbol{A}^{\mathrm{T}}$ and its conjugate transpose by $\boldsymbol{A}^{\mathrm{H}}$. A diagonal matrix with elements $a[i]$ is written as $\mathrm{diag}(\boldsymbol{a})$ and the $Q \times Q$ identity matrix as $\boldsymbol{I}_Q$. The norm of $\boldsymbol{a}$ is denoted through $\|\boldsymbol{a}\|$ and the complex conjugate of b by b^*. The largest (lowest) integer, lower (larger) or equal than $b \in \mathbb{R}$ is denoted by $\lfloor b \rfloor$ ($\lceil b \rceil$).

3. KRYLOV SUBSPACE METHOD

We consider the general linear system $\boldsymbol{A}\boldsymbol{x} = \boldsymbol{b}$, where $\boldsymbol{A}$ is an invertible matrix of size $Q \times Q$, and $\boldsymbol{b}$ is a vector of length Q. The Lanczos algorithm [5] is an iterative algorithm that estimates the solution $\boldsymbol{x}$ of the above linear system in the case where $\boldsymbol{A}$ is symmetric. Thus, it does not compute the matrix $\boldsymbol{A}^{-1}$ explicitly. We give here a description of the algorithm in the general case. In the next section we show its application in the iterative receiver.

This work was funded by the Kplus programm in the I0 project of the ftw. Forschungszentrum Telekommunikation Wien.

The Cayley-Hamilton theorem states that there is a minimum polynomial $\mathcal{R}$ of degree $R \leq Q$ such that $\boldsymbol{I}_Q, \boldsymbol{A}, \ldots, \boldsymbol{A}^{R-1}$ are linearly independent and $\mathcal{R}(\boldsymbol{A}) = \boldsymbol{0}$. This can be rewritten as

$$\boldsymbol{A}^{-1} = \sum_{r=1}^{R} a_r \boldsymbol{A}^{r-1}$$

where $a_0, \cdots, a_{R-1} \in \mathbb{C}$ are given by $\mathcal{R}$. Then we compute and approximate $\boldsymbol{x}$ for $S < R$

$$\boldsymbol{x} = \sum_{r=1}^{R} a_r \boldsymbol{A}^{r-1} \boldsymbol{b} \approx \sum_{r=1}^{S} a_r \boldsymbol{A}^{r-1} \boldsymbol{b} \,.$$

In other words, $\boldsymbol{x}$ is estimated by $\boldsymbol{x}_S$, an element of the Krylov subspace of dimension S

$$\mathcal{K}_S = \text{span}\left\{ \boldsymbol{b}, \boldsymbol{A}\boldsymbol{b}, \ldots, \boldsymbol{A}^{S-1}\boldsymbol{b} \right\} \,.$$

The error $\boldsymbol{r}_S = \boldsymbol{b} - \boldsymbol{A}\boldsymbol{x}_S$ is by constrain orthogonal to $\mathcal{K}_S$.

As an element of $\mathcal{K}_S$, $\boldsymbol{x}_S$ can be written as a linear combination of an orthonormal basis $\boldsymbol{V}_S = [\boldsymbol{v}_1, \ldots, \boldsymbol{v}_S]$ of $\mathcal{K}_S$. This leads to $\boldsymbol{x}_S = \boldsymbol{V}_S \boldsymbol{z}_S$, where $\boldsymbol{z}_S \in \mathbb{C}^S$. $\boldsymbol{V}_S$ is computed by applying the Gram-Schmidt orthonormalization method on the Krylov basis $\boldsymbol{B} = [\boldsymbol{b}, \boldsymbol{A}\boldsymbol{b}, \ldots, \boldsymbol{A}^{S-1}\boldsymbol{b}]$. The condition $\boldsymbol{r}_S \perp \mathcal{K}_S$ writes

$$\boldsymbol{V}_S^{\mathrm{H}} \boldsymbol{r}_S = 0 \Leftrightarrow \boldsymbol{V}_S^{\mathrm{H}} \boldsymbol{b} = \boldsymbol{V}_S^{\mathrm{H}} \boldsymbol{A} \boldsymbol{V}_S \boldsymbol{z}_S \,. \tag{1}$$

Furthermore, the vectors $\boldsymbol{v}_i$ for $i \in \{1, \ldots, S\}$ are such that $\boldsymbol{A}\boldsymbol{v}_i \in \mathcal{K}_{i+1}$, leading to the property

$$\boldsymbol{v}_\ell^{\mathrm{H}} \boldsymbol{A} \boldsymbol{v}_i = 0 \quad \text{if} \quad \ell > i+1$$

. The matrix $\boldsymbol{T}_S = \boldsymbol{V}_S^{\mathrm{H}} \boldsymbol{A} \boldsymbol{V}_S$ has elements $[\boldsymbol{T}_S]_{i,\ell} = \boldsymbol{v}_\ell^{\mathrm{H}} \boldsymbol{A} \boldsymbol{v}_i$. Thus, we can state that it is an upper Hessenberg matrix. $\boldsymbol{A}$ being symmetric, $\boldsymbol{T}_S$ will be tridiagonal symmetric, and we denote its elements on the main diagonal as $\alpha_i \in \mathbb{C}$ and on the secondary diagonals as $\beta_i \in (0; +\infty)$. $(\cdot;\cdot)$ denotes an open interval. Finally, we know by construction of the orthonormal basis $\boldsymbol{V}_S$ that $\boldsymbol{b} = \|\boldsymbol{b}\| \boldsymbol{v}_1$.

Inserting these results into (1), we now need to solve $\boldsymbol{z}_S = \boldsymbol{T}_S^{-1} \|\boldsymbol{b}\| \boldsymbol{e}_1$ where $\boldsymbol{e}_1 = [1, 0, \ldots, 0]^{\mathrm{T}}$ has length S. To compute $\boldsymbol{z}_S$, the first column of $\boldsymbol{T}_S^{-1}$ is needed only. We apply the matrix inversion lemma for partitioned matrices [6] to the iterative relation

$$\boldsymbol{T}_s = \begin{bmatrix} \boldsymbol{T}_{s-1} & \beta_s \tilde{\boldsymbol{e}}_{s-1} \\ \beta_s \tilde{\boldsymbol{e}}_{s-1}^{\mathrm{T}} & \alpha_s \end{bmatrix}$$

where $\tilde{\boldsymbol{e}}_{s-1} = [0, \ldots, 0, 1]^{\mathrm{T}}$ has length $s-1$. This gives the following set of iterative equations

$$\begin{aligned} \boldsymbol{c}_{\text{first}}^{(s)} &= \begin{bmatrix} \boldsymbol{c}_{\text{first}}^{(s-1)} \\ 0 \end{bmatrix} + \gamma_s^{-1} \boldsymbol{c}_{\text{last}}^{(s-1)}[1]^* \begin{bmatrix} \beta_s^2 \boldsymbol{c}_{\text{last}}^{(s-1)} \\ -\beta_s \end{bmatrix} \\ \boldsymbol{c}_{\text{last}}^{(s)} &= \gamma_s^{-1} \begin{bmatrix} -\beta_s \boldsymbol{c}_{\text{last}}^{(s-1)} \\ 1 \end{bmatrix} , \end{aligned} \tag{2}$$

where $\boldsymbol{c}_{\text{first}}^{(s)}$ and $\boldsymbol{c}_{\text{last}}^{(s)}$ denote respectively the first and last columns of $\boldsymbol{T}_s^{-1}$, and $\gamma_s = \alpha_s - \beta_s^2 \boldsymbol{c}_{\text{last}}^{(s-1)}[s-1]$ is a scalar.

The Lanczos algorithm is summarized in Fig. 1. It has been shown that the Lanczos algorithm is equivalent to the multi-stage nested Wiener filter [7].

Define $\boldsymbol{b}$, $\boldsymbol{A}$ and S
$\boldsymbol{v}_1 = \boldsymbol{b}/\|\boldsymbol{b}\|$
$\boldsymbol{u} = \boldsymbol{A}\boldsymbol{v}_1$
$\alpha = \boldsymbol{v}_1^{\mathrm{H}} \boldsymbol{u}$
$\boldsymbol{c}_{\text{first}} = \boldsymbol{c}_{\text{last}} = 1/\alpha$
$\boldsymbol{w} = \boldsymbol{u} - \alpha \boldsymbol{v}_1$
for $s = 2, \ldots, S$
  $\boldsymbol{v}_s = \boldsymbol{w}/\|\boldsymbol{w}\|$
  $\boldsymbol{u} = \boldsymbol{A}\boldsymbol{v}_s$
  $\alpha = \boldsymbol{v}_s^{\mathrm{H}} \boldsymbol{u}$
  $\gamma = \alpha - \beta^2 \boldsymbol{c}_{\text{last}}^{(s-1)}[s-1]$
  $\boldsymbol{c}_{\text{first}}, \boldsymbol{c}_{\text{last}}$ using eq. (2)
  $\boldsymbol{w} = \boldsymbol{u} - \alpha \boldsymbol{v}_s - \beta \boldsymbol{v}_{s-1}$
  $\boldsymbol{V}_S = [\boldsymbol{v}_1, \ldots, \boldsymbol{v}_S]$
  $\boldsymbol{x}_S = \|\boldsymbol{b}\| \boldsymbol{V}_S \boldsymbol{c}_{\text{first}}$

Fig. 1. The Lanczos algorithm.

4. SIGNAL MODEL FOR TIME-VARIANT FREQUENCY-SELECTIVE CHANNELS

The MC-CDMA uplink transmission is block oriented, a data block consists of $M-J$ OFDM data symbols and J OFDM pilot symbols. Each user transmits symbols $b_k[m]$ with symbol rate $1/T_{\mathrm{S}}$. Discrete time is denoted by m. There are K users in the system, the user index is denoted by k. Each symbol is spread by a random spreading sequence $\boldsymbol{s}_k \in \mathbb{C}^N$ with independent identically distributed (i.i.d.) elements chosen from the set $\{\pm 1 \pm \mathrm{j}\}/\sqrt{2N}$. The data symbols $b_k[m]$ result from the binary information sequence $\chi_k[m'']$ of length $2(M-J)R_C$ by convolutional encoding with code rate R_C, random interleaving and quadrature phase shift keying (QPSK) modulation with Gray labelling.

The $M-J$ data symbols are distributed over a block of length M fulfilling $b_k[m] \in \{\pm 1 \pm \mathrm{j}\}/\sqrt{2}$ for $m \notin \mathcal{P}$ and $b_k[m] = 0$ for $m \in \mathcal{P}$ allowing for pilot symbol insertion. The pilot placement is defined through the index set

$$\mathcal{P} = \left\{ \left\lfloor i\frac{M}{J} + \frac{M}{2J} \right\rfloor \;\middle|\; i \in \{0, \ldots, J-1\} \right\} \,.$$

After spreading, pilot symbols $\boldsymbol{p}_k[m] \in \mathbb{C}^N$ are added

$$\boldsymbol{d}_k[m] = \boldsymbol{s}_k b_k[m] + \boldsymbol{p}_k[m] \,. \tag{3}$$

The elements of the pilot symbols $p_k[m,q]$ for $m \in \mathcal{P}$ and $q \in \{0, \ldots, N-1\}$ are randomly chosen from the QPSK symbol set $\{\pm 1 \pm \mathrm{j}\}/\sqrt{2N}$, otherwise $\boldsymbol{p}_k[m] = \boldsymbol{0}_N$ for $m \notin \mathcal{P}$.

Then, an N point inverse discrete Fourier transform (DFT) is performed and a cyclic prefix of length G is inserted. A single OFDM symbol together with the cyclic prefix has length $P = N + G$ chips. After parallel to serial conversion the chip stream with chip rate $1/T_{\mathrm{C}} = P/T_{\mathrm{S}}$ is transmitted over a time-variant multipath fading channel with L resolvable paths.

At the receive antenna the signals of all K users add up. The receiver removes the cyclic prefix and performs a DFT. The received signal vector $\boldsymbol{y}[m] \in \mathbb{C}^N$ after these two operations is given by [1]

$$\boldsymbol{y}[m] = \sum_{k=1}^{K} \operatorname{diag}(\boldsymbol{g}_k[m]) (\boldsymbol{s}_k b_k[m] + \boldsymbol{p}_k[m]) + \boldsymbol{z}[m] \,, \tag{4}$$

where complex additive white Gaussian noise with zero mean and covariance $\sigma_z^2 \boldsymbol{I}_N$ is denoted by $\boldsymbol{z}[m] \in \mathbb{C}^N$ with elements $z[m,q]$ and $\boldsymbol{g}_k[m] \in \mathbb{C}^N$ denotes the time-variant frequency response.

4.1. Iterative Data Detection

We define the time-variant effective spreading sequences $\tilde{\boldsymbol{s}}_k[m] = \operatorname{diag}(\boldsymbol{g}_k[m]) \boldsymbol{s}_k$, and the time-variant effective spreading matrix

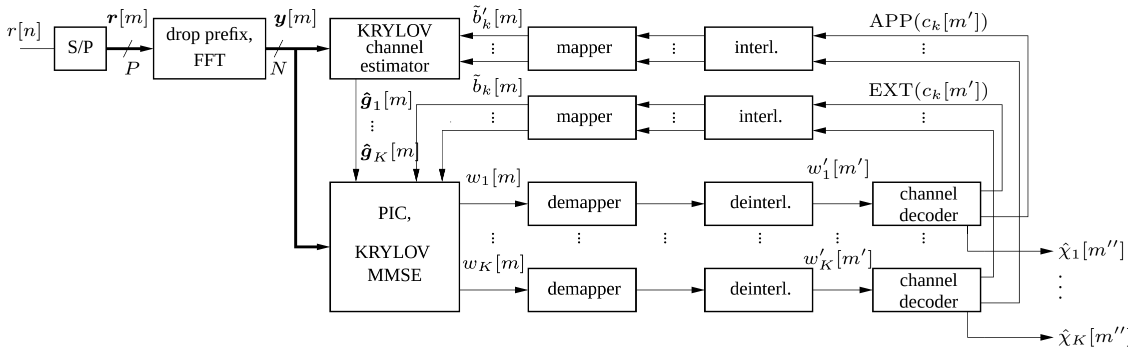

Fig. 2. Model for the MC-CDMA receiver. The MC-CDMA receiver performs joint iterative time-variant channel estimation and multi-user detection.

$\tilde{\boldsymbol{S}}[m] = [\tilde{\boldsymbol{s}}_1[m], \ldots, \tilde{\boldsymbol{s}}_K[m]] \in \mathbb{C}^{N\times K}$. Using these definitions the signal model for data detection writes as

$$\boldsymbol{y}[m] = \tilde{\boldsymbol{S}}[m]\boldsymbol{b}[m] + \boldsymbol{z}[m] \quad \text{for} \quad m \notin \mathcal{P}\,,$$

where $\boldsymbol{b}[m] = [b_1[m], \ldots, b_K[m]]^{\mathrm{T}} \in \mathbb{C}^K$ contains the stacked data symbols for K users.

Fig. 2 shows the structure of the iterative receiver [1, 8]. The receiver detects the data $\boldsymbol{b}[m]$ using the received symbol vector $\boldsymbol{y}[m]$, the spreading matrix $\tilde{\boldsymbol{S}}^{(i)}[m]$, and the feedback extrinsic probability $\mathrm{EXT}(c_k^{(i)}[m'])$ on the code symbols at iteration i.

In order to cancel the multi-access interference, we perform soft parallel interference cancellation for user k

$$\tilde{\boldsymbol{y}}_k^{(i)}[m] = \boldsymbol{y}[m] + \tilde{\boldsymbol{s}}_k^{(i)}[m]\tilde{b}_k^{(i)}[m] - \tilde{\boldsymbol{S}}^{(i)}[m]\tilde{\boldsymbol{b}}^{(i)}[m]\,, \qquad (5)$$

where $\tilde{\boldsymbol{b}}^{(i)}[m]$ contains the soft symbol estimates computed from the extrinsic probability supplied by the decoding stage, see [8] $\tilde{\boldsymbol{b}}_k^{(i)}[m] = 2\mathrm{EXT}(c_k^{(i)}[2m]) - 1 + \mathrm{j}(2\mathrm{EXT}(c_k^{(i)}[2m+1]) - 1)$. We apply unbiased conditional linear MMSE filtering, omitting iteration and time indexes i and m for simplification ([8])

$$\boldsymbol{f}_k = \frac{(\sigma_z^2\boldsymbol{I}_N + \tilde{\boldsymbol{S}}\boldsymbol{V}\tilde{\boldsymbol{S}}^{\mathrm{H}})^{-1}\tilde{\boldsymbol{s}}_k}{\tilde{\boldsymbol{s}}_k^{\mathrm{H}}(\sigma_z^2\boldsymbol{I}_N + \tilde{\boldsymbol{S}}\boldsymbol{V}\tilde{\boldsymbol{S}}^{\mathrm{H}})^{-1}\tilde{\boldsymbol{s}}_k}\,. \qquad (6)$$

The matrix $\boldsymbol{V}$ denotes the error covariance matrix of the soft symbols $\boldsymbol{V} = \mathbb{E}\{(\boldsymbol{b}[m] - \tilde{\boldsymbol{b}}[m])(\boldsymbol{b}[m] - \tilde{\boldsymbol{b}}[m])^{\mathrm{H}}\}$ with diagonal elements $V_{k,k} = \mathbb{E}\{1 - |\tilde{b}_k[m]|^2\}$ that are constant during the iteration. $\boldsymbol{b}$ and $\tilde{\boldsymbol{b}}$ are assumed to be independent, and the other elements are assumed to be zeros. The estimates $w_k[m]$ of the transmitted symbols $b_k[m]$ are then given by

$$w_k[m] = \boldsymbol{f}_k[m]^{\mathrm{H}}\tilde{\boldsymbol{y}}_k[m] \qquad (7)$$

and decoded by a BCJR decoder.

We apply the Lanczos algorithm to the linear MMSE filter (6). In other words, we estimate the product $(\sigma_z^2\boldsymbol{I}_N + \tilde{\boldsymbol{S}}\boldsymbol{V}\tilde{\boldsymbol{S}}^{\mathrm{H}})^{-1}\tilde{\boldsymbol{s}}_k$, denoting

$$\begin{aligned}\boldsymbol{A} &= \sigma_z^2\boldsymbol{I}_N + \tilde{\boldsymbol{S}}\boldsymbol{V}\tilde{\boldsymbol{S}}^{\mathrm{H}} \in \mathbb{C}^{N\times N}\\ \boldsymbol{b} &= \tilde{\boldsymbol{s}}_k \in \mathbb{C}^N\,.\end{aligned} \qquad (8)$$

Due to the parallel interference cancellation (5), the input vector $\tilde{\boldsymbol{s}}_k[m]$ of the linear MMSE filter is different for every user. Thus the Lanczos algorithm must be applied for every user independently. We see in the algorithm given in Fig. 1 that the matrix $\boldsymbol{A} = \sigma_z^2\boldsymbol{I}_N + \tilde{\boldsymbol{S}}\boldsymbol{V}\tilde{\boldsymbol{S}}^{\mathrm{H}}$ does not need to be explicitly computed. Instead, we compute $\boldsymbol{A}\boldsymbol{v}_s = \sigma_z^2\boldsymbol{v}_s + \tilde{\boldsymbol{S}}(\boldsymbol{V}(\tilde{\boldsymbol{S}}^{\mathrm{H}}\boldsymbol{v}_s))$ at every iteration s (lines 3 and 9 of the algorithm), which is equivalent to two matrix-vector products.

We use the definition of floating point operation (FLOP) that is given in [9] to define the computational complexity. We denote C_{Krylov} the number of FLOPs required to compute the filter (6) with the Krylov method. The complexity for the exact linear MMSE filter is denoted by C_{MMSE}. With the Krylov method the filters for all K users must be computed independently (see (8)), resulting in $C_{\mathrm{Krylov}} = K(2SKN + (6S-1)N + SK)$ FLOPs.

For the exact linear MMSE filter, the matrix inverse is computed once and reused for all K filters due to the structure of (6). Thus, the exact linear MMSE filter requires the computation of $\boldsymbol{A}$ (NK^2 FLOPs) and its inversion ($\frac{2}{3}N^3$ FLOPs) only once. This result is then used to calculate the K filters (6). The overall computational complexity is $C_{\mathrm{MMSE}} = KN^2 + \frac{2}{3}N^3 + NK^2$ FLOPs.

Comparing C_{Krylov} and C_{MMSE}, we can see that the Krylov method does not allow to reduce the computational complexity. However, we do gain in terms of storage requirements. The Krylov method needs to store one matrix $\boldsymbol{V}_S \in \mathbb{C}^{N\times S}$ and two vectors $\boldsymbol{u}$ and $\boldsymbol{w} \in \mathbb{C}^N$, instead of two matrices $\boldsymbol{A}$ and $\boldsymbol{A}^{-1} \in \mathbb{C}^{N\times N}$. In contrast to the exact linear MMSE filter, the Krylov method allows the parallel computations of the K linear MMSE filters. This is beneficial for hardware implementation. The time needed for the computation is reduced by a factor of K.

Fig. 3 shows the mean square error (MSE) of the Krylov subspace method versus the subspace dimension S. $\boldsymbol{A}$ and $\boldsymbol{b}$ are chosen as in (8) and results are shown for $E_b/N_0 = 5, 10, 15$ dB and $K = 16, 64$ users. We consider spreading sequences of length $N = 64$. A dimension of the Krylov subspace between 2 (at 5 dB) and 4 (at 15 dB) is sufficient to reach a MSE smaller than 10^{-2} for 16 users. For 64 users, a subspace with dimension between 3 (at 5 dB) and 12 (at 15 dB) is needed.

4.2. Iterative Time-Variant Channel Estimation

The performance of the iterative receiver depends on the channel estimates for the time-variant frequency response $\boldsymbol{g}_k[m]$ since the effective spreading sequence directly depends on the actual channel realization.

We describe the time variation of $g_k[m,q]$ in (9) through the Slepian basis expansion [1, 4]. The Slepian sequences $u_i[m]$ as defined in [1] span an orthogonal basis in which we expand the sequence $g_k[m,q]$

$$g_k[m,q] \approx \tilde{g}_k[m,q] = \sum_{i=0}^{D-1} u_i[m]\psi_k[i,q]\,, \qquad (9)$$

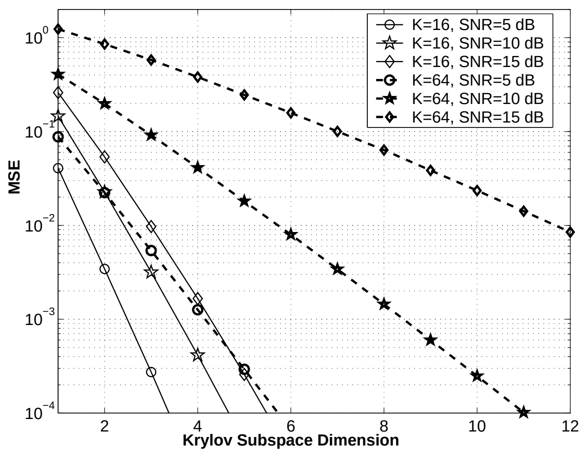

Fig. 3. MSE versus Krylov subspace dimension S for $E_b/N_0 = 5, 10, 15$ dB, $K = 16, 64$ users with spreading sequence length $N = 64$.

for $m \in \{0, \ldots, M-1\}$ and $q \in \{0, \ldots, N-1\}$. The dimension D of this basis expansion fulfills $\lceil 2\nu_{\text{Dmax}} M \rceil + 1 \leq D \leq M-1$. Substituting the basis expansion (9) for the time-variant subcarrier coefficients $g_k[m, q]$ into the system model (4) we obtain

$$y[m, q] = \sum_{k=1}^{K} \sum_{i=0}^{D-1} u_i[m] \psi_k[i, q] d_k[m, q] + z[m, q] , \qquad (10)$$

where $d_k[m, q]$ are the elements of $\boldsymbol{d}_k[m]$ (3).

For channel estimation, J pilot symbols in (3) are known. The remaining $M - J$ symbols are not known. We replace them by soft symbols that are calculated from the a-posteriori probabilities (APP) obtained in the previous iteration from the BCJR decoder output. This enables us to obtain refined channel estimates if the soft symbols get more reliable from iteration to iteration.

An estimate of the subcarrier coefficients $\hat{\psi}_k[i, q]$ can be obtained jointly for all K users but individually for every subcarrier q. We define the vectors

$$\boldsymbol{\psi}_q = [\psi_1[0, q], \ldots, \psi_K[0, q], \psi_1[D-1, q], \ldots, \\ \psi_K[D-1, q]]^{\mathrm{T}} \in \mathbb{C}^{KD}$$

containing the basis expansion coefficients of all K users for subcarrier q. $\boldsymbol{y}_q = [y[0, q], \ldots, y[M-1, q]]^{\mathrm{T}} \in \mathbb{C}^M$ is the received symbol sequence of each single data block on subcarrier q. Using these definitions we write $\boldsymbol{y}_q = \tilde{\boldsymbol{\mathcal{D}}}_q \boldsymbol{\psi}_q + \boldsymbol{z}_q$, where
$\tilde{\boldsymbol{\mathcal{D}}}_q = \left[\operatorname{diag}(\boldsymbol{u}_0)\, \tilde{\boldsymbol{D}}_q, \ldots, \operatorname{diag}(\boldsymbol{u}_{D-1})\, \tilde{\boldsymbol{D}}_q\right] \in \mathbb{C}^{M \times KD}$.

$\tilde{\boldsymbol{D}}_q \in \mathbb{C}^{M \times K}$ contains the transmitted symbols for all K users on subcarrier q

$$\tilde{\boldsymbol{D}}_q = \begin{bmatrix} \tilde{d}_1[0, q] & \ldots & \tilde{d}_K[0, q] \\ \vdots & \ddots & \vdots \\ \tilde{d}_1[M-1, q] & \ldots & \tilde{d}_K[M-1, q] \end{bmatrix} , \qquad (11)$$

and $\tilde{d}_k[m, q] = s_k[q] \tilde{b}_k[m] + p_k[m, q]$.

The linear estimator can be expressed as [1] (omitting the subcarrier index q)

$$\hat{\psi}_{\text{LMMSE}} = \left(\tilde{\boldsymbol{\mathcal{D}}}^{\mathrm{H}} \boldsymbol{\Delta}^{-1} \tilde{\boldsymbol{\mathcal{D}}} + \boldsymbol{I}_{KD}\right)^{-1} \tilde{\boldsymbol{\mathcal{D}}}^{\mathrm{H}} \boldsymbol{\Delta}^{-1} \boldsymbol{y}. \qquad (12)$$

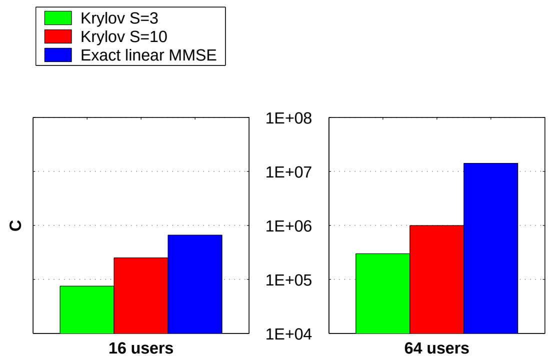

Fig. 4. Computational complexity (FLOPs) C for channel estimation with a Krylov subspace of dimension $S = 3, 10$, for $K = 16, 64$ users, $M = 256$ data symbols and a Slepian basis of dimension $D = 3$.

where $\boldsymbol{\Delta} \triangleq \boldsymbol{\Lambda} + \sigma_z^2 \boldsymbol{I}_M$ and the elements of the diagonal matrix $\boldsymbol{\Lambda}$ are defined as

$$\boldsymbol{\Lambda}_{mm} = \sum_{k=1}^{K} \sum_{i=0}^{D-1} \frac{1}{N} u_i^2[m] (1 - |\tilde{b}_k[m]|^2) .$$

After estimating $\hat{\boldsymbol{\psi}}_q$, an estimate for the time-variant frequency response is given by $\hat{g}'_k[m, q] = \sum_{i=0}^{D-1} u_i[m] \hat{\psi}_k[i, q]$. Further noise suppression is achieved if we exploit the correlation between the subcarriers $\hat{\boldsymbol{g}}_k[m] = \boldsymbol{F}_{N \times L} \boldsymbol{F}_{N \times L}^{\mathrm{H}} \hat{\boldsymbol{g}}'_k[m]$ where $[\boldsymbol{F}_{N \times L}]_{i,\ell} = 1/\sqrt{N}\, \mathrm{e}^{\frac{-\mathrm{j}2\pi i \ell}{N}}$ for $i \in \{0, \ldots, N-1\}$ and $\ell \in \{0, \ldots, L-1\}$.

The Lanczos algorithm is here applied to the linear MMSE estimator (12), with $\boldsymbol{A} = \tilde{\boldsymbol{\mathcal{D}}}^{\mathrm{H}} \boldsymbol{\Delta}^{-1} \tilde{\boldsymbol{\mathcal{D}}} + \boldsymbol{I}_{KD}$ and $\boldsymbol{b} = \tilde{\boldsymbol{\mathcal{D}}}^{\mathrm{H}} \boldsymbol{\Delta}^{-1} \boldsymbol{y}$. Again, the matrix $\boldsymbol{A}$ is not explicitly computed, but at every iteration s: $\boldsymbol{A}\boldsymbol{v}_s = \boldsymbol{v}_s + \tilde{\boldsymbol{\mathcal{D}}}^{\mathrm{H}} (\boldsymbol{\Delta}^{-1} (\tilde{\boldsymbol{\mathcal{D}}} \boldsymbol{v}_s))$.

The algorithm here drastically reduces the computational complexity. Instead of $C_{\text{MMSE}} = (\frac{2}{3}KD + M)(KD)^2$ FLOPs required for computation and inversion of $\boldsymbol{A}$ in the exact linear MMSE filter, we approximate $\hat{\boldsymbol{\psi}}_{\text{LMMSE}}$ with a computational complexity $C_{\text{Krylov}} = 2SKDM + MS + (6S-1)KD \approx 2SKDM$ FLOPs.

Fig. 4 shows $\log_{10}(C)$ for a Krylov subspace of dimension $S \in \{3, 10\}$ and for the exact linear MMSE filter. We consider $K \in \{16, 64\}$ users, spreading sequence length $N = 64$, $M = 256$ data symbols and a Slepian basis dimension $D = 3$. The Lanczos algorithm reduces the computational complexity by one magnitude with 16 users and more than 1.5 magnitudes with 64 users.

5. SIMULATION RESULTS

We use the same simulation setup as in [1]. The realizations of the time-variant frequency-selective channel $h'_k[n, \ell]$, sampled at the chip rate $1/T_{\mathrm{C}}$, are generated using an exponentially decaying power delay profile with root mean square delay spread $T_{\mathrm{D}} = 4T_{\mathrm{C}} = 1\mu$s for a chip rate of $1/T_{\mathrm{C}} = 3.84 \cdot 10^6\,\mathrm{s}^{-1}$ [10]. The autocorrelation for every channel tap is given by the classical Jakes spectrum. The system operates at carrier frequency $f_C = 2$ GHz and $K \in \{32, 64\}$ users move with velocity $v = 100$km/h. These gives Doppler bandwidth $B_{\mathrm{D}} = 190$Hz and $\nu_{\mathrm{D}} = 3.9 \cdot 10^{-3}$. The number of subcarriers $N = 64$ and the OFDM symbol with cyclic prefix has length $P = G + N = 79$. The data block consists of $M = 256$ OFDM symbols with $J = 60$ OFDM pilot symbols. The system is designed for $v_{\max} = 102.5$ km/h which results in $D = 3$ for the Slepian basis expansion.

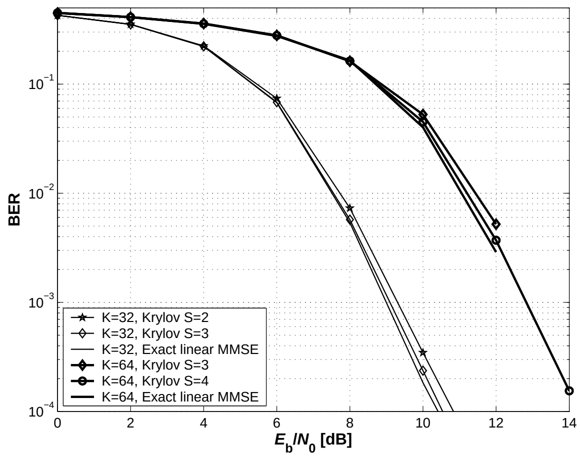

Fig. 5. **Krylov channel estimation:** BER versus SNR after 4 iterations of the receiver. The data detection uses exact linear MMSE filter. The time-variant channel estimation uses the Krylov method with subspace dimension $S \in \{2, 3, 4\}$. We show results for $K \in \{32, 64\}$ users and the random spreading sequences have length $N = 64$.

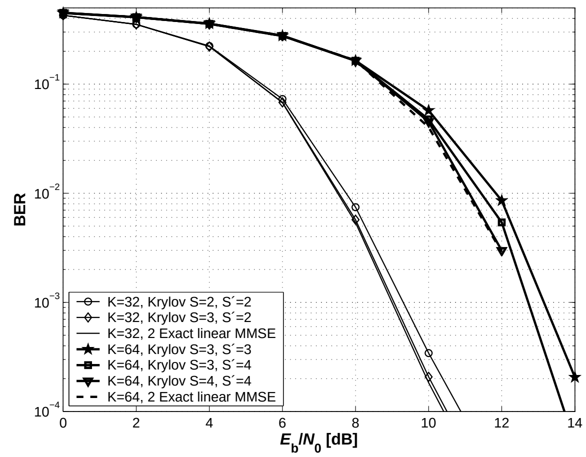

Fig. 6. **Krylov channel estimation and Krylov data detection:** BER versus SNR after 4 iterations of the receiver. The data detection uses the Krylov method with subspace dimension $S' \in \{2, 3, 4\}$. The time-variant channel estimation uses the Krylov method with subspace dimension $S \in \{2, 3, 4\}$. We show results for $K \in \{32, 64\}$ users and the random spreading sequences have length $N = 64$.

For data transmission, a convolutional, non-systematic, non-recursive, 4 state, rate $R_C = 1/2$ code with generator polynomial $(5, 7)_8$ is used. The illustrated results are obtained by averaging over 100 independent channel realizations. The QPSK symbol energy is normalized to 1 and we define $E_b/N_0 = \frac{1}{2R_C\sigma_z^2}\frac{P}{N}\frac{M}{M-J}$ taking into account the loss due to coding, pilots and cyclic prefix.

The noise variance in (6) and (12) is assumed to be known at the receiver. Fig. 5 illustrates the uplink performance with iterative time-variant channel estimation based on the Slepian basis expansion combined with the Krylov subspace method. We depict the bit error rate (BER) versus E_b/N_0 after 4 iterations for the iterative receiver. In this case, the multi-user detection is done using the exact linear MMSE filter solution (6).

Fig. 6 shows the uplink performance when the Krylov subspace method is used for both multi-user detection and time-variant channel estimations. For data detection, a Krylov subspace with dimension $S \leq 4$is used. Referring to Fig. 3, a MSE smaller than 0.3 is thus sufficient to reach the same performance as the exact linear MMSE filter. We are able to reduce the computational complexity for channel estimation by 1.5 magnitudes, and the storage requirements for multi-user detection by one magnitude.

6. CONCLUSION

We use the Krylov subspace method implemented via the Lanczos algorithm in order to approximate the linear MMSE filter for data detection. For time-variant channel estimation we use the same method to approximate the linear MMSE filter output. This enables a computational complexity reduction by 1.5 magnitudes for the time variant channel estimation in the case of a fully-loaded system with $K = N = 64$. For data detection the computational complexity is not reduced, but applying the Lanczos algorithm allows to reduce the storage requirements by about one magnitude. Additionally, the K Krylov data detection MMSE filters are parallelized. The receiver performance is identical to the one that is achieved by calculating the exact linear MMSE filters [1].

7. REFERENCES

[1] T. Zemen, C. F. Mecklenbräuker, J. Wehinger, and R. R. Müller, "Iterative multi-user decoding with time-variant channel estimation for MC-CDMA," in *Fifth International Conference on 3G Mobile Communication Technologies*, London, United Kingdom, 18-20 Oct. 2004, (invited).

[2] A. Lampe and J.B. Huber, "Iterative interference cancellation for DS-CDMA systems with high system loads using reliability-dependent feedback," *IEEE Trans. Veh. Technol.*, vol. 51, no. 3, pp. 445–452, May 2002.

[3] D. Slepian, "Prolate spheroidal wave functions, Fourier analysis, and uncertainty - V: The discrete case," *The Bell System Technical Journal*, vol. 57, no. 5, pp. 1371–1430, May-June 1978.

[4] T. Zemen and C. F. Mecklenbräuker, "Time-variant channel estimation using discrete prolate spheroidal sequences," *IEEE Trans. Signal Processing*, accepted for publication.

[5] Y. Saad, *Iterative Methods for Sparse Linear Systems*, SIAM, 2nd edition, 2003.

[6] T. K. Moon and W.C. Stirling, *Mathematical Methods and Algorithms*, Prentice Hall, 2000.

[7] M. Joham and M.D. Zoltowski, "Interpretation of the multi-stage nested Wiener filter in the Krylov subspace framework," *Technical Report TUM-LNS-TR-00-6, Munich University of Technology*, Nov. 2000.

[8] T. Zemen, *OFDM Multi-User Communication Over Time-Variant Channels*, Ph.D. thesis, Vienna University of Technology, Vienna, Austria, July 2004.

[9] G. H. Golub and C. F. Van Loan, *Matrix Computations*, Johns Hopkins University Press, Baltimore (MD), USA, 2nd edition, 1989.

[10] L. M. Correia, *Wireless Flexible Personalised Communications*, Wiley, 2001.