KRYLOV APPROXIMATION FOR MULTI-USER DETECTION WITH PARALLEL INTERFERENCE CANCELLATION IN CHIP AND USER SPACE

Charlotte Dumard and Thomas Zemen

ftw. Forschungszentrum Telekommunikation Wien
Tech Gate, Donau-City Str. 1/3, A-1220 Vienna, Austria
E-mail: {dumard, zemen}@ftw.at

ABSTRACT

Iterative multi-user detection and time-variant channel estimation in a multi-carrier (MC) code division multiple access (CDMA) uplink requires high computational complexity. This is mainly due to the linear minimum mean square error (MMSE) filters for data detection and time-variant channel estimation. We develop an algorithm based on the Krylov subspace method to solve a linear system with low complexity, trading accuracy for efficiency. It has been shown by Dumard *et al* that this approach enables drastic reduction of computational complexity as well as storage reduction when used for time-variant channel estimation. We compare two scenarios of parallel interference cancellation (PIC), in chip space and in user space. In the case of PIC in chip space, the Krylov subspace method allows parallelization of the computations of the K filters and storage reduction. However, the overall computational complexity remains similar to that of the exact linear MMSE filter. In the case of PIC in user space, it reduces drastically the computational complexity but incurs some loss of performance.

1. INTRODUCTION

An iterative multi-user detector with time-variant channel estimator for a multi-carrier (MC) code division multiple access (CDMA) uplink is described in [1]. Both data detection and time-variant channel estimation are based on linear minimum mean square error (MMSE) filters. The matrix inversion that is necessary to calculate the linear MMSE filters largely determines the computational complexity of the receiver. The Krylov subspace method is an efficient way to solve linear equation systems by trading accuracy for efficiency. We apply the Krylov subspace method in order to implement efficiently the two linear MMSE filters for data detection and time-variant channel estimation.

In iterative receivers, the soft information gained about the transmitted data symbols is used to enhance the channel estimation and data detection in consecutive iterations. For data detection we apply parallel interference cancellation (PIC) and individual linear MMSE filtering [1, 2]. For time-variant channel estimation we exploit the fact that the maximum variation in time of the wireless channel is upper bounded by the maximum (one sided) normalized Doppler bandwidth

$$\nu_{\mathrm{Dmax}} = \frac{v_{\max} f_C}{c_0} T_{\mathrm{S}}\,,$$

where $v_{\max}$ is the maximum supported velocity, T_{S} is the symbol duration, and c_0 denotes the speed of light. MC-CDMA is based on orthogonal frequency division multiplexing (OFDM). Thus each time-variant frequency-flat subcarrier is fully described through a sequence of complex scalars at the OFDM symbol rate $1/T_{\mathrm{S}}$. This sequence is bandlimited by ν_{Dmax}. We make use of Slepian's basic result that time-limited parts (snapshots) of band-limited sequences span a low dimensional subspace of the signals space [3]. The basis functions of this subspace are the discrete prolate spheroidal sequences. Using these results from the theory of time-concentrated and bandlimited sequences we represent a time-variant subcarrier through a Slepian basis expansion of low dimensionality [4, 5].

The application of the Krylov subspace method for channel estimation enables drastic computational complexity reduction of the order of 1.5 magnitudes as well as storage reduction [6]. Our contribution in this paper is the application of the Krylov subspace method to iterative data detection in order to reduce the complexity of the linear MMSE filter. We compare two scenarios of multi-user interference cancellation, in chip space and in user space. In the case of PIC in chip space, the Krylov subspace method allows parallelization of the computations of the K filters and storage reduction. However, the overall computational complexity remains similar to that of the exact linear MMSE filter. In the case of PIC in user space, it reduces drastically the computational complexity but loses performance when the system is fully loaded.

The rest of the paper is organized as follows: We define the notation in Section 2 and introduce the Krylov subspace method in Section 3. The signal model for the multi-user uplink is presented in Section 4. Section 5 outline the iterative data detection, comparing multi-user interference cancellation in chip space and in user space. In both cases, the use of the Lanczos algorithm will be shown. Simulation results are given in Section 6 and conclusions are drawn in Section 7.

2. NOTATION

We denote a column vector by $\boldsymbol{a}$ and its i-th element with $a[i]$. Equivalently, we denote a matrix by $\boldsymbol{A}$, its i,ℓ-th element by $[\boldsymbol{A}]_{i,\ell}$. Its transpose is given by $\boldsymbol{A}^{\mathrm{T}}$ and its conjugate transpose by $\boldsymbol{A}^{\mathrm{H}}$. A diagonal matrix with elements $a[i]$ is written as $\mathrm{diag}(\boldsymbol{a})$ and the $Q \times Q$ identity matrix as $\boldsymbol{I}_Q$. The norm of $\boldsymbol{a}$ is denoted through $\|\boldsymbol{a}\|$ and the complex conjugate of b by b^*. The largest (lowest) integer, lower (larger) or equal than $b \in \mathbb{R}$ is denoted by $\lfloor b \rfloor$ ($\lceil b \rceil$).

3. KRYLOV SUBSPACE METHOD

We consider the general linear system $\boldsymbol{A}\boldsymbol{x} = \boldsymbol{b}$, where $\boldsymbol{A}$ is an invertible matrix of size $Q \times Q$, and $\boldsymbol{b}$ is a vector of length Q.

The Lanczos algorithm [7] is an iterative algorithm that estimates the solution $\boldsymbol{x}$ of the above linear system in the case where $\boldsymbol{A}$ is symmetric. Thus, it does not compute the matrix $\boldsymbol{A}^{-1}$ explicitly. We give here a description of the algorithm in the general case. In the next section we show its application in the iterative receiver.

The Cayley-Hamilton theorem states that there is a minimum polynomial $\mathcal{R}$ of degree $R \leq Q$ such that $\boldsymbol{I}_Q, \boldsymbol{A}, \ldots, \boldsymbol{A}^{R-1}$ are linearly independent and $\mathcal{R}(\boldsymbol{A}) = \mathbf{0}$. This can be rewritten as

$$\boldsymbol{A}^{-1} = \sum_{r=1}^{R} a_r \boldsymbol{A}^{r-1}$$

where $a_0, \cdots, a_{R-1} \in \mathbb{C}$ are given by $\mathcal{R}$. Then we compute and approximate $\boldsymbol{x}$ for $S < R$

$$\boldsymbol{x} = \sum_{r=1}^{R} a_r \boldsymbol{A}^{r-1} \boldsymbol{b} \approx \sum_{r=1}^{S} a_r \boldsymbol{A}^{r-1} \boldsymbol{b} \,.$$

In other words, $\boldsymbol{x}$ is estimated by $\boldsymbol{x}_S$, an element of the Krylov subspace of dimension S

$$\mathcal{K}_S = \text{span}\left\{\boldsymbol{b}, \boldsymbol{A}\boldsymbol{b}, \ldots, \boldsymbol{A}^{S-1}\boldsymbol{b}\right\} .$$

We constrain the residual error $\boldsymbol{r}_S = \boldsymbol{b} - \boldsymbol{A}\boldsymbol{x}_S$ to be uncorrelated to $\mathcal{K}_S$.

As element of $\mathcal{K}_S$, $\boldsymbol{x}_S$ can be written as linear combination of an orthonormal basis $\boldsymbol{V}_S = [\boldsymbol{v}_1, \ldots, \boldsymbol{v}_S]$ of $\mathcal{K}_S$. This becomes in vector notation $\boldsymbol{x}_S = \boldsymbol{V}_S \boldsymbol{z}_S$, where $\boldsymbol{z}_S \in \mathbb{C}^S$. $\boldsymbol{V}_S$ is computed by applying the Gram-Schmidt orthonormalization method on the Krylov basis $\boldsymbol{B} = [\boldsymbol{b}, \boldsymbol{A}\boldsymbol{b}, \ldots, \boldsymbol{A}^{S-1}\boldsymbol{b}]$. The condition $\boldsymbol{r}_S \perp \mathcal{K}_S$ writes

$$\boldsymbol{V}_S^{\mathrm{H}} \boldsymbol{r}_S = 0 \Leftrightarrow \boldsymbol{V}_S^{\mathrm{H}} \boldsymbol{b} = \boldsymbol{V}_S^{\mathrm{H}} \boldsymbol{A} \boldsymbol{V}_S \boldsymbol{z}_S \,. \tag{1}$$

Furthermore, the vectors $\boldsymbol{v}_i$ for $i \in \{1, \ldots, S\}$ are such that $\boldsymbol{A}\boldsymbol{v}_i \in \mathcal{K}_{i+1}$, leading to the property

$$\boldsymbol{v}_\ell^{\mathrm{H}} \boldsymbol{A} \boldsymbol{v}_i = 0 \quad \text{if} \quad \ell > i + 1 \,.$$

The matrix $\boldsymbol{T}_S = \boldsymbol{V}_S^{\mathrm{H}} \boldsymbol{A} \boldsymbol{V}_S$ has elements $[\boldsymbol{T}_S]_{i,\ell} = \boldsymbol{v}_\ell^{\mathrm{H}} \boldsymbol{A} \boldsymbol{v}_i$. Thus, we can state that it is an upper Hessenberg matrix. $\boldsymbol{A}$ being symmetric, $\boldsymbol{T}_S$ will be tridiagonal symmetric, and we denote its elements on the main diagonal as $\alpha_i \in \mathbb{C}$ and on the secondary diagonals as $\beta_i \in (0; +\infty)$. $(\cdot;\cdot)$ denotes an open interval. Finally, we know by construction of the orthonormal basis $\boldsymbol{V}_S$ that $\boldsymbol{b} = \|\boldsymbol{b}\| \boldsymbol{v}_1$.

Inserting these results into (1), we obtain a new equation to solve

$$\boldsymbol{z}_S = \boldsymbol{T}_S^{-1} \|\boldsymbol{b}\| \boldsymbol{e}_1$$

where $\boldsymbol{e}_1 = [1, 0, \ldots, 0]^{\mathrm{T}}$ has length S. To compute $\boldsymbol{z}_S$, the first column of $\boldsymbol{T}_S^{-1}$ is needed only. We apply the matrix inversion lemma for partitioned matrices [8] to the iterative relation

$$\boldsymbol{T}_s = \begin{bmatrix} \boldsymbol{T}_{s-1} & \beta_s \tilde{\boldsymbol{e}}_{s-1} \\ \beta_s \tilde{\boldsymbol{e}}_{s-1}^{\mathrm{T}} & \alpha_s \end{bmatrix}$$

where $\tilde{\boldsymbol{e}}_{s-1} = [0, \ldots, 0, 1]^{\mathrm{T}}$ has length $s - 1$. This gives the following set of iterative equations

$$\begin{aligned} \boldsymbol{c}_{\text{first}}^{(s)} &= \begin{bmatrix} \boldsymbol{c}_{\text{first}}^{(s-1)} \\ 0 \end{bmatrix} + \gamma_s^{-1} \boldsymbol{c}_{\text{last}}^{(s-1)}[1]^* \begin{bmatrix} \beta_s^2 \boldsymbol{c}_{\text{last}}^{(s-1)} \\ -\beta_s \end{bmatrix} \\ \boldsymbol{c}_{\text{last}}^{(s)} &= \gamma_s^{-1} \begin{bmatrix} -\beta_s \boldsymbol{c}_{\text{last}}^{(s-1)} \\ 1 \end{bmatrix} , \end{aligned} \tag{2}$$

1	Define $\boldsymbol{b}$, $\boldsymbol{A}$ and S	7	for $s = 2, \ldots, S$
2	$\boldsymbol{v}_1 = \boldsymbol{b}/\|\boldsymbol{b}\|$	8	$\quad \boldsymbol{v}_s = \boldsymbol{w}/\|\boldsymbol{w}\|$
3	$\boldsymbol{u} = \boldsymbol{A}\boldsymbol{v}_1$	9	$\quad \boldsymbol{u} = \boldsymbol{A}\boldsymbol{v}_s$
4	$\alpha = \boldsymbol{v}_1^{\mathrm{H}} \boldsymbol{u}$	10	$\quad \alpha = \boldsymbol{v}_s^{\mathrm{H}} \boldsymbol{u}$
5	$\boldsymbol{c}_{\text{first}} = \boldsymbol{c}_{\text{last}} = 1/\alpha$	11	$\quad \gamma = \alpha - \beta^2 \boldsymbol{c}_{\text{last}}^{(s-1)}[s-1]$
6	$\boldsymbol{w} = \boldsymbol{u} - \alpha \boldsymbol{v}_1$	12	$\quad \boldsymbol{c}_{\text{first}}, \boldsymbol{c}_{\text{last}}$ using eq. (2)
		13	$\quad \boldsymbol{w} = \boldsymbol{u} - \alpha \boldsymbol{v}_s - \beta \boldsymbol{v}_{s-1}$
		14	$\boldsymbol{V}_S = [\boldsymbol{v}_1, \ldots, \boldsymbol{v}_S]$
		15	$\boldsymbol{x}_S = \|\boldsymbol{b}\| \boldsymbol{V}_S \boldsymbol{c}_{\text{first}}$

Fig. 1. The Lanczos algorithm.

where $\boldsymbol{c}_{\text{first}}^{(s)}$ and $\boldsymbol{c}_{\text{last}}^{(s)}$ denote respectively the first and last columns of $\boldsymbol{T}_s^{-1}$, and $\gamma_s = \alpha_s - \beta_s^2 \boldsymbol{c}_{\text{last}}^{(s-1)}[s-1]$ is a scalar.

The Lanczos algorithm is summarized in Fig. 1.

One can note that if $\boldsymbol{A}$ is defined as product of matrices, which will be the case most of the time, then it does not need to be explicitly computed, since the only place where it appears is in the computation of $\boldsymbol{A}\boldsymbol{b}$ (lines 3 and 10 of the algorithm).

4. SIGNAL MODEL FOR TIME-VARIANT FREQUENCY-SELECTIVE CHANNELS

The MC-CDMA uplink transmission is block oriented, a data block consists of $M - J$ OFDM data symbols and J OFDM pilot symbols. Each user transmits symbols $b_k[m]$ with symbol rate $1/T_{\mathrm{S}}$. Discrete time is denoted by m. There are K users in the system, the user index is denoted by k. Each symbol is spread by a random spreading sequence $\boldsymbol{s}_k \in \mathbb{C}^N$ with independent identically distributed (i.i.d.) elements chosen from the set $\{\pm 1 \pm \mathrm{j}\}/\sqrt{2N}$. The data symbols $b_k[m]$ result from the binary information sequence $\chi_k[m'']$ of length $2(M - J)R_C$ by convolutional encoding with code rate R_C, random interleaving and quadrature phase shift keying (QPSK) modulation with Gray labelling.

The $M - J$ data symbols are distributed over a block of length M fulfilling $b_k[m] \in \{\pm 1 \pm \mathrm{j}\}/\sqrt{2}$ for $m \notin \mathcal{P}$ and $b_k[m] = 0$ for $m \in \mathcal{P}$ allowing for pilot symbol insertion. The pilot placement is defined through the index set

$$\mathcal{P} = \left\{ \left\lfloor i \frac{M}{J} + \frac{M}{2J} \right\rfloor \;\middle|\; i \in \{0, \ldots, J-1\} \right\} .$$

After spreading, pilot symbols $\boldsymbol{p}_k[m] \in \mathbb{C}^N$ are added

$$\boldsymbol{d}_k[m] = \boldsymbol{s}_k b_k[m] + \boldsymbol{p}_k[m] \,. \tag{3}$$

The elements of the pilot symbols $p_k[m, q]$ for $m \in \mathcal{P}$ and $q \in \{0, \ldots, N-1\}$ are randomly chosen from the QPSK symbol set $\{\pm 1 \pm \mathrm{j}\}/\sqrt{2N}$, otherwise $\boldsymbol{p}_k[m] = \mathbf{0}_N$ for $m \notin \mathcal{P}$.

Then, an N point inverse discrete Fourier transform (DFT) is performed and a cyclic prefix of length G is inserted. A single OFDM symbol together with the cyclic prefix has length $P = N + G$ chips. After parallel to serial conversion the chip stream with chip rate $1/T_{\mathrm{C}} = P/T_{\mathrm{S}}$ is transmitted over a time-variant multipath fading channel with L resolvable paths.

At the receive antenna the signals of all K users add up. The receiver removes the cyclic prefix and performs a DFT. The received signal vector $\boldsymbol{y}[m] \in \mathbb{C}^N$ after these two operations is given by [1]

$$\boldsymbol{y}[m] = \sum_{k=1}^{K} \text{diag}\left(\boldsymbol{g}_k[m]\right) \left(\boldsymbol{s}_k b_k[m] + \boldsymbol{p}_k[m]\right) + \boldsymbol{z}[m] \,, \tag{4}$$

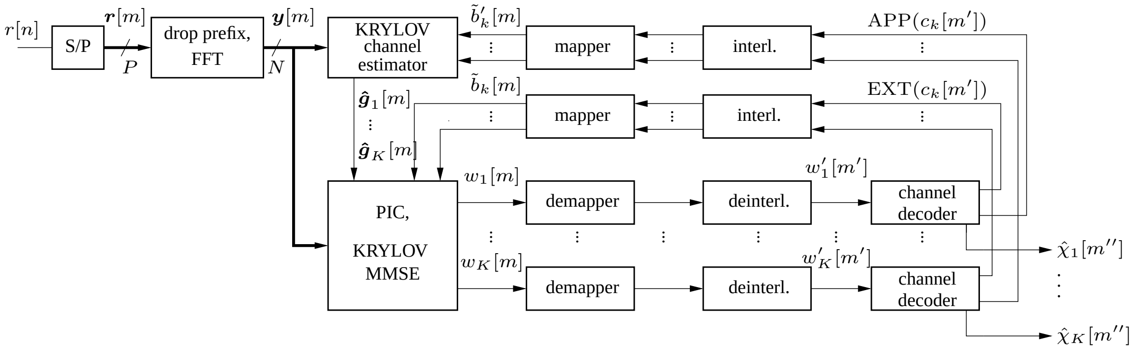

Fig. 2. Model for the MC-CDMA receiver. The MC-CDMA receiver performs joint iterative time-variant channel estimation and multi-user detection.

where complex additive white Gaussian noise with zero mean and covariance $\sigma_z^2\boldsymbol{I}_N$ is denoted by $\boldsymbol{z}[m]\in\mathbb{C}^N$ with elements $z[m,q]$ and $\boldsymbol{g}_k[m]\in\mathbb{C}^N$ denotes the time-variant frequency response.

We define the time-variant effective spreading sequences $\tilde{\boldsymbol{s}}_k[m] = \mathrm{diag}\,(\boldsymbol{g}_k[m])\,\boldsymbol{s}_k$, and the time-variant effective spreading matrix $\tilde{\boldsymbol{S}}[m] = [\tilde{\boldsymbol{s}}_1[m],\ldots,\tilde{\boldsymbol{s}}_K[m]]\in\mathbb{C}^{N\times K}$. Using these definitions the signal model for data detection writes as

$$\boldsymbol{y}[m] = \tilde{\boldsymbol{S}}[m]\boldsymbol{b}[m] + \boldsymbol{z}[m] \quad \text{for} \quad m\notin\mathcal{P}\,,$$

where $\boldsymbol{b}[m] = [b_1[m],\ldots,b_K[m]]^{\mathrm{T}}\in\mathbb{C}^K$ contains the stacked data symbols for K users.

Fig. 2 shows the structure of the iterative receiver [1,9]. The receiver detects the data $\boldsymbol{b}[m]$ using the received symbol vector $\boldsymbol{y}[m]$, the spreading matrix $\tilde{\boldsymbol{S}}^{(i)}[m]$, and the feedback extrinsic probability $\mathrm{EXT}(c_k^{(i)}[m'])$ on the code symbols at iteration i. The spreading matrix depends directly on the channel realization, which thus has a strong influence on the iterative receiver performance. The use of the Lanczos algorithm to approximate the linear MMSE filter at channel estimation has been developed in [6]. It has been shown that a maximal Krylov subspace dimension of 4 gives the same performances as the exact linear MMSE filter, and allows drastic computational complexity reduction up to 1.5 magnitude as well as storage reduction. In the following section, we will apply the Lanczos algorithm to data detection, and compare two cases of parallel interference cancellation.

5. ITERATIVE DATA DETECTION

Data detection is performed using the soft symbol estimates $\tilde{b}_k^{(i)}[m]$. They are computed from the extrinsic probability supplied by the decoding stage, see [9]

$$\tilde{\boldsymbol{b}}_k^{(i)}[m] = \frac{1}{\sqrt{2}}(2\mathrm{EXT}(c_k^{(i)}[2m]){-}1{+}\mathrm{j}(2\mathrm{EXT}(c_k^{(i)}[2m{+}1]){-}1))\,.$$

We define $\boldsymbol{V}$ the error covariance matrix of the soft symbols given by $\boldsymbol{V} = \mathbb{E}\{(\boldsymbol{b}[m]-\tilde{\boldsymbol{b}}[m])(\boldsymbol{b}[m]-\tilde{\boldsymbol{b}}[m])^{\mathrm{H}}\}$ with diagonal elements $V_{k,k} = \mathbb{E}\{1-|\tilde{b}_k[m]|^2\}$ that are constant during the iteration. $\boldsymbol{b}$ and $\tilde{\boldsymbol{b}}$ are supposed to be independent and the other elements of $\boldsymbol{V}$ are assumed to be zeros.

We consider in this paper two models of soft multi-user interference cancellation. In the first case, we perform interference cancellation in the chip space on the received signal $\boldsymbol{y}$. In the second cases, we will apply a matched filter on $\boldsymbol{y}$, and perform soft interference cancellation in the user space.

5.1. Soft Parallel Interference Cancellation in the Chip space

We perform soft parallel interference cancellation for user k

$$\tilde{\boldsymbol{y}}_k^{(i)}[m] = \boldsymbol{y}[m] + \tilde{\boldsymbol{s}}_k^{(i)}[m]\tilde{b}_k^{(i)}[m] - \tilde{\boldsymbol{S}}^{(i)}[m]\tilde{\boldsymbol{b}}^{(i)}[m]\,, \qquad (5)$$

and apply unbiased conditional linear MMSE filtering, omitting iteration and time indexes i and m for simplification (see [9])

$$\boldsymbol{f}_k = \frac{(\sigma_z^2\boldsymbol{I}_N + \tilde{\boldsymbol{S}}\boldsymbol{V}\tilde{\boldsymbol{S}}^{\mathrm{H}})^{-1}\tilde{\boldsymbol{s}}_k}{\tilde{\boldsymbol{s}}_k^{\mathrm{H}}(\sigma_z^2\boldsymbol{I}_N + \tilde{\boldsymbol{S}}\boldsymbol{V}\tilde{\boldsymbol{S}}^{\mathrm{H}})^{-1}\tilde{\boldsymbol{s}}_k}\,. \qquad (6)$$

The estimates $w_k[m]$ of the transmitted symbols $b_k[m]$ are then given by

$$w_k[m] = \boldsymbol{f}_k[m]^{\mathrm{H}}\tilde{\boldsymbol{y}}_k[m]$$

and decoded by a BCJR decoder.

We apply the Lanczos algorithm to the linear MMSE filter (6). In other words, we estimate the product $(\sigma_z^2\boldsymbol{I}_N + \tilde{\boldsymbol{S}}\boldsymbol{V}\tilde{\boldsymbol{S}}^{\mathrm{H}})^{-1}\tilde{\boldsymbol{s}}_k$, denoting

$$\begin{aligned}\boldsymbol{A} &= \sigma_z^2\boldsymbol{I}_N + \tilde{\boldsymbol{S}}\boldsymbol{V}\tilde{\boldsymbol{S}}^{\mathrm{H}} \in\mathbb{C}^{N\times N}\\ \boldsymbol{b} &= \tilde{\boldsymbol{s}}_k\in\mathbb{C}^N\,.\end{aligned} \qquad (7)$$

Due to the parallel interference cancellation (5), the linear MMSE filter (6) is different for every user. Thus the Lanczos algorithm must be applied for every user independently.

In terms of computational complexity, using the Lanczos algorithm we do not need to compute the matrix $\boldsymbol{A}$. Instead, we compute $\boldsymbol{A}\boldsymbol{v}_s = \sigma_z^2\boldsymbol{v}_s + \tilde{\boldsymbol{S}}(\boldsymbol{V}(\tilde{\boldsymbol{S}}^{\mathrm{H}}\boldsymbol{v}_s))$ at every iteration s. Since the Lanczos algorithm has to be computed for all K users, the overall computational complexity using the Krylov method is $C_{\mathrm{Krylov}} = K(2SKN + (6S-1)N + SK)$ FLOPs.

In comparison, the exact linear MMSE filter requires the computation of $\boldsymbol{A} = \sigma_z^2\boldsymbol{I}_N + \tilde{\boldsymbol{S}}\boldsymbol{V}\tilde{\boldsymbol{S}}^{\mathrm{H}}$ (KN^2 FLOPs), its inversion ($\frac{2}{3}N^3$ FLOPs) and the calculation of the K filters given by (6), resulting in a computational complexity of $C_{\mathrm{MMSE}} = KN^2 + \frac{2}{3}N^3 + NK^2$ FLOPs.

The inverse $\boldsymbol{A}^{-1}$ is the same for all K filters. For this reason, the computational complexity in both cases remains similar. However, we do gain in terms of storage requirements. The Lanczos algorithm needs to store one matrix $\boldsymbol{V}_S\in\mathbb{C}^{N\times S}$ and two vectors $\boldsymbol{u}$ and $\boldsymbol{w}\in\mathbb{C}^N$, instead of two matrices $\boldsymbol{A}$ and $\boldsymbol{A}^{-1}\in\mathbb{C}^{N\times N}$. Besides, it allows parallelization of the computations of the K linear MMSE filters.

We can see in Fig. 3 that despite the low Krylov dimension required, having to compute K filters instead of one inverse matrix does not reduce the computational complexity of the data detector.

In the following section, we will consider a different model of detection, performing PIC in the user space.

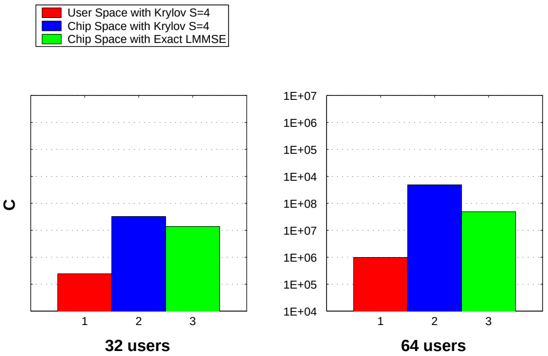

Fig. 3. **Computational Complexity for Data Detection:** for PIC in user space with Krylov approximated linear MMSE of dimension 4, PIC in chip space with Krylov approximated linear MMSE of dimension 4 and with exact linear MMSE, for $K \in \{32, 64\}$ users.

5.2. Soft Parallel Interference Cancellation in the User space

We apply a matched filter $\tilde{\boldsymbol{S}}$ to the input signal $\boldsymbol{y}$. The interference cancellation is then performed on $\boldsymbol{x} = \tilde{\boldsymbol{S}}^{\mathrm{H}}\boldsymbol{y}$

$$\tilde{\boldsymbol{x}} = \boldsymbol{x} - (\tilde{\boldsymbol{S}}^{\mathrm{H}}\tilde{\boldsymbol{S}} - \tilde{\boldsymbol{S}}_D)\tilde{\boldsymbol{b}}\,,$$

where $\tilde{\boldsymbol{S}}_D$ denotes the diagonal matrix whose elements are the diagonal elements of $\tilde{\boldsymbol{S}}^{\mathrm{H}}\tilde{\boldsymbol{S}}$. The vector $\tilde{\boldsymbol{x}} = [\tilde{\boldsymbol{x}}(1), \ldots, \tilde{\boldsymbol{x}}(K)]$ contains the received signal for all K users after interference cancellation. We apply then linear MMSE filtering $\boldsymbol{F}$ defined by

$$\boldsymbol{F}^{\mathrm{H}} = \operatorname{argmin}_F \mathbb{E}\{\|\boldsymbol{F}^{\mathrm{H}}\tilde{\boldsymbol{x}} - \boldsymbol{b}\|^2\}\,,$$

which leads to

$$\boldsymbol{F} = (\boldsymbol{V}\boldsymbol{R}+\tilde{\boldsymbol{S}}_D(\boldsymbol{I}_K-\boldsymbol{V}))(\boldsymbol{R}\boldsymbol{V}\boldsymbol{R}+\sigma_z^2\boldsymbol{R}+\tilde{\boldsymbol{S}}_D(\boldsymbol{I}_K-\boldsymbol{V})\tilde{\boldsymbol{S}}_D)^{-1}\,, \quad (8)$$

where $\boldsymbol{R} = \tilde{\boldsymbol{S}}^{\mathrm{H}}\tilde{\boldsymbol{S}}$ denotes the covariance matrix of the effective spreading sequence matrix. The estimates w_k of the transmitted symbols b_k are then given by the vector $\boldsymbol{w} = [w_1, \ldots, w_K] = \boldsymbol{F}^{\mathrm{H}}\tilde{\boldsymbol{x}}$ and decoded by a BCJR decoder. We apply the Lanczos algorithm to estimate the product $\boldsymbol{A}^{-1}\boldsymbol{b}$ with

$$\begin{aligned}\boldsymbol{A} &= \boldsymbol{R}\boldsymbol{V}\boldsymbol{R}+\sigma^2\boldsymbol{R}+\tilde{\boldsymbol{S}}_D(\boldsymbol{I}_K-\boldsymbol{V})\tilde{\boldsymbol{S}}_D \in \mathbb{C}^{K\times K}\\ \boldsymbol{b} &= \tilde{\boldsymbol{S}}^{\mathrm{H}}\boldsymbol{y} \in \mathbb{C}^{K}\,.\end{aligned} \quad (9)$$

In this situation, all K signals are taken into account in the MMSE filtering. Thus, we only need to perform the Lanczos algorithm once to obtain all K symbols. The corresponding value of the computational complexity is given by $C_{\text{Krylov}} = 6KN + 5K$ FLOPs. Fig. 3 compares the computational complexity for one iteration of the receiver in case of PIC in chip space with exact linear MMSE and with Krylov approximated linear MMSE. Additionally, we show the complexity of PIC in user space with Krylov approximated linear MMSE. Per iteration of the receiver, we can reduce the computational complexity by more than 1.5 magnitudes.

6. SIMULATION RESULTS

We use the same simulation setup as in [1]. The realizations of the time-variant frequency-selective channel $h'_k[n, \ell]$, sampled at the chip rate $1/T_{\mathrm{C}}$, are generated using an exponentially decaying power delay profile with root mean square delay spread $T_{\mathrm{D}} = 4T_{\mathrm{C}} = 1\mu$s for a chip rate of $1/T_{\mathrm{C}} = 3.84 \cdot 10^6\,\mathrm{s}^{-1}$ [10]. The autocorrelation for every channel tap is given by the classical Jakes spectrum. The system operates at carrier frequency $f_C = 2\,$GHz and $K \in \{32, 64\}$ users move with velocity $v = 100$km/h. These gives Doppler bandwidth $B_{\mathrm{D}} = 190$Hz and $\nu_{\mathrm{D}} = 3.9 \cdot 10^{-3}$. The number of subcarriers $N = 64$ and the OFDM symbol with cyclic prefix has length $P = G + N = 79$. The data block consists of $M = 256$ OFDM symbols with $J = 60$ OFDM pilot symbols. The system is designed for $v_{\max} = 102.5\,$km/h which results in $D = 3$ for the Slepian basis expansion.

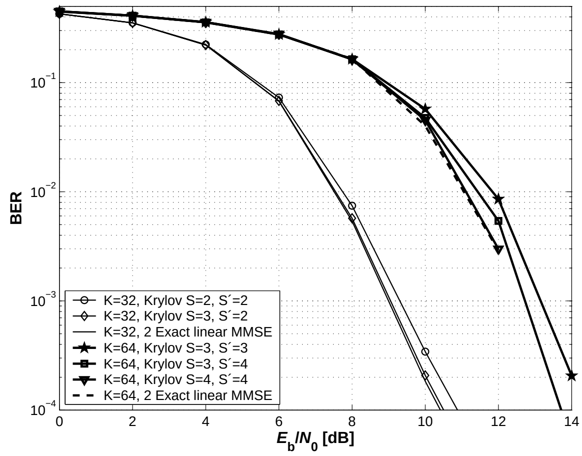

Fig. 4. **Krylov PIC in chip space:** BER versus SNR after 4 iterations of the receiver, time-variant channel estimation with a Krylov subspace of dimension $S \in \{2, 3, 4\}$ and data detection with a Krylov subspace of dimension $S' \in \{2, 3, 4\}$, for $K \in \{32, 64\}$ users.

For data transmission, a convolutional, non-systematic, non-recursive, 4 state, rate $R_C = 1/2$ code with generator polynomial $(5, 7)_8$ is used. The illustrated results are obtained by averaging over 100 independent channel realizations. The QPSK symbol energy is normalized to 1 and we define $E_b/N_0 = \frac{1}{2R_C\sigma_z^2}\frac{P}{N}\frac{M}{M-J}$ taking into account the loss due to coding, pilots and cyclic prefix. The noise variance in (6) and (8) is assumed to be known at the receiver. Fig. 4 shows the uplink performance when the Krylov subspace method is used for both multi-user detection in chip space and time-variant channel estimation. We depict the bit error rate (BER) versus E_b/N_0 after 4 iterations for the iterative receiver. For data detection, a Krylov subspace with dimension $S \leq 4$ is used. We are able to reduce the storage requirements for multi-user detection by one magnitude and to parallelize the computation of all K filters.

Fig. 5 compares the iterative receiver performances for data detection in chip space and in user space for 4 to 6 iterations of the receiver. Both data detection and channel estimation are performed using exact linear MMSE filter. We define the load of the system as $\beta = \frac{K}{N}$. For 32 users ($\beta = 0.5$) the system performs better with PIC in user space than in chip space with 4 iterations. However, for 64 users ($\beta = 1$)it needs 6 iterations to reach the performances of PIC in chip space.

Fig. 6 shows the performance obtained using the Lanczos algorithm for data detection with interference cancellation in user space. We consider a Krylov subspace with dimension $S \in \{4, 8\}$. For channel estimation, we assume that the Krylov dimension $S' = 4$ with 64 users and $S' = 3$ with 32 users obtained in [6] is sufficient. With 32 users ($\beta = 0.5$), good performances are reached with only 4 iterations of the receiver and a Krylov dimension 3 for

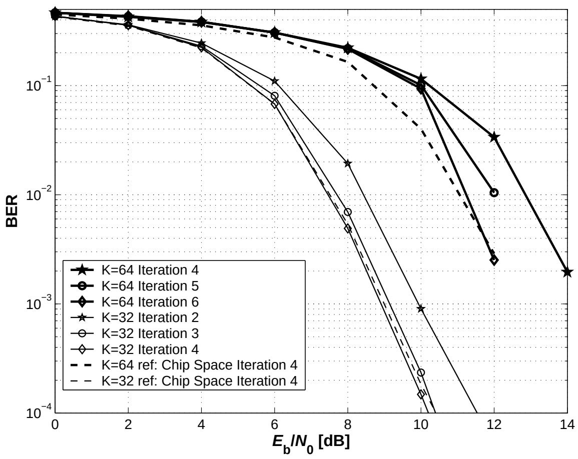

Fig. 5. **Iterative receiver performance with PIC in user space:** BER versus SNR after 4,5 and 6 iterations of the receiver for $K \in \{32, 64\}$ users. Both data detection and channel estimation are performed using the exact linear MMSE filter. The doted lines show the performances with PIC in chip space that we want to achieve.

data detection and channel estimation. In the case of 64 users ($\beta = 1$), the Lanczos algorithm used in data detection never reaches the performances of the linear MMSE filter but rather becomes unstable. This is mainly due to the instability of the Gram-Schmidt orthonormalization implemented in the Lanczos algorithm.

7. CONCLUSION

We use the Lanczos algorithm based on the Krylov subspace method in order to approximate a linear MMSE filter output for data detection in an iterative multi-user receiver. We consider two scenarios of multi-user detection: we apply first parallel interference cancellation in the chip space and then in the user space. Using the Lanczos algorithm allows to reduce storage requirements in all cases by one magnitude. For data detection with PIC in chip space, the computational complexity is rather increased because we compute K Krylov data detection MMSE filters instead of one inverse matrix. However, it allows parallelization of the computations of these K filters. For data detection with PIC in user space, the iterative receiver will need more iterations to reach equivalent performances, but complexity is still reduced by 1 magnitude in case of half load. When the system is fully loaded, the Lanczos algorithm becomes rather unstable and will not reach identical performances as the exact linear MMSE filter [1].

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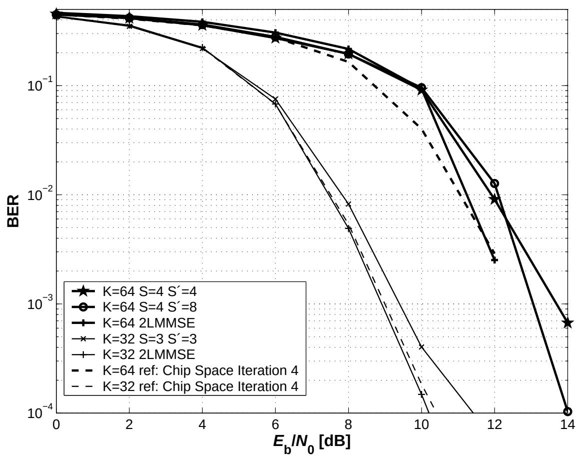

Fig. 6. **Krylov PIC in user space:** BER versus SNR after 6 iterations of the receiver. Channel estimation is using a Krylov subspace with dimension $S \in \{3, 4\}$, data detection is performed in user space with a Krylov subspace of dimension $S' \in \{4, 8\}$ for $K \in \{32, 64\}$ users. The doted lines show the performances with PIC in chip space that we want to achieve.

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