INTEGRATION OF THE KRYLOV SUBSPACE METHOD IN AN ITERATIVE MULTI-USER DETECTOR FOR TIME-VARIANT CHANNELS

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ABSTRACT

Iterative multi-user detection and time-variant channel estimation in a multi-carrier (MC) code division multiple access (CDMA) uplink requires high computational complexity. This is mainly due to the linear minimum mean square error (LMMSE) filters that are used for multi-user detection and time-variant channel estimation. Krylov subspace methods allow for an efficient implementation of the LMMSE filter. We show that a suitable chosen starting value, exploiting the iterative receiver structure, allows for a further speedup of the Krylov method. We achieve a complexity reduction by more than one order of magnitude. The Krylov subspace method allows a parallelization of the computations of the multi-user detector, while keeping the receiver performance constant. Numerical simulation results for a fully loaded system with $K = 64$ users are presented.

1. INTRODUCTION

The Krylov subspace method allows to trade efficiency for accuracy due to a stepwise approximation of the LMMSE filter output. We use the Krylov subspace method for complexity reduction of an iterative receiver with parallel interference cancelation (PIC) in [1]. The Krylov subspace method converges towards the LMMSE performance within as few as four steps for both multi-user detection and time-variant channel estimation. Due to PIC the computational complexity of the multi-user detector remains similar to the one using an LMMSE. In this paper, we make use of the structure of the iterative receiver to accelerate further the convergence of the Krylov algorithm and thus reduce the computational complexity.

Our contributions are: (i) Due to the iterative receiver structure we can use results from the previous iteration as a starting point in the next iteration. Such a tight integration of the Krylov subspace method in an iterative multi-user detector allows substantial complexity savings. (ii) We analyze the convergence properties of the Krylov subspace method in an iterative receiver structure by using properties of random matrices.

The paper is organized as follow: The system model and the multi-user detection is described in Section 2. We briefly review the Krylov methods in Section 3. Different initialization possibilities for the Krylov method are described in Section 3.1. The convergence speed of the Krylov subspace method is analyzed in Section 3.2. We compare the computational complexity of the different methods in Section 3.2. Simulation results are presented in Section 4 and conclusions are drawn in Section 5.

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2. ITERATIVE MULTI-USER DETECTOR

The MC-CDMA uplink transmission [2] is based on orthogonal frequency division multiplexing (OFDM). A data block consists of $M-J$ OFDM data symbols and J OFDM pilot symbols. We consider a system with K users. User k transmits symbols $b_k[m]$ with symbol rate $1/T_{\rm S}$, where m denotes discrete time. Each symbol is spread by a random spreading sequence[1] $\boldsymbol{s}_k \in \mathbb{C}^N$ with independent identically distributed (i.i.d.) elements chosen from the set $\{\pm 1 \pm {\rm j}\}/\sqrt{2N}$.

The $M-J$ data symbols are distributed over a block of length M fulfilling $b_k[m] \in \{\pm 1 \pm {\rm j}\}/\sqrt{2}$ for $m \notin \mathcal{P}$ and $b_k[m] = 0$ for $m \in \mathcal{P}$, where $\mathcal{P} = \{\lfloor M/J(i+1/2)\rfloor \mid i = 0, \ldots, J-1\}$ defines the pilot symbol positions. After spreading, J pilot symbols $\boldsymbol{p}_k[m] \in \mathbb{C}^N$ are added $\boldsymbol{d}_k[m] = \boldsymbol{s}_k b_k[m] + \boldsymbol{p}_k[m]$. The elements $p_k[m,q]$, $q \in \{0, \ldots, N-1\}$, of $\boldsymbol{p}_k[m]$ are randomly chosen from the QPSK symbol set $\{\pm 1 \pm {\rm j}\}/\sqrt{2N}$ for $m \in \mathcal{P}$. Otherwise $\boldsymbol{p}_k[m] = \boldsymbol{0}_N$.

Then, an N point inverse discrete Fourier transform (DFT) is performed and a cyclic prefix of length G is inserted. A single OFDM symbol together with the cyclic prefix has length $P = N+G$ chips. After parallel to serial conversion the chip stream with chip rate $1/T_{\rm C} = P/T_{\rm S}$ is transmitted over a time-variant multipath fading channel with L resolvable paths.

At the receive antenna the signals of all K users add up. The receiver removes the cyclic prefix and performs a DFT. The received signal vector after these two operations is given by [2]

$$\boldsymbol{y}[m] = \sum_{k=1}^{K} \operatorname{diag}\left(\boldsymbol{g}_k[m]\right)\left(\boldsymbol{s}_k b_k[m] + \boldsymbol{p}_k[m]\right) + \boldsymbol{z}[m]\,, \qquad (1)$$

where complex additive white Gaussian noise with zero mean and covariance $\sigma_z^2 \boldsymbol{I}_N$ is denoted by $\boldsymbol{z}[m] \in \mathbb{C}^N$ with elements $z[m,q]$ and $\boldsymbol{g}_k[m] \in \mathbb{C}^N$ denotes the time-variant frequency response. Fig. 1 shows the structure of the iterative receiver [2].

In this paper we focus on the multi-user detector. The time-variant channel estimator using prolate spheroidal sequences [2, 6] and its low complexity implementation using the Krylov subspace method can be found in [1]. We define the time-variant channel estimate $\hat{\boldsymbol{g}}_k[m]$ and the time-variant effective spreading sequence

$$\tilde{\boldsymbol{s}}_k[m] = \operatorname{diag}\left(\hat{\boldsymbol{g}}_k[m]\right)\boldsymbol{s}_k\,. \qquad (2)$$

[1] We denote a column vector by $\boldsymbol{a}$ and its i-th element with $a[i]$. The transpose of a matrix $\boldsymbol{A}$ is given by $\boldsymbol{A}^{\rm T}$ and its conjugate transpose by $\boldsymbol{A}^{\rm H}$. A diagonal matrix with elements $a[i]$ is written as $\operatorname{diag}(\boldsymbol{a})$ and the $Q \times Q$ identity matrix as $\boldsymbol{I}_Q$. The vector of size Q containing zeros is denoted $\boldsymbol{0}_Q$. The norm of $\boldsymbol{a}$ is denoted through $\|\boldsymbol{a}\|$ and its norm with respect to a matrix $\boldsymbol{A}$ through $\|\boldsymbol{a}\|_{\boldsymbol{A}}$. The largest integer, lower than or equal to $b \in \mathbb{R}$ is denoted by $\lfloor b \rfloor$. The superscript $^{(i)}$ denotes the i-th iteration of the receiver.

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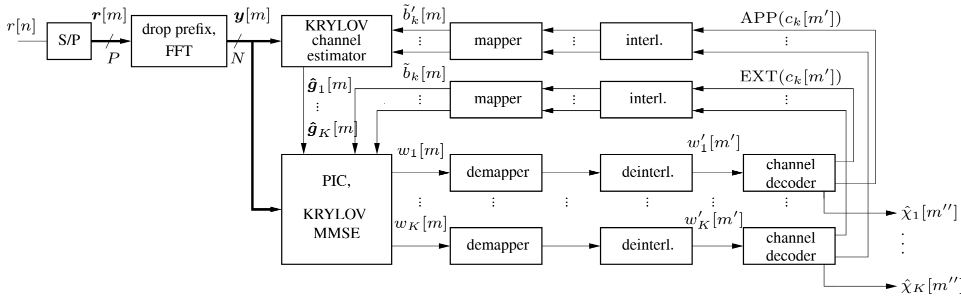

Fig. 1. Model for the MC-CDMA receiver. The MC-CDMA receiver performs joint iterative time-variant channel estimation and multi-user detection.

The corresponding time-variant effective spreading matrix is defined as $\tilde{\boldsymbol{S}}[m] = [\tilde{\boldsymbol{s}}_1[m], \ldots, \tilde{\boldsymbol{s}}_K[m]]$. Using these definitions the signal model for multi-user detection writes $\boldsymbol{y}[m] = \tilde{\boldsymbol{S}}[m]\boldsymbol{b}[m] + \boldsymbol{z}[m]$ for $m \notin \mathcal{P}$ where $\boldsymbol{b}[m] = [b_1[m], \ldots, b_K[m]]^{\mathrm{T}} \in \mathbb{C}^K$ contains the stacked data symbols for K users.

The receiver detects the data $\boldsymbol{b}[m]$ using the received symbol vector $\boldsymbol{y}[m]$, the spreading matrix $\tilde{\boldsymbol{S}}[m]$, and the soft symbol estimates $\tilde{\boldsymbol{b}}_k^{(i)}[m]$ computed from the feedback extrinsic probability $\mathrm{EXT}(c_k^{(i)}[m'])$ on the code symbols at iteration (i) with

$$\tilde{\boldsymbol{b}}_k^{(i)}[m] = 2\mathrm{EXT}(c_k^{(i)}[2m]) - 1 + \mathrm{j}(2\mathrm{EXT}(c_k^{(i)}[2m+1]) - 1)\,. \quad (3)$$

In order to cancel the multi-access interference, we perform soft PIC for user k (iteration(i) and time m are omitted for clarity): $\tilde{\boldsymbol{y}}_k = \boldsymbol{y} + \tilde{\boldsymbol{s}}_k \tilde{b}_k - \tilde{\boldsymbol{S}}\tilde{\boldsymbol{b}}$, and apply unbiased conditional LMMSE filtering:

$$\boldsymbol{f}_k = \frac{(\sigma_z^2 \boldsymbol{I}_N + \tilde{\boldsymbol{S}}\boldsymbol{V}\tilde{\boldsymbol{S}}^{\mathrm{H}})^{-1}\tilde{\boldsymbol{s}}_k}{\tilde{\boldsymbol{s}}_k^{\mathrm{H}}(\sigma_z^2 \boldsymbol{I}_N + \tilde{\boldsymbol{S}}\boldsymbol{V}\tilde{\boldsymbol{S}}^{\mathrm{H}})^{-1}\tilde{\boldsymbol{s}}_k}\,. \quad (4)$$

The matrix $\boldsymbol{V} = \mathbb{E}\{(\boldsymbol{b} - \tilde{\boldsymbol{b}})(\boldsymbol{b} - \tilde{\boldsymbol{b}})^{\mathrm{H}}\}$ denotes the error covariance matrix of the soft symbols, assumed diagonal with elements $V_{k,k} = \mathbb{E}\{1 - |\tilde{b}_k|^2\}$. The estimates w_k of the transmitted symbols b_k are then given by $w_k = \boldsymbol{f}_k^{\mathrm{H}}\tilde{\boldsymbol{y}}_k$ and decoded by a BCJR decoder.

The matrix inversion in (4) can be efficiently approximated by the Krylov subspace method. In the next section we give a short overview on the mathematics involved and we discuss the parameters that will influence the convergence speed of the Krylov algorithm in the context of an iterative receiver.

3. KRYLOV SUBSPACE METHOD FOR MULTI-USER DETECTION

The Krylov method [3] approximates the solution of a linear system $\boldsymbol{A}\boldsymbol{x} = \boldsymbol{a}$ where $\boldsymbol{A}$ is a known matrix with size $Q \times Q$ and $\boldsymbol{a}$ is a known vector with size $Q \times 1$. An initial value $\boldsymbol{x}_0$ is projected onto the Krylov subspace defined by

$$\mathcal{K}_s = \mathrm{span}\left\{\tilde{\boldsymbol{a}}, \boldsymbol{A}\tilde{\boldsymbol{a}}, \ldots, \boldsymbol{A}^{s-1}\tilde{\boldsymbol{a}}\right\} \quad (5)$$

where $\tilde{\boldsymbol{a}} = \boldsymbol{a} - \boldsymbol{A}\boldsymbol{x}_0$. The Krylov subspace dimension s is increased stepwise.

We consider the Ritz-Galerkin approach [4], which requires the residual vector $\boldsymbol{r}_s = \tilde{\boldsymbol{a}} - \boldsymbol{A}\boldsymbol{x}_s$ to be uncorrelated (or orthogonal) to $\mathcal{K}_s$. An orthonormal basis $\boldsymbol{W}_s$ of $\mathcal{K}_s$ is computed by applying the Gram-Schmidt orthonormalization onto the Krylov basis. The Ritz-Galerkin condition can be written as

$$\boldsymbol{W}_s^{\mathrm{H}}\boldsymbol{r}_s = 0 \Leftrightarrow \boldsymbol{W}_s^{\mathrm{H}}\tilde{\boldsymbol{a}} = \boldsymbol{W}_s^{\mathrm{H}}\boldsymbol{A}\boldsymbol{W}_s\boldsymbol{z}_s\,, \quad (6)$$

where $\boldsymbol{x}_s = \boldsymbol{W}_s\boldsymbol{z}_s$. Furthermore, $\boldsymbol{W}_s^{\mathrm{H}}\tilde{\boldsymbol{a}} = ||\tilde{\boldsymbol{a}}||\boldsymbol{e}_1$, where $\boldsymbol{e}_1 = [1, 0, \ldots, 0]^{\mathrm{T}}$ has length s. The matrix $\boldsymbol{T}_s = \boldsymbol{W}_s^{\mathrm{H}}\boldsymbol{A}\boldsymbol{W}_s$ is tri-diagonal and the first column of its inverse only is needed to obtain $\boldsymbol{z}_s = ||\tilde{\boldsymbol{a}}||\boldsymbol{T}_s^{-1}\boldsymbol{e}_1$. Finally, we get $\boldsymbol{x}_s = \boldsymbol{x}_0 + \boldsymbol{W}_s\boldsymbol{z}_s$ [3, 1].

We apply the Krylov algorithm to approximate the LMMSE filter (4). Here $\boldsymbol{A}$ and $\boldsymbol{a}$ are defined as $\boldsymbol{A} = \sigma_z^2\boldsymbol{I}_N + \tilde{\boldsymbol{S}}^{(i)}\boldsymbol{V}^{(i)}\tilde{\boldsymbol{S}}^{(i)\mathrm{H}}$ and $\boldsymbol{a} = \tilde{\boldsymbol{s}}_k^{(i)}$. In other words, we estimate the product

$$\boldsymbol{\pi}_k^{(i)}[m] = (\sigma_z^2\boldsymbol{I}_N + \tilde{\boldsymbol{S}}^{(i)}[m]\boldsymbol{V}^{(i)}[m]\tilde{\boldsymbol{S}}^{(i)\mathrm{H}}[m])^{-1}\tilde{\boldsymbol{s}}_k^{(i)}[m] \quad (7)$$

at every iteration (i) for each user k and each symbol m. We know that the error made at step[2] s of the Krylov subspace algorithm is upper bounded by [5]

$$\|\boldsymbol{x}_s - \boldsymbol{x}\|_{\boldsymbol{A}} \leq 2\|\boldsymbol{x}_0 - \boldsymbol{x}\|_{\boldsymbol{A}}\left(\frac{\sqrt{k_{\boldsymbol{A}}} - 1}{\sqrt{k_{\boldsymbol{A}}} + 1}\right)^s\,, \quad (8)$$

where the condition number $k_{\boldsymbol{A}} = \frac{\lambda_{\max}(\boldsymbol{A})}{\lambda_{\min}(\boldsymbol{A})} > 1$ of matrix $\boldsymbol{A}$ is defined as the ratio of the largest to the smallest eigenvalue. Hence convergence is assured, but the convergence speed depends strongly on the spectrum of $\boldsymbol{A}$ and on the choice of the initial value $\boldsymbol{x}_0$.

By using an appropriate initial value $\boldsymbol{x}_0$ the Krylov subspace algorithm converges faster or in other words we need a smaller subspace dimension s to achieve a given mean square error (MSE). Thus the complexity reported in [1] can be further reduced by a suitable initialization method which we discuss in the next subsection.

3.1. Initialization methods

We are considering an iterative receiver, hence we can exploit the information available from the previous iteration. We consider the following three possibilities for the integration of the Krylov subspace method in the iterative multi-user detector:

1. **Zeros** method: $\boldsymbol{x}_0^{(i)}[m] = \boldsymbol{0}_N$ for all iterations (i) and data symbols m. This is the standard method used in [1].
2. **Loop**-adaptive method: We use the result from the previous receiver iteration $(i-1)$ and define $\boldsymbol{x}_0^{(i)}[m] = \tilde{\boldsymbol{\pi}}_k^{(i-1)}[m]$, where $\tilde{\boldsymbol{\pi}}_k$ is the estimate of (7). For $i = 1$ $\boldsymbol{x}_0^{(1)}[m] = \boldsymbol{0}_N$.
3. **Time**-adaptive method: The variation of the channel's frequency response $\boldsymbol{g}_k[m]$ in time is bandlimited by the maximum Doppler frequency [6]. This is also true for the effective spreading sequence (2). Thus we use $\boldsymbol{x}_0^{(i)}[m] = \tilde{\boldsymbol{\pi}}_k^{(i)}[m-1]$. The initial value is given by $\boldsymbol{x}_0^{(i)}[0] = \boldsymbol{0}$.

[2] We use the term 'iteration' (i) for the outer feedback loop from the decoder to the PIC and the channel estimator in Fig. 1. The term 'step' s is used for the inner loop in the Krylov algorithm for data detection and channel estimation.

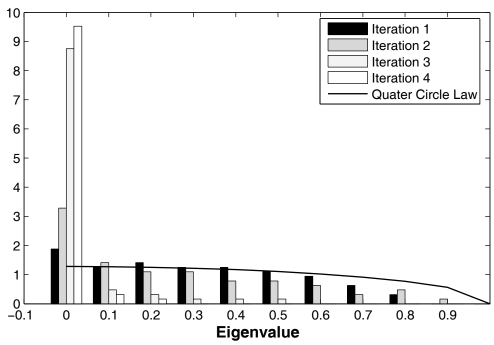

Fig. 2. Eigenvalue distribution of $\tilde{\boldsymbol{S}}\boldsymbol{V}^{(i)}\tilde{\boldsymbol{S}}^{\mathrm{H}}$ for receiver iteration $(i) \in \{1, 2, 3, 4\}$, averaged over 100 data blocks.

3.2. On the condition number

In this section we consider perfect channel knowledge. Then the effective spreading matrix $\tilde{\boldsymbol{S}}$ does not depend on the receiver iteration index (i). Considering the form of $\boldsymbol{A}$ and assuming that the smallest eigenvalue of $\tilde{\boldsymbol{S}}\boldsymbol{V}^{(i)}\tilde{\boldsymbol{S}}^{\mathrm{H}}$ is zero, we obtain $k_{\boldsymbol{A}} = 1 + \rho/\sigma_z^2$ where the spectral radius $\rho = \lambda_{\max}(\tilde{\boldsymbol{S}}\boldsymbol{V}^{(i)}\tilde{\boldsymbol{S}}^{\mathrm{H}})$. Let us assume that $\tilde{\boldsymbol{S}}$ has i.i.d. elements with zero mean and variance $1/2N$ which allows to utilize the quarter circle-law from random matrices theory [7]. This law states that the eigenvalue distribution of $(\tilde{\boldsymbol{S}}\tilde{\boldsymbol{S}}^{\mathrm{H}})^{1/2}$ converges to a density represented by a quarter circle if N grows to infinity. The eigenvalues $0 \leq \lambda \leq 1$ are distributed as

$$p(x) = \begin{cases} \frac{1}{\pi}\sqrt{1-x^2} & \text{if} \quad 0 \leq x \leq 1\,, \\ 0 & \text{elsewhere}\,. \end{cases} \tag{9}$$

We plot the eigenvalue distribution of $(\tilde{\boldsymbol{S}}\boldsymbol{V}^{(i)}\tilde{\boldsymbol{S}}^{\mathrm{H}})^{1/2}$ for $i \in \{1, \ldots, 4\}$ in Fig. 2. For the first iteration $\boldsymbol{V}^{(1)} = \boldsymbol{I}_N$ and we obtain a distribution very similar to the quarter circle with maximum value $\rho \leq 1$. With each iteration (i) of the receiver, the diagonal covariance matrix $\boldsymbol{V}^{(i)}$ becomes closer to the zero matrix due to PIC, making the eigenvalue distribution of $(\tilde{\boldsymbol{S}}\boldsymbol{V}^{(i)}\tilde{\boldsymbol{S}}^{\mathrm{H}})^{1/2}$ moving as well towards zero. From (8), we can assume that the required number of steps in the Krylov method will decrease with increasing iteration number in the receiver. In [8] a polynomial expansion is considered for reducing the LMMSE filter complexity. However, no interference cancelation is performed in [8] thus our approach requires less steps.

The empirical density of the eigenvalue distribution for increasing receiver iteration count is shown in Fig. 2. From this result we can tell that $\rho \leq 1$ and, for the practical implementation, that the necessary steps of the Krylov algorithm s decreases with increasing iteration count (i). In future work we will analyze an adaptively chosen Krylov subspace dimension $s(i)$.

3.3. Computational complexity of the Krylov detector

We compare the computational complexity of the LMMSE filter and the Krylov filter with **Zeros**, **Loop** and **Time** initialization. The system is fully loaded, $N = K$. For the multi-user detector (MUD) and the time-variant channel estimator (TVCE) the following result were obtained in [1]:

$$C_{\text{Krylov}}^{\text{MUD}} \approx K(2SK^2 + 7SK) \text{ and } C_{\text{LMMSE}}^{\text{MUD}} \approx \tfrac{8}{3}K^3\,,$$

$$C_{\text{Krylov}}^{\text{TVCE}} \approx 2S'KDM \text{ and } C_{\text{LMMSE}}^{\text{TVCE}} \approx \tfrac{2}{3}(KD+M)(KD)^2\,,$$

where S and S' is the Krylov subspace dimension for data detection and channel estimation respectively. The dimension of the Slepian subspace used for the channel model [2, 6] is denoted by D.

The computational complexity of the Krylov and the LMMSE multi-user detectors are of the same order. This is due to PIC, since all users have different filters requiring the same matrix inverse. However, using the Krylov subspace method allow parallelization of the computations for the K filters.

The different initialization methods for the multi-user Krylov filter have the following properties: the **Zeros** method allows saving of one matrix-vector product at the initialization ($\tilde{\boldsymbol{a}} = \boldsymbol{a}$). The **Loop** method has similar computational complexity as the **Time** method, but needs to remember the values $\tilde{\boldsymbol{\pi}}_k^{(i)}[m]$, which means saving a large matrix of size $N \times K \times M$ over the whole receiver iteration.

4. SIMULATION RESULTS

We use the simulation setup from [2]. The realizations of the time-variant frequency-selective channel $h'_k[n, \ell]$, sampled at the chip rate $1/T_{\mathrm{C}}$, are generated using an exponentially decaying power delay profile with root mean square delay spread $T_{\mathrm{D}} = 4T_{\mathrm{C}} = 1\mu$s for a chip rate of $1/T_{\mathrm{C}} = 3.84 \cdot 10^6\,\mathrm{s}^{-1}$ [9]. The autocorrelation for every channel tap is given by the classical Jakes spectrum. The system operates at carrier frequency $f_C = 2\,\mathrm{GHz}$ and $K = 64$ users move with velocity $v = 70\mathrm{kmh}^{-1}$. The number of subcarriers $N = 64$ (the system is fully loaded) and the OFDM symbol with cyclic prefix has length $P = G + N = 79$. The data block consists of $M = 256$ OFDM symbols with $J = 60$ OFDM pilot symbols. The system is designed for $v_{\max} = 102.5\,\mathrm{kmh}^{-1}$ which results in $D = 3$ for the Slepian basis expansion. The Doppler bandwidth $B_{\mathrm{D}} = 190$Hz and $\nu_{\mathrm{D}} = 3.9 \cdot 10^{-3}$.

For data transmission, a convolutional, non-systematic, non-recursive, 4 state, rate $R_C = 1/2$ code with generator polynomial $(5, 7)_8$ is used. The illustrated results are obtained by averaging over 100 independent channel realizations. The QPSK symbol energy is normalized to 1 and we define $E_b/N_0 = \frac{1}{2R_C\sigma_z^2}\frac{P}{N}\frac{M}{M-J}$ taking into account the loss due to coding, pilots and cyclic prefix. The noise variance is assumed to be known at the receiver.

First we show the

$$\mathrm{MSE}[m] = ||\boldsymbol{\pi}_k^{(i)}[m] - \tilde{\boldsymbol{\pi}}_k^{(i)}[m]||^2 \tag{10}$$

resulting from the approximation of the filter (7) by the Krylov filter with **Zeros**, **Loop** and **Time** initialization. Fig. 3 plots $\mathrm{MSE}[m]$ over discrete time m, for the receiver iteration $(i) \in \{1 \ldots, 4\}$ using a Krylov subspace dimension $S = 3$. At the first iteration of the receiver, the **Zeros** and **Loop** initialization are equal, both being initialized with $\mathbf{0}_N$. With consecutive receiver iterations the MSE of the **Time** and **Zeros** method becomes slightly larger. The MSE using the **Loop** model decreases monotonically with each receiver iteration. The **Loop** method is the only one directly benefiting from the precious receiver iteration: at every iteration, $\tilde{\boldsymbol{\pi}}_k[m]$ becomes more accurate and thus the initial value $\boldsymbol{x}_0$ becomes closer to the exact value $\boldsymbol{x} = \boldsymbol{\pi}_k[m]$.

Fig. 4 shows the BER using LMMSE and the three Krylov methods for multi-user detection (MUD), with perfect channel knowledge. We reach the LMMSE performance with the **Time** method after the first Krylov step. The **Loop** method allows a considerable gain over the **Zeros** method, but does not perform as well as the **Time** method. The **Zeros** Krylov model reaches the LMMSE performance with a Krylov dimension $S = 2$.

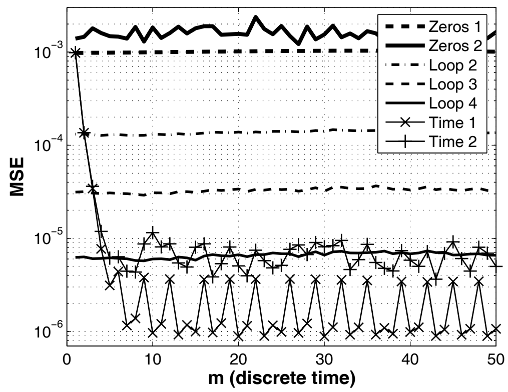

Fig. 3. $\mathrm{MSE}[m]$ (10) for the Krylov approximation of the LMMSE filter. The **Zeros**, **Loop** and **Time** initialization method are compared for subspace dimension $S = 3$, at an $E_b/N_0 = 10\,\mathrm{dB}$ and for receiver iteration $(i) \in \{1, \ldots, 4\}$ iterations.

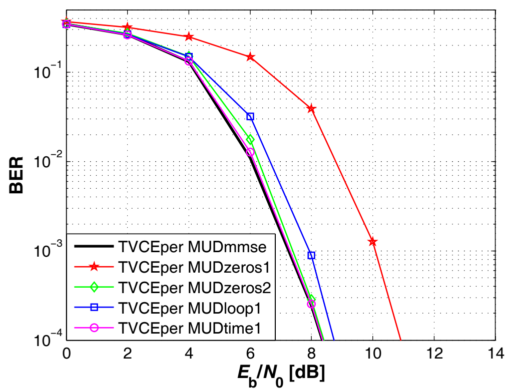

Fig. 4. Receiver performance in terms of BER versus E_b/N_0. We compare the LMMSE filter and the Krylov filter with **Zeros**, **Loop** and **Time** initialization for multi-user detection (MUD). The Krylov subspace has dimension $S \in \{1, 2\}$. The time-variant channel is known perfectly (TVCEper).

In Fig. 5 we show the combined use of different Krylov scenarios for both channel estimation and multi-user detection. To analyze these results, we need to keep in mind the following facts: (i) One Krylov step for channel estimation costs less computational complexity than one for multi-user detection. The **Loop** initialization for the channel estimation need a storage of $D \times K$ only [1]. (ii) The **Loop** initialization for the multi-user detector needs saving of a large matrix. This depreciates this method for a hardware implementation.

Thus the best combination is to use the **Loop** initialization with Krylov dimension 4 for the time-variant channel estimator and the **Time** initialization with dimension 2 for the multi-user detector. With this setup we are able to reduce the complexity (channel estimation and multi-user detection) to 6.53% of the complexity using two LMMSE filters [2].

5. CONCLUSION

We apply the Krylov subspace method for iterative time-variant channel estimation and multi-user detection for the uplink of an MC-CDMA system. Exploiting the information that is available from the previous receiver iteration we can speed up the Krylov subspace

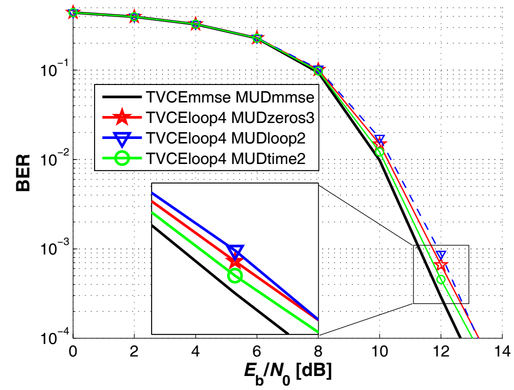

Fig. 5. Receiver performance in terms of BER versus E_b/N_0. We compare the LMMSE filter and the Krylov filter with **Zeros**, **Loop** and **Time** initialization with dimension $S \in \{1, 2, 3\}$ for multi-user detection (MUD). For time-variant channel estimation (TVCE) an LMMSE filter and a Krylov filter with **Loop** initialization and dimension $S' \in \{2, 3\}$ is used.

method. Due to the tight integration of the Krylov subspace method in the iterative receiver structure we are able to reduce the computational complexity by more than one order of magnitude. Due to the used Krylov subspace method the computations of the multi-user detector can be computed in K parallel branches which is highly beneficial for a low latency hardware implementation.

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