Large Aperture Antenna Array Design for Cellular LOS Massive MIMO

Stefan Pratschner*†, David Löschenbrand‡, Stefan Schwarz*†, Thomas Zemen‡ and Markus Rupp†
‡Security & Communication Technologies, Center for Digital Safety & Security,
AIT Austrian Institute of Technology, Vienna, Austria
*Christian Doppler Laboratory for Dependable Wireless Connectivity for the Society in Motion
†Institute of Telecommunications, TU Wien, Austria, Email: stefan.pratschner@tuwien.ac.at

***Abstract*—Massive multiple-input multiple-output (MIMO) achieves enhanced data rates with linear precoding schemes, making it an interesting PHY scheme for future mobile communications systems. The deployment of a high number of antennas facilitates reliable communication to many users in a multi-user MIMO system. However, reliable low-latency communication is only maintained, if all mobile users are served simultaneously in a multi-user MIMO fashion. In case of high inter-user correlation, for example, when two users are spatially close to each other, the user rate with linear precoding decreases. In the worst case, either of the two users is dropped by the scheduler in order to maintain a certain rate to at least one of them. We aim to improve a massive MIMO system's reliability while keeping the number of antennas constant. To decrease user correlation of closely spaced users, we design an antenna array with a large aperture. We consider designing the antenna array by array thinning, employing a genetic algorithm. This leads to a non-uniform spacing of antennas. We show that the proposed array design maintains a certain user rate more reliably for linear precoding schemes compared to a uniform linear array.**

***Index Terms*—array thinning, massive MIMO, multi-user MIMO**

I. Introduction

Massive multiple-input multiple-output (MIMO) promises increased spectral efficiency as well as high reliability [1]. Ultra-reliable and low-latency communication (uRLLC) is facilitated through the effects of channel hardening and favorable propagation [2]. The effect of channel hardening means an effective wireless channel without small scale fading effects. Favorable propagation refers to the fact that interference-free transmission is asymptotically achieved with linear precoding schemes for an increasing number of antennas. Both effects usually assume an iid. Rayleigh fading channel, modeling a very dense scattering environment. In this case, the aforementioned propagation effects prove advantageous, even for a finite number of antennas.

In the case of line of sight (LOS) propagation, inter-user interference between served users is mainly determined by the user positions and the employed antenna array. When two users are getting spatially close to each other, they will suffer from high inter-user interference, originating in the antenna array's finite main lobe width or high side lobes. In this situation, both users experience a low rate when applying a linear precoding scheme, due to their undesirable channel conditions. In order to maintain a certain target rate for at least one of the two users, one might consider scheduling of users. This method is also commonly employed in order to increase the sum rate of a massive MIMO system [3]. In the context of uRLLC, however, dropping users is disadvantageous as this strategy leads to increased transmission delay.

Therefore, we aim to increase reliability, in the sense of maintaining a certain minimal user rate, by reducing the inter-user interference via antenna array design for massive MIMO. Specific array configurations were found to be advantageous in terms of sum spectral efficiency (SE) [4]. Large aperture antenna array configurations for massive MIMO were investigated in [5], [6]. Distributed massive MIMO measurements with sub-array configurations were performed in [7].

Applying (finite) filter design methods to obtain an antenna array with a desired pattern is not a beneficial design strategy due to the exponential structure of the problem. A well known strategy to achieve specific array pattern properties, such as narrow main beam width, with a reduced number of elements compared to a conventional uniform linear array (ULA) is array thinning [8]. Rather than aiming to find the position of antenna elements within an array as the solution of an optimization problem, array thinning starts from a certain given antenna array configuration and tries to achieve a desired pattern while reducing the number of active elements. Considering a ULA with a large number of elements as starting point, an array thinning algorithm reduces the number of elements (by turning them off) iteratively until a certain thinning is reached, e.g., turn off 30 out of 100 elements. Due to the non-convex nature of this problem, finding the optimal solution by trying all possible combinations is infeasible for antenna arrays with a large number of elements. There exist many heuristic algorithms, inspired by biological or economical processes to solve non-convex optimization problems in the context of array design, such as pattern search [9], genetic algorithm (GA) [10], particle swarm optimization [11], grey wolf optimization and imperialist competitive algorithm [8].

Contribution: We propose to employ a well known array thinning algorithm, known as GA, to design antenna arrays for massive MIMO communications systems. By comparing

This work has been funded by the Christian Doppler Laboratory for Dependable Wireless Connectivity for the Society in Motion. The financial support by the Austrian Federal Ministry for Digital and Economic Affairs and the National Foundation for Research, Technology and Development is gratefully acknowledged.

to existing array designs, we show that the proposed antenna array is beneficial in terms of minimum achievable user SE and therefore improves reliability. We consider scheduling of users with beneficial channel conditions, aiming to increase the minimum achievable SE, as benchmark in terms of simulation. We show that the proposed thinned array configuration allows to maintain a higher minimal user SE compared to conventional array designs.

II. System Model

We consider a multi-user massive MIMO cellular downlink scenario with a large antenna array at the base station (BS). We assume that all N antenna elements of the array experience the same large scale fading. There are U users with a single isotropic antenna per user. The received signal of user u is

$$y_u = \sum_{j=1}^{U} \sqrt{\beta_u} \mathbf{h}_u^{\mathrm{H}} \mathbf{f}_j \sqrt{\rho_j} x_j + w_u. \tag{1}$$

where w_u is additive white Gaussian noise with $w_u \sim \mathcal{CN}\left(0, \sigma_w^2\right)$. The real positive power scaling factors are denoted by ρ_u for users $u \in \{1, \ldots, U\}$. The parameter β_u denotes the large scale fading coefficient and $\mathbf{h}_u$ denotes the small scale fading channel vector of user u. Here, x_u denotes the independent random transmit symbol with $\mathbb{E}\{xx^*\} = 1$ and $\mathbf{f}_u$ is the precoding vector of user u. To fulfill a sum transmit power constraint of the transmit signal $\mathbb{E}\left\{\| \sum_{u=1}^{U} \mathbf{f}_u \sqrt{\rho_u} x_u \|_2^2\right\} \leq P_T$, we require $\|\mathbf{f}_u\|_2^2 = 1$ and $\sum_{u=1}^{U} \rho_u = P_T$ where P_T is the sum transmit power. Therefore, the precoding matrix $\mathbf{F} = (\mathbf{f}_1, \mathbf{f}_2, \ldots, \mathbf{f}_U) \in \mathbb{C}^{N \times U}$ is normalized per-user such that $\|\mathbf{f}_u\|_2 = 1$, $\forall u \in \{1, \ldots, U\}$.

User u has the signal to interference and noise ratio (SINR)

$$\mathrm{SINR}_u = \frac{\beta_u \rho_u \left|\mathbf{h}_u^{\mathrm{H}} \mathbf{f}_u\right|^2}{\sum_{\substack{j \neq u \\ j=1}}^{U} \beta_u \rho_j \left|\mathbf{h}_u^{\mathrm{H}} \mathbf{f}_j\right|^2 + \sigma_w^2}. \tag{2}$$

The achievable SE of user u in bits/ s/ Hz is then given by

$$\mathrm{SE}_u = \log_2\left(1 + \mathrm{SINR}_u\right). \tag{3}$$

A. Precoding and Power Allocation

In this work, we consider maximum ratio transmission (MRT) precoding as well as minimum mean square error (MMSE) precoding. For a MRT precoder the precoding vectors are chosen as the corresponding channel vectors

$$\tilde{\mathbf{f}}_u^{\mathrm{MRT}} = \mathbf{h}_u \quad \forall u \in \{1, \ldots, U\}. \tag{4}$$

For an MMSE precoder, the precoding matrix is calculated as

$$\widetilde{\mathbf{F}}^{\mathrm{MMSE}} = \mathbf{H}\left(\mathbf{H}^{\mathrm{H}}\mathbf{H} + \frac{U\sigma_w^2}{P_T}\mathbf{I}_U\right)^{-1}, \tag{5}$$

where the channel matrix $\mathbf{H}$ is given by $\mathbf{H} = (\mathbf{h}_1, \ldots, \mathbf{h}_U)$ and $\widetilde{\mathbf{F}}$ is the precoding matrix $\widetilde{\mathbf{F}} = \left(\widetilde{\mathbf{f}}_1, \ldots, \widetilde{\mathbf{f}}_U\right)$. In order to fulfill the introduced sum power constraint, the precoding matrix is normalized for any precoding scheme according to

$$\mathbf{f}_u = \frac{\widetilde{\mathbf{f}}_u}{\left\|\widetilde{\mathbf{f}}_u\right\|_2} \quad \forall u \in \{1, \ldots, U\}. \tag{6}$$

Through this precoder normalization, precoding vectors for all users are of equal Euclidean norm. The transmit power allocation is therefore described by the power scaling parameters ρ_u for $u \in \{1, \ldots, u\}$. There exist sophisticated power allocation schemes for multi-user downlink scenarios, especially in the context of maximizing the minimum SINR across users. However, in order to focus on the effect of the array pattern on the minimum achievable SE we apply equal power to all users, that is, we choose $\rho_u = {}^{P_T}/_{U} \, \forall u \in \{1, \ldots, u\}$.

III. Antenna Array Design

We aim to increase the minimum achievable SE among users by smart antenna array design. Considering the system's SINR (2) and SE (3), the interference power plays a major role for the minimal SE. Let us assume MRT precoding and an equal path loss of one as well as equal power allocation for all users. In this case, the signal power is identical for all users. The minimal SINR is then determined by the maximum interference power

$$\max_{u=\{1,\ldots,U\}} \frac{P_T}{U} \sum_{\substack{j \neq u \\ j=1}}^{U} \left|\mathbf{h}_u^{\mathrm{H}} \mathbf{h}_j\right|^2. \tag{7}$$

The inner product of channel vectors however, is very much related to the antenna array pattern. To get a better insight in this problem, let us assume a ULA and a far-field approximation for now. Then, the channel vector of user u is given by

$$\mathbf{h}_u^{\mathrm{ULA}} = \left(e^{-i\frac{N}{2}kd\sin(\alpha_u)}, \ldots, e^{i\frac{N}{2}kd\sin(\alpha_u)}\right)^{\mathrm{T}}, \tag{8}$$

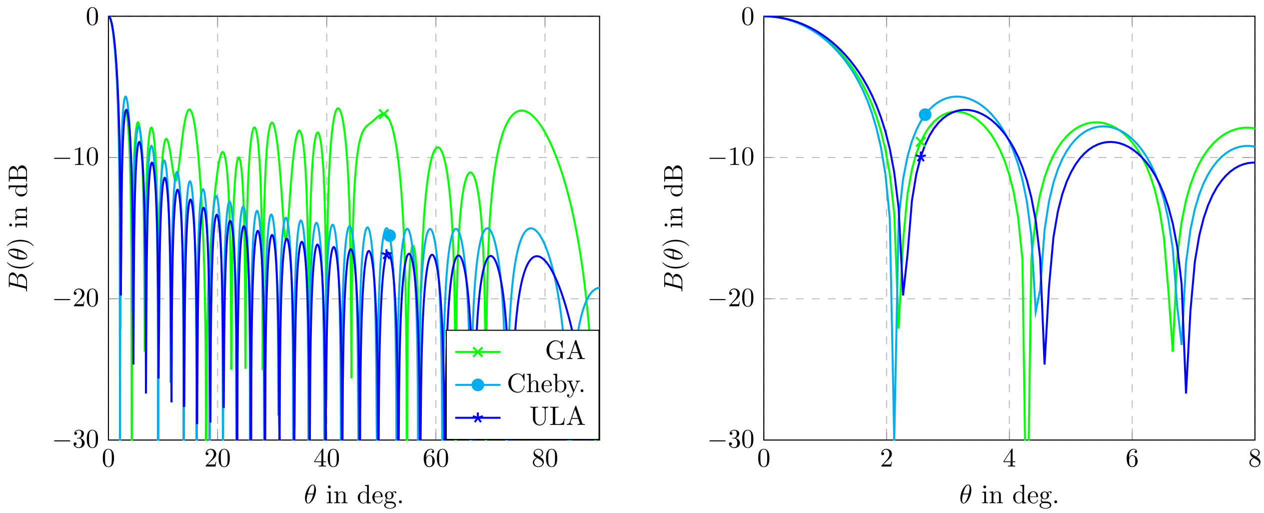
with inter-antenna distance d and the direction to user u, α_u. The inner product of channel vectors of two users is then

$$\left|\mathbf{h}_u^{\mathrm{H}} \mathbf{h}_j\right|^2 = \frac{\sin^2\left(Nkd\sin(\frac{\alpha_u - \alpha_j}{2})\cos(\frac{\alpha_u + \alpha_j}{2})\right)}{\sin^2\left(kd\sin(\frac{\alpha_u - \alpha_j}{2})\cos(\frac{\alpha_u + \alpha_j}{2})\right)}. \tag{9}$$

This equation is very similar to the well known array factor for ULA but depends on the difference of user angles $\alpha_u - \alpha_j$. Equation (9) emphasizes that the inter-user interference depends on the distance between users and also on the antenna array configuration. Assuming a uniform distribution of user angles implies that is is much more likely to have two users with a small angle between them than two users with a large angle between them. Therefore, we aim to design an antenna array with small main beam width and limited side lobe level (SLL) to decrease the inter-user interference.

Genetic Algorithm: We consider antenna array design by array thinning via the GA. Although we basically implement the GA from [10], we provide a brief description for readability and tractability.

To apply a GA to the problem of array thinning, we identify the so called chromosomes as strings or vectors of ones and zeros, indicating which antenna element is active or not. We assume an initial number of $M = 50$ elements in an ULA configuration with ${}^{\lambda}/_{2}$ element spacing and consider a fixed thinning rate such that $N = 32$ elements remain



(a) Antenna array pattern.

(b) Main beam pattern detail.

Fig. 1. Antenna array pattern $B(\theta)$ over azimuth angle θ for different array types.

active after thinning. Since this results in a minimal distance of $\lambda/2$ between antenna elements, mutual coupling is not an issue for this array design [12]. We consider only symmetric antenna array configurations to reduce the complexity of the optimization problem.

The GA aims to find the global optimum of a non convex optimization problem in an iterative fashion. In each iteration, the fitness of the current population (set of solutions) is evaluated. Common choices for fitness functions for array thinning are the half-power beam width (HPBW), the beam width variation factor (BWVF) or the SLL. The BWVF is defined as $\mathrm{HPBW} - \mathrm{HPBW}_{\mathrm{FULL}}$, where $\mathrm{HPBW}_{\mathrm{FULL}}$ denotes the HPBW of an array with all elements active. We consider a weighted combination of those two metrics in our work, as proposed by [9], and define the fitness function (FF) as

$$\mathrm{FF} = w_1\,\mathrm{BWVF} + w_2\,\mathrm{SLL}, \tag{10}$$

where we chose $w_1 = 1$ and $w_2 = 0.1$. The BWVF is in degree and the SLL is in dB. The choice of parameters w_1 and w_2 reflects the fact that the shape of the beam pattern close to the main lobe has the biggest impact on the maximum inter-user interference. In each iteration, the following tasks are performed.

- **Generate random population:** Generate a random matrix with M rows as initial population with exactly $N = 32$ active elements per row.
- **Evaluate Fitness:** Calculate the FF for the whole population. According to the "survival of the strongest" idea, we discard the worse half of the population.
- **Mating:** We consider mating according to the tournament method to let "healthy" chromosomes generate offspring. For this, we pick two subsets of size 20 from the better half of the population and consider the solution with the lowest FF from each set as parent. This is repeated $M/2$ times.
- **Generate offspring:** Offspring is generated from the $M/2$ pairs of parents by a simple single point crossover technique with a random crossover point.
- **Perform random mutation:** In order to prevent the iterative algorithm from getting stuck in a local minimum, a random mutation for all elements in the population with a mutation probability of 0.02 is performed.
- **Correct number of active elements:** Since the number of active elements can be changed by the processes of mating and mutation, we correct the number of active elements for all possible solutions of the population. This is done by randomly turning off elements in a solution with too many active elements or randomly turning on elements in a solution with too few active elements until the desired number of $N = 32$ active elements is restored.

We do not consider a breaking condition but perform the algorithm for a fixed number of 40 iterations. Employing this algorithm, we found the following antenna array configuration:

$$(1,1,0,1,1,1,0,1,0,1,1,0,0,1,0,1,1,1,1,0,0,1,0,1,1,\\ 1,1,0,1,0,0,1,1,1,1,0,1,0,0,1,1,0,1,0,1,1,1,0,1,1),$$

which achieves a BWVF of $-0.072°$ and an SLL of $-6.51\,\mathrm{dB}$. Please note that a negative BWVF means a beam width that is smaller than the beam width of the full array. This is possible due to the rather high SLL but necessary to improve user separability, that is, to decrease the inter-user interference of closely located users. Antenna array patterns $B(\theta)$ of an equidistantly spaced ULA, an array with a Chebychev configuration and the proposed solution for a thinned array are shown in Fig. 1. All antenna arrays in this comparison have the same length of $L = (M-1)d = 49\lambda/2$ and consist of $N = 32$ elements. The SLL of the ULA is $-6.62\,\mathrm{dB}$ while the SLL of the Chebychev array is $-5.68\,\mathrm{dB}$. The array obtained from array thinning with the GA has a SLL comparable to

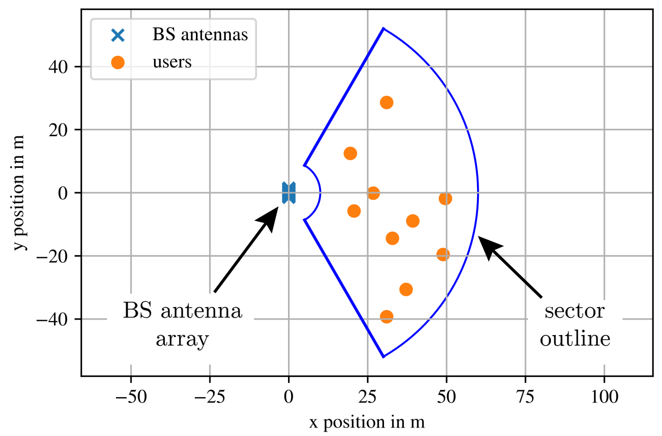


Fig. 2. Simulation scenario geometry with users uniformly random distributed within the sector.

the one of the ULA and a beam width that is in between the Chebychev array and the ULA.

IV. Simulation Results

We perform Monte Carlo simulations to evaluate mean performance indicators. To obtain results for an average system performance we perform 5 000 realizations with random users positions. Simulations are performed with a geometry as shown in Fig. 2 at a center frequency of 2.5 GHz. We consider a BS at the center, equipped with a one dimensional antenna array with $N = 32$ elements parallel to the y-axis. User positions are limited to a circular sector of $120°$ with a minimum distance to the BS of 10 m and a maximum distance to the BS of 60 m. User are randomly positioned according to a two dimensional uniform distribution within this sector. Each user has a single omni-directional antenna.

There are 10 random user positions generated per realization, out of which either $U = 9$ or $U = 10$ users are scheduled. We consider a min. max. scheduling strategy which aims to minimize the maximum mutual inner product between users. The set of scheduled users is denoted by $\mathcal{S}$ with $|\mathcal{S}| = U$.

$$\mathcal{S} = \arg\min_{\mathcal{S}'} \max_{\substack{k,j\in\mathcal{S}' \\ k\neq j}} \left|\mathbf{h}_k^{\mathrm{H}}\mathbf{h}_j\right| \tag{11}$$

The scheduled channel $\mathbf{H}$ consists of channel vectors of scheduled users $\mathbf{h}_k$ with $k \in \mathcal{S}$.

We assume a LOS channel, with the channel vector of user u being

$$\mathbf{h}_u = \left(e^{-ikd_{u,1}}, \dots, e^{-ikd_{u,N}}\right)^{\mathrm{T}}, \tag{12}$$

where k is the wave vector magnitude $k = {}^{2\pi}/_{\lambda}$ with the wavelength λ. Here, $d_{u,n}$ denotes the distance from user u to antenna element n. Please note that this channel model does not apply a far-field approximation in the sense that it does not assume planar wave fronts.

Large scale fading effects are considered via the free space path loss coefficient for user $u \in \{1, \dots, U\}$

$$\beta'_u = \left(\frac{\lambda}{4\pi d'_u}\right)^2, \tag{13}$$

where d'_u denotes the distance from user u to the BS. In order to obtain large scale fading coefficients with a mean of one, we employ the normalization $\beta_u = {}^{\beta'_u}/_{\overline{\beta}}$ where $\overline{\beta}$ denotes the mean path loss over all possible user positions within the specified sector. Employing this normalization of large scale fading, we may define the signal to noise ratio (SNR) as

$$\mathrm{SNR} = \frac{P_T}{U\sigma_w^2}, \tag{14}$$

which is the mean received SNR without array gain.

We show SE over SNR in Fig. 3. As result metric we consider the minimum SE over all scheduled users

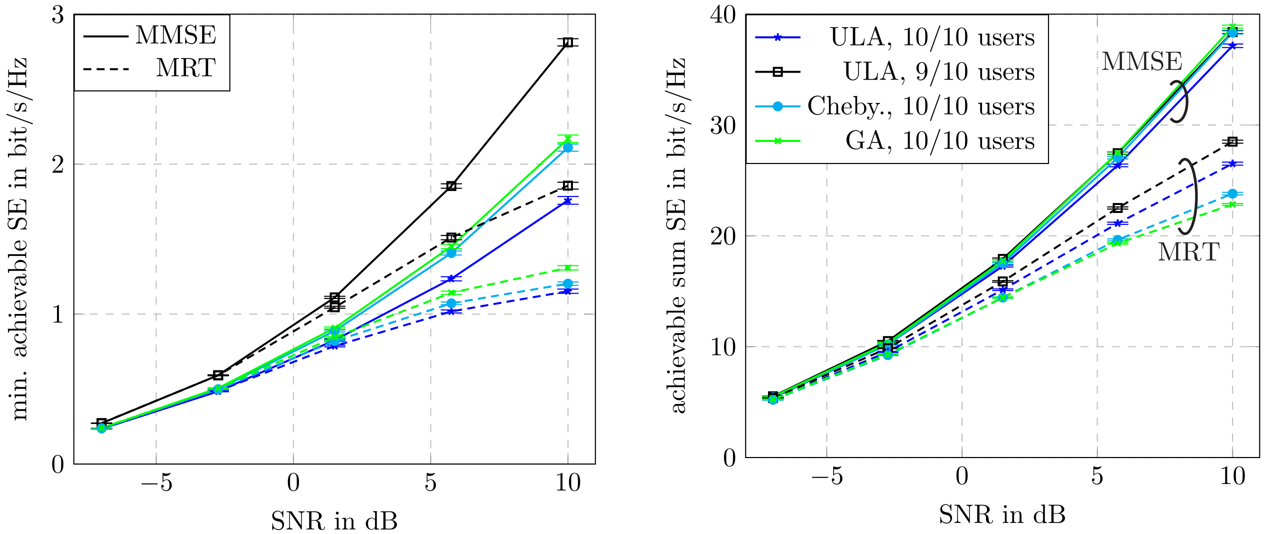
$$\mathrm{SE}_{\min} = \min_{u=\{1,\dots,U\}} \mathrm{SE}_u. \tag{15}$$

and the achievable sum SE

$$\mathrm{SE}_{\mathrm{sum}} = \sum_{u=1}^{U} \log_2\left(1 + \mathrm{SINR_u}\right). \tag{16}$$

Results in terms of minimum SE over SNR are shown in Fig. 3a and results for the achievable sum SE are shown in Fig. 3b. In both cases, we compare an MRT precoder and an MMSE precoder for different antenna array geometries.

For the minimum SE case, results for MRT precoding and for MMSE precoding show the same behavior: The *Chebychev* array outperforms the ULA in terms of minimum SE, while the proposed GA thinned array outperforms both of them. This shows that the arrays' narrower main beam width compared to the ULA indeed reduces the maximum inter-user interference. However, although the proposed thinned array has a wider beam width than the *Chebychev* array, the GA thinned array still performs better in terms of minimum SE. The lower SLL, see Fig. 1b, of the GA thinned array provides an even better performance with respect to inter-user interference. In Fig. 3a we also provide a comparison to the case of a ULA with only $U = 9$ users scheduled out of 10 available users according to (11). As this scheduling strategy leads to a minimization of the maximum inter-user interference power for equal power allocation among users, the minimum SE is significantly improved. However, the comparison is not a fair one with respect to fairness among users. In case there are only $9/10$ users scheduled, one user is not assigned any resources. Therefore, this comparison serves as a lower bound in terms of fairness among users and therefore as upper bound in terms of gain of minimum SE via scheduling. Still, considering antenna array design instead of user scheduling leads to a minimum SE in between the conventional ULA case and the scheduling $9/10$ approach. Further, the array design approach does not require channel knowledge and achieves perfect fairness without introducing transmission latency.



(a) Minimal achievable SE.

(b) Achievable sum SE.

Fig. 3. Achievable downlink SE with $N = 32$ assuming perfect channel knowledge for different array configurations with MRT and MMSE precoding. Simulation results are plotted with 95% confidence intervals.

Although the array design goal of this contribution is increased reliability in terms of increased minimum SE, the sum SE is also provided in Fig. 3b. Interestingly, both, the GA thinned and the *Chebychev* antenna array achieve the same sum SE as the ULA with user scheduling for MMSE precoding. Still, for this precoding scheme all those three strategies outperform the conventional ULA case. Unfortunately, the sum SE decreases when applying the *Chebychev* or the thinned array compared to the conventional ULA for MRT precoding. While the MMSE is able to exploit the narrow main lobe of the array in therms of sum SE, the MRT precoder suffers from the high SLLs for both, the *Chebychev* and the GA array.

V. Conclusion

By designing the antenna array of a massive MIMO system, we achieve a gain in minimum SE efficiency and thereby increase the system's reliability. This gain comes without cost of computational complexity, hardware complexity or energy consumption. Compared to maintaining a certain minimal user rate by means of scheduling, our proposed method facilitates uRLLC as it does not increase the transmission delay. The results show that even small improvements in the antenna array pattern lead to a significant impact in system performance. In terms of antenna pattern, the region around the main beam, and the width of the main beam, are especially important for the maximum inter-user interference power. We show that the proposed array design leads to improvements compared to other conventional designs.

References

[1] E. Björnson, E. G. Larsson, and T. L. Marzetta, "Massive MIMO: ten myths and one critical question," *IEEE Communications Magazine*, vol. 54, no. 2, pp. 114–123, Feb. 2016.

[2] P. Popovski, J. J. Nielsen, C. Stefanovic, E. D. Carvalho, E. Strom, K. F. Trillingsgaard, A.-S. Bana, D. M. Kim, R. Kotaba, J. Park *et al.*, "Wireless access for ultra-reliable low-latency communication: Principles and building blocks," *IEEE Network*, vol. 32, no. 2, pp. 16–23, 2018.

[3] M. Matthaiou, P. J. Smith, H. Q. Ngo, and H. Tataria, "Does massive MIMO fail in ricean channels?" *IEEE Wireless Communications Letters*, vol. 8, no. 1, pp. 61–64, 2018.

[4] D. Pinchera, M. D. Migliore, F. Schettino, and G. Panariello, "Antenna arrays for line-of-sight massive MIMO: Half wavelength is not enough," *MDPI Electronics*, vol. 6, no. 3, p. 57, 2017.

[5] C.-M. Chen, V. Volskiy, A. Chiumento, L. V. der Perre, G. A. Vandenbosch, and S. Pollin, "Exploration of user separation capabilities by distributed large antenna arrays," in *IEEE Globecom Workshops*, Washington, DC, USA, 2016, pp. 1–6.

[6] S. Pratschner, E. Zöchmann, H. Groll, S. Caban, S. Schwarz, and M. Rupp, "Does a large array aperture pay off in line-of-sight massive MIMO?" in *IEEE International Workshop on Signal Processing Advances in Wireless Communications (SPAWC 2019)*, Cannes, France, Jul. 2019, pp. 1–5.

[7] D. Löschenbrand, M.Hofer, L. Bernadó, G. Humer, B. Schrenk, S. Zelenbaba, and T. Zemen, "Distributed massive MIMO channel measurements in urban vehicular scenario," in *13th European Conference on Antennas and Propagation (EuCAP)*, Mar. 2019, pp. 1–5.

[8] H. Rezagholizadeh and D. Gharavian, "A thinning method of linear and planar array antennas to reduce SLL of radiation pattern by GWO and ICA algorithms," *AUT Journal of Electrical Engineering*, vol. 50, no. 2, pp. 135–140, 2018.

[9] A. Razavi and K. Forooraghi, "Thinned arrays using pattern search algorithms," *Progress In Electromagnetics Research*, vol. 78, pp. 61–71, 2008.

[10] R. Jain and G. S. Mani, "Solving antenna array thinning problem using genetic algorithm," *Applied Computational Intelligence and Soft Computing*, vol. 2012, p. 24, 2012.

[11] K. C. Lee and J. Y. Jhang, "Application of particle swarm algorithm to the optimization of unequally spaced antenna arrays," *Journal of Electromagnetic Waves and Applications*, vol. 20, no. 14, pp. 2001–2012, 2006.

[12] S. Pratschner, S. Caban, D. Schützenhöfer, M. Lerch, E. Zöchmann, and M. Rupp, "A fair comparison of virtual to full antenna array measurements," in *IEEE International Workshop on Signal Processing Advances in Wireless Communications (SPAWC 2018)*, Kalamata, Greece, Jun. 2018, pp. 1–5.