

# On Channel Estimators for Iterative CDMA Multiuser Receivers in Flat Rayleigh Fading

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**Abstract**—In this work we compare channel estimation algorithms for use in an iterative CDMA receiver in a block fading environment. The receiver consists of a soft multiuser data estimator, a bank of single user decoders, and a multiuser channel estimator. The multiuser data estimator is implemented as parallel interference canceler with unconditional post-MMSE filtering (PIC-MMSE) and the decoder is a soft-in soft-out MAP decoder. In the channel estimator we make use of dedicated pilot symbols and fed back soft-code symbols which are exploited as additional soft pilot symbols when the iterations proceed. We show that using *extrinsic* information increases the receiver performance significantly compared to using *a posteriori* information in the feedback for channel estimation. We introduce a linear MMSE (LMMSE) estimator which takes into account the variances of fed back code symbols and compare it to approximations of the least-squares (ALS) estimator and the linear minimum-mean-square-error (ALMMSE) estimator. Performance results are illustrated in terms of bit error rate (BER) and average normalized square error (ANSE) of the channel estimators. They show that the newly proposed LMMSE algorithm outperforms the ALS and ALMMSE algorithms.

## I. INTRODUCTION

Iterative receivers with joint detection and decoding for coded CDMA systems achieve very good performance results which outperform separated detection and decoding [1], [2], [3]. When extending them with a channel estimators, as illustrated in Fig.1, the receiver can operate successfully in realistic propagation scenarios [4]. Such a setup was first used in [5]. Estimation is based on pilot symbols and supported by soft updates of code symbols that serve as additional pilots. The aim of the present work is to give a structured approach to different channel estimation algorithms and compare them in terms of achievable BER and ANSE of the channel estimation. We will develop the following schemes: (A) approximated least-squares (ALS) and (B) approximated linear MMSE (ALMMSE) where we assume to have deterministic feed back symbols in both cases; (C) linear MMSE (LMMSE) approach which takes into account the variance of fed back soft symbols. We present the signal model in Section II and elaborate on the multiuser data estimator in Section III. The soft decoding issue is illuminated in Section IV before we derive the channel estimation algorithms in Section V. Finally, we illustrate simulation results in Section VI and draw the conclusions in Section VII.

## II. SIGNAL MODEL

We consider the uplink of a symbol synchronous DS-CDMA system with  $K$  users as illustrated in Fig.2.  $K$  users send blocks of  $M$  QPSK-modulated symbols  $b_k(m)$  where each symbol is spread by a random spreading sequence  $s_k$  of length  $N$ . The elements of the spreading sequences are i.i.d. and belong to the QPSK constellation set  $\{\pm 1 \pm j\}/\sqrt{2N}$  satisfying  $\sum_{n=1}^N |s_k[n]|^2 = 1$ . The first  $J$  of the  $M$  QPSK symbols in each block are training symbols. All training symbols  $b_{k,\text{pilot}}(m)$  have the value  $p = (1+j)/\sqrt{2}$ . The remaining  $M - J$  symbols  $b_{k,\text{code}}(m)$  are obtained by Gray mapping of encoded data streams  $c_k(m')$  onto a QPSK constellation. The encoded data stream  $c_k(m')$  is obtained from coding the raw data stream  $d_k(m'')$  with a non-systematic, non-recursive, convolutional code with rate  $R = 1/2$ . The encoder has generator polynomials  $(5, 7)_8$  and is followed by a random interleaver.

We assume that every user is exposed to flat Rayleigh fading.  $h_k$  denotes the  $k$ th user's channel and we assume that it remains constant over  $M$  symbols.

The received signal  $\mathbf{y}(m)$  at symbol time  $m$  can be written as

$$\mathbf{y}(m) = \mathbf{S}\mathbf{B}(m)\mathbf{h} + \mathbf{v}(m) \quad (1)$$

where

- $\mathbf{S}$  [ $N \times K$ ] is the spreading matrix. The  $k$ th column represents the spreading sequence  $s_k$  of user  $k$ .
- $\mathbf{B}(m)$  [ $K \times K$ ] is a diagonal matrix with code or pilot symbols of the  $K$  users at time  $m$ .
- $\mathbf{h}$  [ $K \times 1$ ] denotes the vector of all the users' fading taps with variance  $\sigma_h^2$ .
- $\mathbf{v}(m)$  [ $N \times 1$ ] denotes the complex additive white Gaussian noise vector with covariance matrix  $\sigma_v^2 \mathbf{I}_N$ .

## III. DATA ESTIMATION

For multiuser data estimation we deploy parallel-interference cancellation with post LMMSE filtering. Interference cancellation is a feasible approach since the output of the single user decoders can be used to create an estimate of the multiple-access interference. The feedback structure is illustrated in Fig. 1. Multiuser receivers of this kind have a moderate complexity, achieving, at the same time,

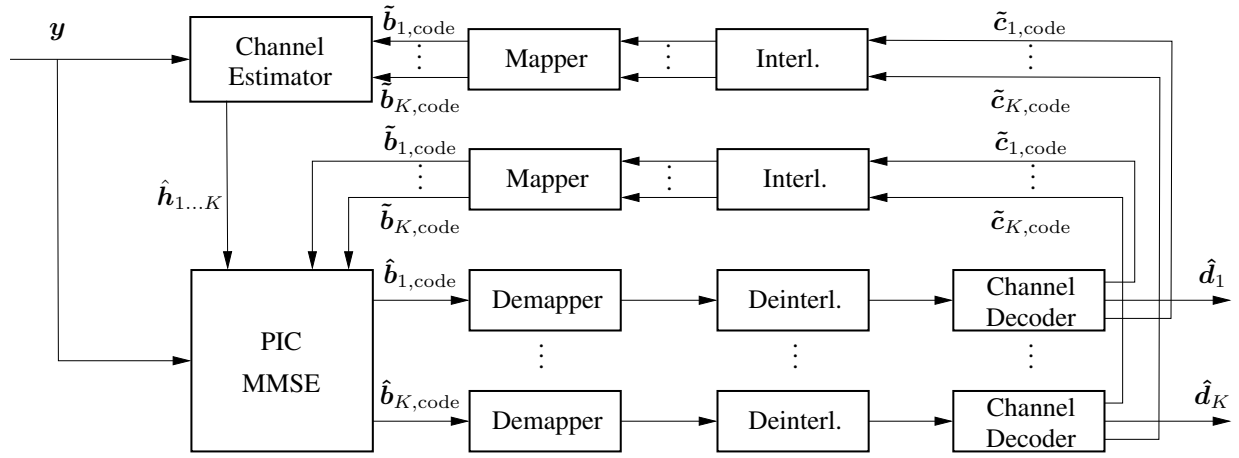


Fig. 1. Iterative receiver structure.

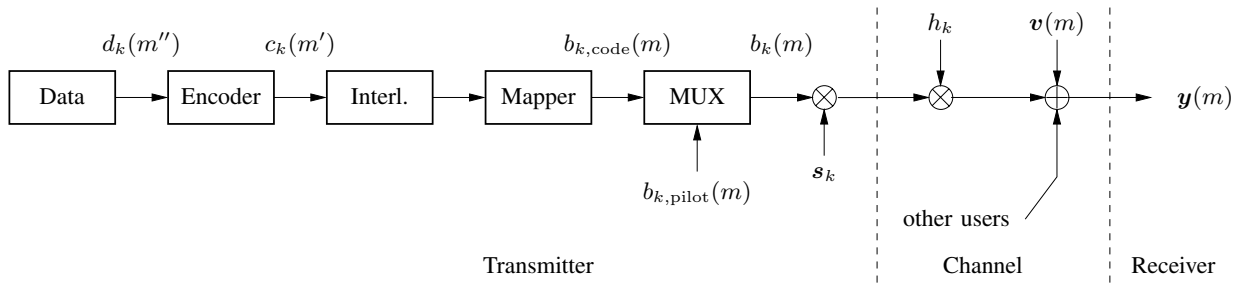


Fig. 2. Transmit situation in the synchronous CDMA uplink.

very good performance. Hence, they offer a practical way to overcome the defying complexity of optimum maximum-likelihood multiuser detection. We employ an LMMSE filter after interference cancellation since it is the optimum way to increase the output SINR.

We formulate the soft interference-canceled observation vector for user  $k$  at iteration  $i$  as

$$\tilde{\mathbf{y}}_k^{(i)}(m) = \mathbf{y}(m) - \tilde{\mathbf{S}}^{(i)} \tilde{\mathbf{b}}^{(i)}(m) + \tilde{\mathbf{s}}_k^{(i)} \tilde{b}_k^{(i)}(m)$$

and apply it to the  $M - J$  code symbols. Here the effective spreading sequence is

$$\tilde{\mathbf{S}}^{(i)} = \mathbf{S} \text{diag}(\hat{\mathbf{h}}^{(i)}) \quad (2)$$

with  $\hat{\mathbf{h}}^{(i)}$  being the vector of channel estimates that we receive from the channel estimator.  $\tilde{\mathbf{b}}^{(i)}(m)$  is the vector with estimates  $\tilde{b}_k^{(i)}(m)$  of the code symbols and is obtained by mapping the soft decoder output probability. Details will be given in Section IV. After cancellation we pass  $\tilde{\mathbf{y}}_k^{(i)}(m)$  through a filter

$$\hat{b}_k^{(i+1)}(m) = (\mathbf{w}_k^{(i)})^H \tilde{\mathbf{y}}_k^{(i)}(m)$$

to obtain a soft code symbol estimate. The linear filter is the solution to the MMSE criterion

$$\mathbf{w}_k^{(i)}(m) = \underset{\mathbf{w}}{\text{argmin}} \mathbb{E} \left\{ \left| b_k(m) - \mathbf{w}^H \tilde{\mathbf{y}}_k^{(i)}(m) \right|^2 \right\}.$$

The *conditional* [6] unbiased representation becomes

$$\mathbf{w}_k^{(i)}(m) = \frac{\tilde{\mathbf{s}}_k^H (\tilde{\mathbf{S}} \mathbf{V}(m) \tilde{\mathbf{S}}^H + \sigma_v^2 \mathbf{I})^{-1}}{\tilde{\mathbf{s}}_k^H (\tilde{\mathbf{S}} \mathbf{V}(m) \tilde{\mathbf{S}}^H + \sigma_v^2 \mathbf{I})^{-1} \tilde{\mathbf{s}}_k}$$

where the diagonal covariance matrix  $\mathbf{V}(m)$  is defined as  $\mathbb{E} \left\{ (\mathbf{b}(m) - \tilde{\mathbf{b}}^{(i)}(m)) (\mathbf{b}(m) - \tilde{\mathbf{b}}^{(i)}(m))^H \right\}$  with dimension  $[K \times K]$ . Instead of computing  $\mathbf{V}(m)$  for every symbol instance individually, we can use  $V_{k,k} = \mathbb{E}\{1 - |\tilde{b}_k^{(i)}|^2\}$  for the diagonal entries of  $\mathbf{V}$ . The resulting filter is termed *unconditional*. It has to be computed once for every iteration and each user only.

#### IV. DECODING

The soft feedback values supplied to the channel and the data estimator are computed from *a posteriori* probabilities (APP) and *extrinsic* probabilities (EXT). A SISO decoder for binary convolutional codes, implemented using the BCJR algorithm [7], supplies these measures. The decoder inputs are the channel values  $z_k(i)$  which are obtained after we demap, deinterleave, and rescale the  $\hat{b}_k(m)$  by  $\sqrt{2}$  to sequential BPSK symbols. They represent the received values for the  $2(M - J)$  polar code bits  $c_k(m')$ . In the decoder we assume that the code bits were distorted by AWGN which includes the effects

of MAI and thermal noise. The channel is characterized by the estimated noise variance  $\nu_k^2$ .

Let  $\{-1, +1\}$  be the possible real signal points for the code bits. Then the APP for having the value  $+1$  as code bit when observing the channel value  $z_k(i)$  is given as  $\text{APP}_k(i) = \Pr(c_k(i) = +1 | z_k)$ . The link between *a posteriori* probability and *extrinsic* information is established via the relation

$$\text{APP}_k(m') \propto \text{EXT}_k(m') f(z_k(m') | c_k(m') = +1)$$

where the last expression is approximated by a Gaussian channel transition function

$$f(z_k(m') | c_k(m') = +1) = \frac{1}{\sqrt{2\pi\nu_k^2}} \exp\left(-\frac{|z_k(m') - 1|^2}{2\nu_k^2}\right).$$

The feedback symbols for the data estimator and the channel estimator are computed from the *a posteriori* and/or *extrinsic* probabilities (see more in Section VI-A). The mapping of *a posteriori* probabilities to soft symbols is done via

$$\tilde{b}_{k,\text{code}}(m) = \frac{1}{\sqrt{2}} \left( \begin{array}{l} 2 \text{APP}_k(2m') - 1 + \\ j(2 \text{APP}_k(2m' + 1) - 1) \end{array} \right)$$

and this works analogously with EXTs measures.

## V. CHANNEL ESTIMATION

The importance of having a good channel estimate is made obvious by (2). The filter matched to the effective spreading sequence directly depends on the quality of the channel estimate. We perform channel estimation based on dedicated pilot symbols and fed back code symbols. The pilot symbols are located in a preamble and take the first  $J$  symbols of the complete symbol block of length  $M$ . After the first iteration the soft decisions on the code symbols are also taken into account in the channel estimator. The linear model (1) can be transferred into a super-matrix representation

$$\mathbf{y} = \mathbf{S}\mathbf{B}\mathbf{h} + \mathbf{v} \quad (3)$$

which is convenient for developing the three channel estimation algorithms illustrated in the sequel. The variables are:

- $\mathbf{y}$  [ $NM \times 1$ ] the vertically stacked observation vectors  $\mathbf{y}(m)$ .
- $\mathbf{S}$  [ $NM \times KM$ ] a block diagonal matrix with the spreading matrix  $\mathbf{S}$  along the main diagonal.
- $\mathbf{B}$  [ $KM \times K$ ] the vertically stacked code symbol matrices  $\mathbf{B}(m)$ .
- $\mathbf{v}$  [ $NM \times 1$ ] the  $M$  vertically stacked noise vectors  $\mathbf{v}(m)$ .

### A. Approximated Least-Squares (ALS) Estimation

For this approach the matrix  $\mathbf{B}$  is replaced by the pilots and *a posteriori* based soft code symbols  $\tilde{\mathbf{B}}$ . No assumptions are made about the statistics of the fading nor of the additive white Gaussian noise. When we introduce

$$\mathbf{A} = \mathbf{S}\tilde{\mathbf{B}}$$

the ALS solution becomes

$$\hat{\mathbf{h}}_{\text{ALS}} = (\mathbf{A}^H \mathbf{A})^{-1} \mathbf{A}^H \mathbf{y}. \quad (4)$$

### B. Approximated Linear MMSE Estimation (ALMMSE)

We assume i.i.d. and zero mean Rayleigh fading taps and a deterministic matrix  $\mathbf{A}$ . Latter assumption is an approximation since the soft feed back elements are stochastic. If the transmitted symbols were correctly at hand we would get the true MMSE estimator. The approximate MMSE solution is

$$\hat{\mathbf{h}}_{\text{ALMMSE}} = (\mathbf{A}^H \mathbf{A} + \sigma_v^2 \mathbf{I})^{-1} \mathbf{A}^H \mathbf{y}. \quad (5)$$

In a real-world system this estimator requires an estimate for the noise variance. In our comparison we assume to know the exact noise power.

### C. Linear MMSE Estimation (LMMSE)

A more advanced method than the ALS and ALMMSE takes into account the variance of the soft feedback code symbols [8],[9]. This is the objective of the linear estimator

$$\hat{\mathbf{h}}_{\text{LMMSE}} = \mathbf{C}_{yh}^H \mathbf{C}_{yy}^{-1} \mathbf{y}. \quad (6)$$

Under the assumption that  $\mathbf{y}$ ,  $\mathbf{v}$  are zero mean, the covariance matrices are given by

$$\begin{aligned} \mathbf{C}_{yy} &= \mathbb{E}_{\mathbf{B}, \mathbf{h}, \mathbf{v}} \{ \mathbf{y} \mathbf{y}^H \} \\ &= \mathbb{E}_{\mathbf{B}, \mathbf{h}, \mathbf{v}} \{ \mathbf{S} \mathbf{B} \mathbf{h} \mathbf{h}^H \mathbf{B}^H \mathbf{S}^H + \mathbf{v} \mathbf{v}^H \} \\ &= \mathbf{S} \mathbb{E}_{\mathbf{B}} \{ \mathbf{B} \mathbf{T} \mathbf{B}^H \} \mathbf{S}^H + \sigma_v^2 \mathbf{I}, \end{aligned}$$

$$\begin{aligned} \mathbf{C}_{yh} &= \mathbb{E}_{\mathbf{B}, \mathbf{h}, \mathbf{v}} \{ \mathbf{y} \mathbf{h}^H \} \\ &= \mathbb{E}_{\mathbf{B}, \mathbf{h}, \mathbf{v}} \{ \mathbf{S} \mathbf{B} \mathbf{h} \mathbf{h}^H + \mathbf{v} \mathbf{h}^H \} \\ &= \mathbf{S} \tilde{\mathbf{B}} \mathbf{T}, \end{aligned}$$

with  $\mathbf{T} = \text{diag}(\sigma_{h,1}^2, \sigma_{h,2}^2, \dots, \sigma_{h,K}^2)$ . In order to come up with the solutions we need to compute the expectation  $\mathbb{E} \{ \mathbf{B} \mathbf{T} \mathbf{B}^H \}$ . If we assume independency among the individual code symbols it holds:

$$\mathbb{E} \{ b_p(m) b_q(n) \} = \begin{cases} \tilde{b}_p(m) \tilde{b}_q(n) & \text{for } p \neq q, m \neq n \\ 1 & \text{for } i = j, m = n. \end{cases}$$

$p, q \in \{1 \dots K\}$  denote the user indices;  $m, n \in \{J+1 \dots M\}$  stand for the symbol indices. The product  $\mathbb{E} \{ \mathbf{B} \mathbf{T} \mathbf{B}^H \}$  can be expressed as

$$\mathbb{E} \{ \mathbf{B} \mathbf{T} \mathbf{B}^H \} = \mathbb{E} \{ \mathbf{B} \} \mathbf{T} \mathbb{E} \{ \mathbf{B}^H \} + \mathbf{\Lambda} = \tilde{\mathbf{B}} \mathbf{T} \tilde{\mathbf{B}}^H + \mathbf{\Lambda}$$

where the matrix  $\mathbf{\Lambda} \in \mathbb{R}^{KM \times KM}$  denotes a diagonal matrix. The individual entries on the diagonals are the symbol variances weighted with the variance of the corresponding fading tap:  $\sigma_{h,k}^2 \sigma_{b,k}^2(m) = \sigma_{h,k}^2 (1 - |\tilde{b}_k(m)|^2)$ . There are  $MK$  diagonal entries and in vector form they become

$$[\sigma_{h,1}^2 \sigma_{b,1}^2(1), \sigma_{h,2}^2 \sigma_{b,2}^2(1), \dots, \sigma_{h,K}^2 \sigma_{b,K}^2(M)].$$

## VI. SIMULATION RESULTS

The combined iterative channel estimator and detector/decoder is tested in a system where all users have spreading sequence length  $N = 8$  for the uplink in a flat Rayleigh fading environment. The overall power for one user is  $\mathbb{E}\{|h_k|^2\} = 1$ . The number of iterations is limited to 7. The single-user bound ('SUBound') is taken as a reference for the receiver performance and different parameter settings. In this context the single-user bound is defined as the receiver performance with one user  $K = 1$  with perfect channel knowledge. The simulation results were obtained by averaging over 100 different channel realizations. The  $E_b/N_0$  is defined as

$$\frac{E_b}{N_0} = \frac{1}{\sigma_v^2 R} \frac{M}{M - J}.$$

We assume an overloaded system with load  $\alpha = K/N = 12/8 = 1.5$  and block length  $M = 100$ . The number of dedicated pilot symbols is  $J = 10$ .

### A. Comparison of Feedback Information

In a great number of papers soft symbols were derived by mapping *a posteriori* information to soft bits for MUD-PIC processing as well as channel estimation. It was observed that this leads to cancellation of useful signal parts and several ways of processing *a posteriori* information in a better way were sought. The work by Boutros and Caire [10] shows that the iterative receiver (without channel estimation) can be viewed as an instance of the belief propagation algorithms which computes exact *a posteriori* information in the case when the corresponding factor graph does not have cycles. This is approximately fulfilled when the block length  $M$  is sufficiently large. The crucial requirement is that *extrinsic* rather than *a posteriori* information is fed back to the multiuser-detector. We show that this also holds for feeding back information to the channel estimator. Fig.3 shows the performance of the iterative receiver with the LMMSE channel estimator derived in Section V. We observe that after the 3rd and the 7th iteration the channel estimator with *extrinsic* information outperforms the *a posteriori* based estimation by far. This becomes even more pronounced when we keep the amplitude constant and assign just a random phase to the individual taps, i.e.  $h_k = e^{j\phi_k}$ . Fig.4 shows that *extrinsic* feedback becomes superior to *a posteriori* feedback for values of  $E_b/N_0$  greater than 5 dB. In the two graphs we have also plotted the performance for a one-shot structure where channel estimation is based on pilot symbols only ('No FB'). In this case we do not iterate at all. Further we also plot the BER in the case where we feed back soft information only to the data estimator but not to the channel estimator. As channel estimate we use the value that we obtained from the MMSE pilot based estimation ('Pilots'). We see that there is hardly any improvement when we compare the BER after iteration 1 and iteration 3.

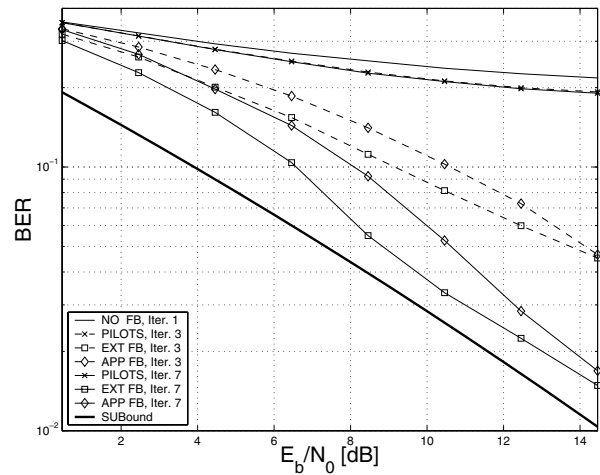


Fig. 3. BER with *a posteriori* and *extrinsic* feedback in combination with the LMMSE estimator in Rayleigh fading.  $\alpha = 1.5$ ,  $J/M = 10\%$ .

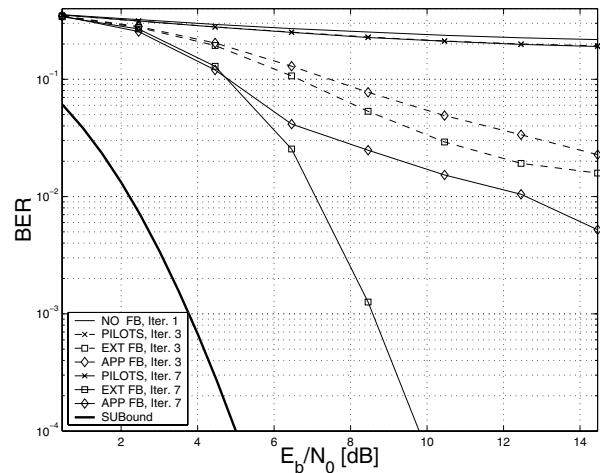


Fig. 4. BER with *a posteriori* and *extrinsic* feedback in combination with the LMMSE estimator with  $h_k = e^{j\phi_k}$ .  $\alpha = 1.5$ ,  $J/M = 10\%$ .

### B. Comparison of Channel Estimators

We compare the performance of the iterative receiver with different channel estimators in Rayleigh fading with respect to BER and ANSE. Figs.5 and 6 show the BER of the receivers after the 3rd and the 7th iteration, respectively. The considered estimators make use of soft-data information as additional pilot symbols. Interestingly, feeding back soft-symbols to the ALS estimator deteriorates the performance after the 3rd iteration. The weak feedback values are enhanced and contribute a lot while strong feedback values with low uncertainty are not enhanced. This does not happen with the ALMMSE and LMMSE schemes. For these we notice a significant drop in BER already after the 3rd iteration. The LMMSE scheme which considers the symbol variances approaches the SUB in the fastest way.

We measure the quality of the channel estimation in terms of average normalized square error (ANSE) and define it as  $\text{ANSE} = \mathbb{E}_h \left\{ \|\mathbf{h} - \hat{\mathbf{h}}\|^2 / \|\mathbf{h}\|^2 \right\}$ . The ANSE is plotted vs. the



iterations in Fig.7. We illustrate the ANSE of all estimation schemes at 0.458 dB with dashed lines and at 14.458 dB with solid lines. We can observe that all estimation schemes except the LMMSE make wrong use of the feedback information since the ANSE always increases at the 2nd iteration. As for the BER the LMMSE shows the fastest convergence towards a low ANSE. The ALMMSE is able to reach the same performance after 7 iterations at a rather high  $E_b/N_0$  of 14.458 dB.

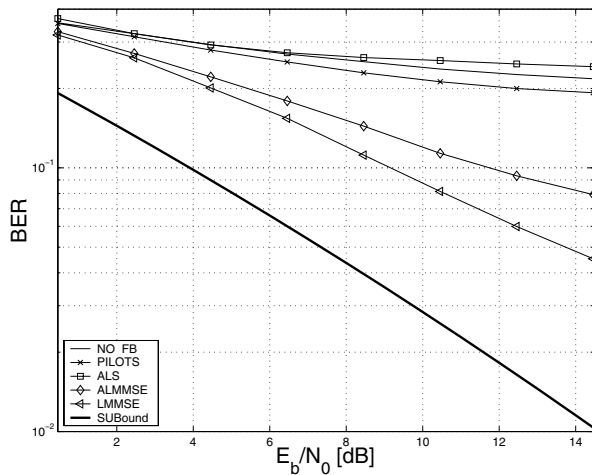


Fig. 5. BER after 3rd iteration.  $\alpha = 1.5$ ,  $J/M = 10$  %.

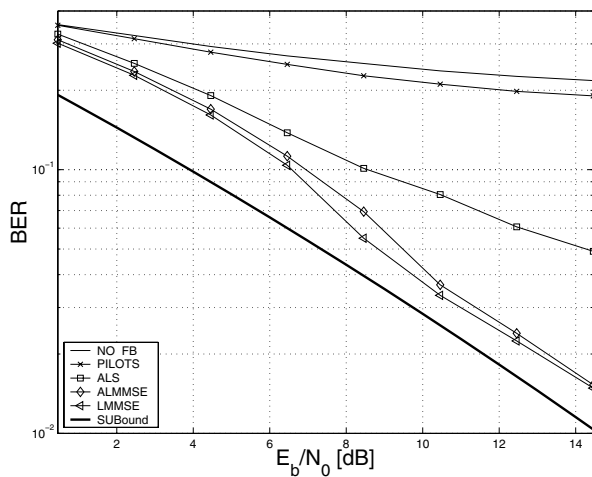


Fig. 6. BER after 7th iteration.  $\alpha = 1.5$ ,  $J/M = 10$  %.

## VII. CONCLUSIONS

In this work we have considered several channel estimation methods suitable for iterative CDMA receiver concepts which include joint channel estimation, multiuser data estimation and decoding. The channel is supposed to be a flat Rayleigh fading channel and stays constant during the transmission of the considered symbol block. In addition to the dedicated pilot symbols, the estimators exploit the soft values of feedback code bits which can be used as additional pilot symbols.

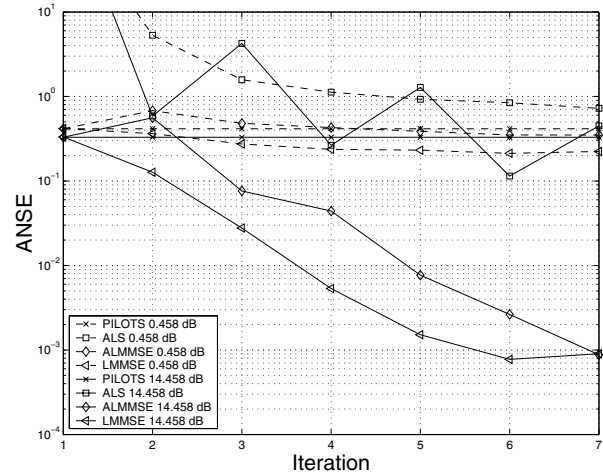


Fig. 7. ANSE vs. iteration.  $\alpha = 1.5$ ,  $J/M = 10$  %.

The results show that for channel estimation the feedback symbols should be mapped from *extrinsic* information rather than from *a posteriori* information. Furthermore, we find that the proposed LMMSE estimator, which makes use of the soft-symbol variances and the fading tap variances, achieves a faster convergence towards the single user bound than the other considered schemes, ALMMSE and ALS.

## VIII. ACKNOWLEDGMENT

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