

# LOW-COMPLEXITY JOINT DATA DETECTION AND CHANNEL ESTIMATION IN TIME-VARYING FLAT-FADING CHANNELS WITHIN THE SAGE FRAMEWORK

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## ABSTRACT

We derive a multi-user receiver that performs joint data detection and channel estimation (JDE) of DS-CDMA signals. The proposed sub-optimal receiver is formulated within the framework of the space-alternating generalized expectation-maximization (SAGE) algorithm. The time-varying channel is represented by discrete prolate spherical (DPS) sequences. The resulting receiver iterates between MMSE based channel estimation in the subspace spanned by the DPS sequences and successive interference cancellation at significantly less time complexity than previously proposed SAGE based JDE schemes. The reduction is in the order of 200 for the investigated system parameters. Numerical examples show that the proposed receiver works efficiently in a wide range of velocities and system loads.

## 1. INTRODUCTION

The jointly optimum multiuser detector performs maximum likelihood sequences decisions [1]. Its non-polynomial time complexity in the number of users, however, prevents any practical application. Iterative signal processing, in contrast, combines computationally low-complex blocks that exchange soft information in a very efficient way. Lively examples in communications are iterative multiuser detection, iterative (Turbo)-equalization, and iterative (Turbo)-decoding. Focusing on the first two issues, several heuristic iterative receivers, iterating between channel estimation and multiuser detection, have been proposed in the past. An overview can be found in [2].

The space-alternating generalized expectation-maximization (SAGE) algorithm [3] is an iterative method to approximate the maximum likelihood estimate of a parameter when its direct computation is computationally prohibitive. At the same time, the SAGE algorithm exhibits the so-called monotonicity property that ensures convergence to a fix-point of the likelihood function. A near-far resistant and robust JDE scheme of time-varying signals for DS-CDMA based on the SAGE algorithm is investigated in [2, 4]. This paper addresses a low-complex version of that receiver in order to make it implementable on a digital signal processing device in a mobile terminal.

In mobile communication systems the time-selective fading process is highly oversampled. Time-limited snapshots of the fading process with length of a data block span a subspace with small dimension. The same subspace is also spanned by index-limited discrete prolate spheroidal (DPS) sequences [5]. The energy of the DPS sequences is time-concentrated in an interval equal to the length of a data

block [6]. The band-limitation of the DPS sequences is chosen according to the maximum support of the power spectral density of the time-selective fading process. The subspace dimension for practical communication systems is in the order of two to five only. This allows drastic complexity reduction for time-varying channel estimation as we will show in the sequel.

## Contributions of the Paper

- We apply subspace-based channel estimation that utilizes Slepian sequences in the iterative SAGE receiver.
- The design of the sequences only depends on the maximum relative velocity  $v_{\max}$  and assumes a flat Doppler spectrum. This design proves to be robust against mismatches in the second-order statistics of actual channel realizations.
- We show that this approach reduces complexity considerably compared to the time-domain implementation presented in [2], while achieving similar performance.

## Notation

We denote a column vector by  $\mathbf{a}$  and its  $i$ -th element with  $a[i]$ . Equivalently, we denote a matrix by  $\mathbf{A}$  and its  $(i, m)$ -th element by  $[\mathbf{A}]_{i,m}$ . The transpose of  $\mathbf{A}$  is given by  $\mathbf{A}^T$ , its conjugate transpose by  $\mathbf{A}^H$ . A diagonal matrix with elements  $a[i]$  is written as  $\text{diag}(\mathbf{a})$  and the  $Q \times Q$  identity matrix as  $\mathbf{I}_Q$ . The absolute value of  $a$  is denoted by  $|a|$  and its complex conjugate by  $a^*$ . The largest (smallest) integer lower (greater) or equal to  $a \in \mathbb{R}$  is denoted by  $\lfloor a \rfloor$  ( $\lceil a \rceil$ ). The symbol  $\otimes$  denotes the Kronecker product. The expectation operator is denoted by  $\mathbb{E}\{\cdot\}$ . The operator  $\text{col}\{\cdot\}$  stacks all elements in a column vector. We denote the set of all integers by  $\mathbb{Z}$ , the set of real numbers by  $\mathbb{R}$  and the set of complex numbers by  $\mathbb{C}$ .

## 2. SYSTEM MODEL

We consider the uplink of a synchronous CDMA system with  $K$  active users each having assigned long signature waveforms  $s_{k,\ell}(t) \in \mathbb{R}$  that are normalized to have unit energy on signaling interval  $\ell$ , i.e.,  $\int_{\ell T_s}^{(\ell+1)T_s} |s_{k,\ell}(t)|^2 dt = 1$ ,  $k = 1, \dots, K$ . Different users transmit equiprobable data sequences  $\mathbf{d}_k$  of length  $L$  over different time-varying frequency-flat Rayleigh fading channels. The normalized frequency shift caused by the time-varying channel is upper bounded by the maximum normalized Doppler bandwidth

$$\nu_{D\max} = \frac{v_{\max}}{c_0} f_C T_s \quad (1)$$

with  $f_C$  denoting the carrier frequency,  $v_{\max}$  being the maximum relative speed between the mobile and the base-station and  $c_0$  standing for the speed of light.

The complex baseband representation of the received signal, obtained by an omni-directional antenna, reads

$$r(t) = \sum_{k=1}^K \sum_{\ell=0}^{L-1} \alpha_k(t) d_k[\ell] s_k(t - \ell T_S) + w(t).$$

In the above expression  $d_k[\ell] \in \{-1, +1\}$  denotes the symbol transmitted by the  $k$ -th user during the  $\ell$ -th signaling interval. The complex gain  $\alpha_k(t)$  characterizes the  $k$ -th user's flat fading channel. We model  $\alpha_k(t)$  as i.i.d. complex, circularly symmetric, Gaussian random processes with variance  $\text{var}\{\alpha_k\} = 1$  and equal second-order statistics. Finally,  $w(t)$  represents complex-valued white Gaussian noise with one sided power spectral density  $N_0$ .

Let the column vector  $\mathbf{z}[\ell] \triangleq \text{col}\{z_1[\ell], \dots, z_K[\ell]\} \in \mathbb{C}^K$  contain the output samples of  $K$  matched filters (MF) in signaling interval  $\ell$  corresponding to the sample instant  $(\ell + 1)T_S$ . It follows from (2) that

$$\mathbf{z}[\ell] \triangleq \mathbf{R}[\ell] \text{diag}(\mathbf{d}[\ell]) \mathbf{a}[\ell] + \mathbf{n}[\ell]. \quad (2)$$

In this expression, the matrix  $\mathbf{R}[\ell] \in \mathbb{R}^{K \times K}$  is of the form

$$\mathbf{R}[\ell] \triangleq \begin{pmatrix} 1 & \dots & \rho_{1,K}[\ell] \\ \vdots & \ddots & \vdots \\ \rho_{K,1}[\ell] & \dots & 1 \end{pmatrix} \quad (3)$$

where

$$\rho_{j,k}[\ell] \triangleq \int_{\ell T_S}^{(\ell+1)T_S} s_{j,\ell}(t - \ell T_S) s_{k,\ell}(t - \ell T_S) dt$$

denotes the cross-correlation between the signature waveforms  $s_{j,\ell}(t)$  of user  $j$  and  $s_{k,\ell}(t)$  of user  $k$  in time interval  $\ell$ . The vector

$$\mathbf{d}[\ell] = \text{col}\{d_1[\ell], \dots, d_K[\ell]\} \in \mathbb{C}^K$$

described the data symbols of the users and the vector

$$\mathbf{a}[\ell] = \text{col}\{a_1[\ell], \dots, a_K[\ell]\} \in \mathbb{C}^K$$

contains the samples in signaling interval  $\ell$  of the complex channel gains  $a_k[\ell] = \alpha_k(\ell T_S)$ ,  $\ell = 0, \dots, L-1$ . Finally,  $\mathbf{n}[\ell] \in \mathbb{C}^K$  is a complex zero-mean Gaussian random vector with covariance matrix  $N_0 \mathbf{R}[\ell]$  [1]. Because of the above assumptions, the channel of each user has the same covariance function  $R_a[\Delta\ell] = \mathbf{E}\{a_k[\ell]^* a_k[\ell + \Delta\ell]\}$ ,  $k = 1, \dots, K$  with power spectral density

$$S_a(\nu) = \sum_{\Delta\ell=-\infty}^{\infty} R_a(\Delta\ell) e^{-j2\pi\Delta\ell\nu}$$

in the fundamental interval  $|\nu| \leq 1/2$ . For the sake of convenience, we collect the  $k$ -th user's channel coefficients of a single data block,  $\ell \in \mathcal{I}_L$ ,  $\mathcal{I}_L = \{0, \dots, L-1\}$  in the vector

$$\mathbf{a}_k = \text{col}\{a_k[0], \dots, a_k[L-1]\}.$$

Hence, the entries of the covariance matrix for  $\mathbf{a}_k$  read

$$[\boldsymbol{\Sigma}_a]_{\ell, \ell + \Delta\ell} = R_a[\Delta\ell] \quad \ell = 0, \dots, L-1.$$

In [2, 7] it was shown that channel estimation dominates the computational complexity in the SAGE based JDE receiver. To reduce complexity, we consider a low dimensional subspace-based approximation of the time-varying channel  $\mathbf{a}_k$ . We project the vector  $\mathbf{a}_k$  onto  $D \ll L$  orthonormal basis vectors  $\mathbf{u}_m = \text{col}\{u_m[0], \dots, u_m[L-1]\} \in \mathbb{R}$ ,  $m = 0, \dots, D-1$ :

$$\mathbf{a}_k \approx \sum_{m=0}^{D-1} g_{k,m} \mathbf{u}_m, \quad (4)$$

where  $g_{k,m}$  are the basis expansion coefficients. It was shown in [5, 8] that index limited DPS sequences, subsequently referred to as *Slepian basis functions*, are ideally suited for expansion of the time-varying channel vector  $\mathbf{a}_k$ . The DPS sequences  $\{u_m[\ell]\}$ ,  $\ell \in \mathbb{Z}$  are band-limited to the interval  $(-\nu_{D\max}, \nu_{D\max})$  and have maximum energy concentration within  $\mathcal{I}_L$  [6]. The  $m$ -th Slepian basis function  $\mathbf{u}_m$  satisfies

$$\mathbf{C} \mathbf{u}_m = \lambda_m \mathbf{u}_m$$

where  $\lambda_m$  denotes its corresponding eigenvector and the elements of the matrix  $\mathbf{C} \in \mathbb{R}^{L \times L}$  are given by

$$[\mathbf{C}]_{i,j} = \frac{\sin(2\pi(i-j)\nu_{D\max})}{\pi(i-j)}, \quad i, j \in \mathcal{I}_L.$$

Notice that  $\mathbf{C}$  results when the Doppler spectrum of the path weights is constant. This spectrum achieves maximum entropy among the families of Doppler spectra bandlimited to  $(-\nu_{D\max}, +\nu_{D\max})$ . The eigenvalue spectrum  $\lambda_m$ ,  $m = 0, \dots, L-1$  has exponential decay with essentially  $D$  dominant values [6]. In fact, the dimension  $D$  of the subspace is a function of the prevailing SINR in the channel estimator, and can be lower bounded by [5, 8]

$$D' = \lceil 2\nu_{D\max} L \rceil + 1.$$

Inserting the right-hand-side of (4) into (2), it follows for the output signal of the MF bank

$$\mathbf{z}[\ell] = \mathbf{R}[\ell] \mathbf{D}[\ell] \mathbf{g} + \mathbf{n}[\ell]. \quad (5)$$

The short-cut  $\mathbf{D}[\ell] \in \mathbb{C}^{K \times KD}$  stands for  $\mathbf{D}[\ell] \triangleq [u_0[\ell], \dots, u_{D-1}[\ell]] \otimes \text{diag}(\mathbf{d}[\ell])$  and

$$\mathbf{g} = \text{col}\{g_{1,0}, \dots, g_{K,0}, \dots, g_{1,D-1}, \dots, g_{K,D-1}\} \in \mathbb{C}^{KD}$$

contains the basis expansion coefficients, subsequently referred as channel coefficients in the Slepian-domain. Their covariance matrix reads

$$\boldsymbol{\Sigma}_g = \frac{1}{2\nu_{D\max}} \text{diag}([\lambda_0, \dots, \lambda_{D-1}]) \otimes \mathbf{I}_K.$$

The off-diagonal elements of  $\boldsymbol{\Sigma}_g$  are zero since we assume no knowledge about the second-order statistics of the channels at the receive side (other than  $v_{\max}$ ).

### 3. THE SLEPIAN-DOMAIN SAGE-JDE RECEIVER

The SAGE algorithm [3] is used to iteratively approximate the maximum-likelihood (ML) estimate of  $\mathbf{d} \triangleq \text{col}\{\mathbf{d}_1, \dots, \mathbf{d}_K\} \in \{-1, +1\}^{KL}$ . At iteration  $i$ , the SAGE algorithm only re-estimates the symbol vector  $\mathbf{d}_k$  of user  $k = k[i] = i \bmod (K+1)$  while the estimate of the vector  $\mathbf{d}_{\bar{k}}$ , containing the symbols of the other users, is not updated. The SAGE algorithm relies on the concept of a (hypothetical) admissible hidden data  $\mathbf{x}_k$  with respect to  $\mathbf{d}_k$  to which

Table 1: NUMBER OF CMACs PER USER AND BIT FOR THE TIME-DOMAIN (TD) SAGE-JDE RECEIVER [2].

Operation	CMAC per user and information bit
$\mathbf{a}^{[0]}$ [2, (6.24)]	$\frac{L_P^3}{3(L-L_P)}K^2 + \frac{L_P^2}{(L-L_P)}K + \frac{5L_P}{2(L-L_P)}K + \frac{L_P^2}{2(L-L_P)} + \frac{5L_P}{2(L-L_P)} - \frac{1}{3(L-L_P)}K^{-1}$
E-step [2, (6.24)]	$\frac{s}{3(L-L_P)}K^3L^3 + \frac{s}{L-L_P}K^2L^2 + \frac{5s}{2(L-L_P)}K^2L + \frac{s}{2(L-L_P)}KL^2 + \frac{5s}{2(L-L_P)}KL - \frac{s}{3(L-L_P)}L$
Total $\bar{C}_{TD}$	$\frac{s}{3(L-L_P)}K^3L^3 + \mathcal{O}(K^2L^2/(L-L_P))$

Table 2: NUMBER OF CMACs PER USER AND BIT FOR THE SLEPIAN-DOMAIN (SD) SAGE-JDE RECEIVER.

Operation	CMAC per user and information bit
$\mathbf{g}^{[0]}$ (15)	$\frac{1}{3(L-L_P)}D^3K^2 + \frac{6+L_P}{4(L-L_P)}D^2K + \frac{1}{4(L-L_P)}D^2 + \frac{5L_P-2}{4(L-L_P)}DK + \frac{5L_P+10}{4(L-L_P)}D - \frac{1}{3(L-L_P)}K^{-1}$
$\mathbf{a}^{[0]}$ (17)	$\frac{DL}{L-L_P}$
$(a_j[\ell]a_k[\ell^*])^{[i]}$ (10)	$\frac{3sD^2L(K-1)}{L-L_P}$
E-step (11)	$\frac{s}{3(L-L_P)}D^3K^3 + \frac{s(6+L)}{4(L-L_P)}D^2K^2 + \frac{s}{4(L-L_P)}D^2KL + \frac{s(5L-2)}{4(L-L_P)}DK^2 + \frac{s(5K+10)}{4(L-L_P)}DK - \frac{s}{L-L_P}$
$\mathbf{a}^{[0]}$ (4)	$\frac{sL}{L-L_P}DK$
Total $\bar{C}_{SD}$	$\frac{s}{3(L-L_P)}D^3K^3 + \mathcal{O}(D^3K^2/(L-L_P))$

the incomplete data  $\mathbf{z}$  is related by a deterministic mapping  $\mathcal{X}_k \mapsto \mathbf{z}(\mathcal{X}_k)$ . Following a standard approach we incorporate the nuisance parameter  $\mathbf{g}$  in the admissible data and choose  $\mathcal{X}_k = \{\mathbf{z}, \mathbf{g}\}$ . Starting from an initial estimate  $\mathbf{d}^{[0]}$ , the SAGE algorithm iterates between two steps, the expectation step (E-step), and the maximization step (M-step). In the *E-step* the algorithm computes an estimate

$$Q(\mathbf{d}_k | \mathbf{d}^{[i]}) \triangleq \mathbb{E} \left\{ \Lambda(\mathcal{X}_k; \mathbf{d}_k, \mathbf{d}_k^{[i]}) \mid \mathbf{z}, \mathbf{d}^{[i]} \right\} \quad (6)$$

of the log-likelihood function  $\Lambda(\cdot)$  for the observation  $\mathcal{X}_k$  based on  $\mathbf{z}$  and a current estimate  $\mathbf{d}^{[i]}$  of  $\mathbf{d}$ . In the *M-step* the algorithm updates  $\mathbf{d}_k$  as the value that maximizes this estimated log-likelihood function. In the iterative process the absolute value of the sequence of likelihood estimates  $\left\{ \Lambda(\mathbf{z} | \mathbf{d}^{[i]}) \right\}_{i=0}^{\infty}$  is non-decreasing. We start with the log-likelihood function of  $\mathbf{d}$  for the observation  $\mathcal{X}_k$ :

$$\Lambda(\mathcal{X}_k | \mathbf{d}) = \Lambda(\mathbf{z}, \mathbf{g} | \mathbf{d}) = \Lambda(\mathbf{z} | \mathbf{g}, \mathbf{d}) + \Lambda(\mathbf{g} | \mathbf{d}). \quad (7)$$

The second summand in (7) does not depend on  $\mathbf{d}$  and hence, can be discarded. Using (5), the first summand in (7) becomes

$$\Lambda(\mathbf{z} | \mathbf{g}, \mathbf{d}) \propto \sum_{\ell=0}^{L-1} \left\{ (\mathbf{z}[\ell] - \mathbf{R}[\ell] \mathbf{D}[\ell] \mathbf{g})^H \mathbf{R}[\ell]^{-1} \right. \\ \left. \times (\mathbf{z}[\ell] - \mathbf{R}[\ell] \mathbf{D}[\ell] \mathbf{g}) \right\}. \quad (8)$$

Further, inserting (7) and (8) in (6), it follows for the *E-step* of the SAGE algorithm after some straightforward algebra

$$Q(\mathbf{d}_k | \mathbf{d}^{[i]}) = \sum_{\ell=0}^{L-1} d_k[\ell] \Re \left\{ (a_k[\ell]^{[i]})^* z_k[\ell] - \right. \\ \left. \sum_{j \neq k} \rho_{k,j}[\ell] d_j[\ell]^{[i]} (a_j[\ell] a_k[\ell^*])^{[i]} \right\}, \quad (9)$$

where the estimated channel weights and their second moments read

$$a_k[\ell]^{[i]} \triangleq \sum_{m=0}^{D-1} \mathbb{E} \left\{ g_{k,m} \mid \mathbf{z}, \mathbf{d}^{[i]} \right\} u_m[\ell], \\ (a_j[\ell] a_k[\ell^*])^{[i]} \triangleq \sum_{m=0}^{D-1} \sum_{n=0}^{D-1} \mathbb{E} \left\{ g_{j,m} g_{k,n}^* \mid \mathbf{z}, \mathbf{d}^{[i]} \right\} u_m[\ell] u_n[\ell]. \quad (10)$$

The conditional distribution of  $\mathbf{g}$  given  $\mathbf{z}$  and assuming  $\mathbf{b}^{[i]}$  is transmitted is Gaussian with conditional expectation [9]

$$\mathbf{g}_{k,m}^{[i]} = \mathbb{E} \left\{ g_{k,m} \mid \mathbf{z}, \mathbf{b}^{[i]} \right\} \\ = \left[ N_0^{-1} \boldsymbol{\Sigma}_g \sum_{\ell=0}^{L-1} (\mathbf{D}[\ell]^{[i]})^H \mathbf{z}[\ell] \right]_{mD+k} \quad (11)$$

and covariance matrix

$$\boldsymbol{\Sigma}_g^{[i]} \triangleq N_0 \left( N_0 \mathbf{I}_{DK} + \right. \\ \left. \boldsymbol{\Sigma}_g \sum_{\ell=0}^{L-1} (\mathbf{D}[\ell]^{[i]})^H \mathbf{R}[\ell] \mathbf{D}[\ell]^{[i]} \right)^{-1} \boldsymbol{\Sigma}_g. \quad (12)$$

From (11) and (12), the conditional expectation in (10) reads

$$\mathbb{E} \left\{ g_{j,m} g_{k,n}^* \mid \mathbf{z}, \mathbf{d}^{[i]} \right\} = g_{j,m}^{[i]} (g_{k,n}^{[i]})^* + \\ \left[ \boldsymbol{\Sigma}_g^{[i]} \right]_{mD+j, nD+k}. \quad (13)$$

In the so-called *M-step* of the SAGE algorithm the estimate of  $\mathbf{d}_k$  is updated as the argument maximizing  $Q(\mathbf{d}_k | \mathbf{d}^{[i]})$  in (9). Due to our particular system set-up, where symbols are independent of each other, the resulting expression breaks down to the following individual symbol updating algorithm

$$d_k[\ell]^{[i+1]} = \text{sgn} \left\{ (a_k[\ell]^{[i]})^* z_k[\ell] - \right. \\ \left. \sum_{j \neq k} \rho_{k,j}[\ell] d_j[\ell]^{[i]} (a_j[\ell] a_k[\ell^*])^{[i]} \right\}. \quad (14)$$

The operator  $\text{sgn}\{\cdot\}$  denotes the signum of the argument. Note that the bit estimates of one user only are updated in the M-step. After all users have been processed once, a so-called “stage” is completed. The SAGE-JDE scheme iterates over several stages between *E-step* (9) and *M-step* (14) until convergence is achieved.

## 4. IMPLEMENTATION ISSUES

### 4.1 Initialization

To initialize the Slepian-domain SAGE-JDE scheme,  $L_P$  pilot symbols are inserted in every user’s data sequence. Their corresponding symbol positions  $\mathcal{P} \triangleq \{l_0, \dots, l_{P-1}\}$  are regularly distributed over the entire block of length  $L$ . The positions are computed as  $l_i = i\Delta$  where  $\Delta = \lceil (L-1)/(L_P-1) \rceil$  denotes the distance between two pilot symbols. The initial estimate  $\mathbf{g}^{[0]}$  is the MMSE estimate of  $\mathbf{g}$  given the observation  $\mathbf{z}[\ell]$ ,  $\ell \in \mathcal{P}$  and the pilot symbols  $\mathbf{d}[\ell]$ ,  $\ell \in \mathcal{P}$ :

$$\mathbf{g}_{k,m}^{[0]} = \left[ N_0^{-1} \boldsymbol{\Sigma}_{\mathbf{g}}^{[0]} \sum_{\ell \in \mathcal{P}} (\mathbf{D}[\ell])^H \mathbf{z}[\ell] \right]_{mD+k}. \quad (15)$$

The covariance matrix of  $\mathbf{g}^{[0]}$

$$\boldsymbol{\Sigma}_{\mathbf{g}}^{[0]} = N_0 \left( N_0 \mathbf{I}_{DK} + \boldsymbol{\Sigma}_{\mathbf{g}} \sum_{\ell \in \mathcal{P}} (\mathbf{D}[\ell])^H \mathbf{R}[\ell] \mathbf{D}[\ell] \right)^{-1} \boldsymbol{\Sigma}_{\mathbf{g}}. \quad (16)$$

Inserting (16) into (11), we obtain an initial guess for the channel weights

$$\mathbf{a}_k^{[0]} = \sum_{m=0}^{D-1} \mathbf{g}_{k,m}^{[0]} \mathbf{u}_m. \quad (17)$$

The initial symbol estimate  $\mathbf{d}[\ell]^{[0]}$  is the conditional linear MMSE estimate of  $\mathbf{d}$  on the initial channel estimate  $\mathbf{a}^{[0]}$

$$\mathbf{d}[\ell]^{[0]} = \text{sgn}\{\Re\{\boldsymbol{\lambda}[\ell]\}\} \quad (18)$$

where

$$\boldsymbol{\lambda}[\ell] \triangleq (\mathbf{a}[\ell]^{[0]})^H \left( \mathbf{R}[\ell] \mathbf{a}[\ell]^{[0]} (\mathbf{a}[\ell]^{[0]})^H + N_0 \mathbf{I}_K \right)^{-1} \mathbf{z}[\ell].$$

### 4.2 Sorting of the Users

Within each stage  $s$ , consisting of  $K$  iterations, all users are processed in ascending order of their energy

$$\mathcal{E}_k = \sum_{\ell=0}^{L-1} |a_k[\ell]^{[i]}|^2.$$

### 4.3 Pseudo-Code of the Receiver

The pseudo-code in Fig. 1 shall ease understand the behavior of the SAGE-JDE scheme.

### 4.4 Computational Complexity

In this subsection we compare the time complexity of the SAGE-JDE derived in Slepian-domain with that one in time-domain when the channel is time-varying [2]. In the following we determine the number of complex multiplications and accumulations (CMACs) that are required per user and bit. The matrix inversions are computed by means of an LU-factorization with back-substitution [10]. The particular structures of diagonal and hermitian matrices have been exploited in order to keep the number of computations at a minimum.

```

% Place pilot symbols
1: - for each user, place  $L_P$  pilot bits in
   signaling interval  $l_0, \dots, l_{P-1}$  in  $\mathbf{D}_P$ .
2: - all other entries in  $\mathbf{D}_P$  are set equal zero.
% Initial MMSE channel estimation
3: - compute matrix  $\boldsymbol{\Sigma}_{\mathbf{g}}^{[0]}$  (16)
4: - compute channel vector  $\mathbf{g}^{[0]}$  (15)
% Initial LMMSE data estimation
5: - compute data vector  $\mathbf{d}^{[0]}$  (18)
% Data detection and channel estimation
6: for  $s = 1 : \text{number of stages}$ 
7:   for  $i = 1 : K$ 
   % E-step
8:     - compute covariance matrix  $\boldsymbol{\Sigma}_{\mathbf{g}}^{[i]}$  (12)
9:     - compute  $\mathbf{g}^{[i]}$  (11)
   % M-step - update user  $k = i \bmod K + 1$ 
10:    for  $\ell = 1 : L$ 
11:      - compute  $d_k[\ell]^{[i+1]}$  (14)
12:    end
   % Update symbol matrix
13:     $\mathbf{d}^{[i+1]} = \{\mathbf{d}_k^{[i+1]}, \mathbf{d}_k^{[i]}\}$ 
14:  end
15: end

```

Figure 1: PSEUDOCODE OF THE SAGE-JDE SCHEME IN SLEPIAN-DOMAIN.

Table 1 presents the time complexity per user and bit  $C_{\text{TD}}$  of the time-domain SAGE-JDE receiver for transmission over time-varying channels [2]. We have evaluated the exact number of CMACs and list them in Table 1. Analogously, Table 2 lists the time complexity of the proposed Slepian-domain SAGE-JDE receiver  $C_{\text{SD}}$ . It can be readily seen that the complexity ratio  $C_{\text{TD}}/C_{\text{SD}}$  is dominated by the term  $L^3/D^3$  for  $K \rightarrow \infty$ . To illustrate the reduction in computational complexity, let us consider the scenario  $L = 40$ ,  $P = 4$ ,  $D = 5$ ,  $K = 16$ , and  $s = 3$  described in Table 3. Exact evaluation of  $C_{\text{TD}}/C_{\text{SD}}$  gives 240. For  $K \rightarrow \infty$  it can be seen that the proposed Slepian-domain SAGE-JDE scheme is 512 times less complex than the time-domain SAGE-JDE receiver.

## 5. NUMERICAL RESULTS

The general system settings are summarized in Table 3. We assume Clarke’s model for the channel weights of each user. Hence, the covariance function is given by  $R_a[n] = J_0(2\pi\nu_D n)$ . Here,  $J_0\{\cdot\}$  designates the Bessel function of the first kind of zeroth order. The channel weights are generated via spectral filtering of a white random sequence. The long signature sequences are randomly generated and their individual chips are BPSK modulated. We assume a symbol duration  $T_S$  of 54  $\mu\text{s}$ . The Slepian sequences are designed for a maximum velocity of  $v_{\text{max}} = 250$  kmph. This corresponds to a normalized Doppler frequency of  $25 \times 10^{-3}$ .

In Fig. 2 we illustrate the bit-error-rate (BER) curves of the proposed Slepian-domain SAGE receiver and the time-domain SAGE receiver [2, Section 6.3]. To test the receiver performance versus speed we consider relative velocities  $v \in \{12.5, 125, 250\}$  kmph. The lowest curve is the single-user BER performance. It indicates the bit error rate that is achieved with one single user and assuming that the receiver has access to the exact channel coefficients  $a_i[\ell]$ ,  $\ell = 0, \dots, L-1$ . The results show that the BER curves of both the low-complexity SD-receiver and the TD-receiver are very close. The deviation in case of  $v = 12.5$  kmph is largest due the fact that the sequences were designed for

Table 3: SIMULATION PARAMETERS.

PARAMETER	VALUE
Number of users $K$	$\{8, 12, 16\}$
Block length $L$	40
Number of pilot symbols $P$	4
Spreading sequence	long
Chip modulation	BPSK
Spreading factor $N_C$	16
Symbol modulation	BPSK
Symbol duration $T_S$	$54 \mu\text{s}$
Max. velocity $v_{\max}$	250 kmph
Subspace dimension $D$	5
Max. Doppler bandwidth $\nu_{D\max}$	$25 \times 10^{-3}$
User velocities $v$	$\{12.5, 125, 250\}$ kmph
Number of stages $s$	3

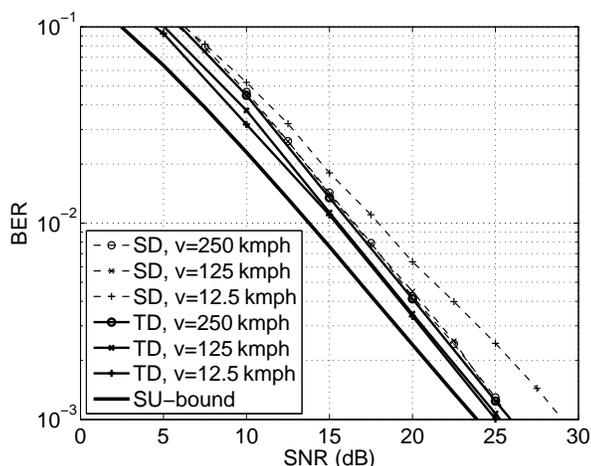


Figure 2: BERS FOR DIFFERENT VELOCITIES AND  $K = 12$ .

$v_{\max} = 250$  kmph. Note that the TD-receiver utilizes full knowledge of the second-order statistics of the channel while the SD-receiver knows only  $v_{\max}$ .

In Fig. 3 we illustrate the dependency of the BER on the system load  $\beta \triangleq K/N_C$ . The load  $\beta$  ranges in  $\{0.5, 0.75, 1.0\}$ . The results indicate that the loss of the SD-receiver to the TD-scheme is less than 1 dB.

## 6. CONCLUSIONS

We designed an iterative SAGE-receiver that performs channel estimation in the Slepian-subspace rather than in the time-domain directly. This allows to tremendously reduce the inherent complexity, mainly brought in by the channel estimator. The observed reduction in numbers of complex multiplications and accumulations (CMACs) is in the order of several hundreds for practical system configurations. The observed performance is very close to the time-domain based implementation. Furthermore, the Slepian-SAGE does not require precise second-order statistics of the actual channel realizations, neither in the design of the sequences nor in the channel estimator.

## 7. ACKNOWLEDGMENTS

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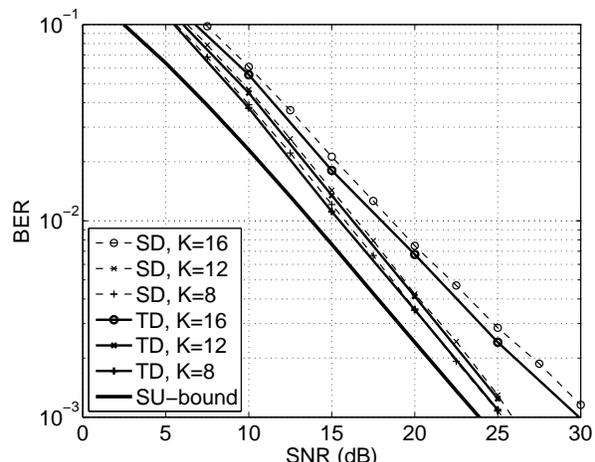


Figure 3: BERS FOR DIFFERENT LOADS AND  $v = 250$  kmph.

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