ESTIMATORS FOR TIME-VARIANT CHANNELS APPLIED TO UMTS-HSDPA

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ABSTRACT

In this work we analyze the *mean-square error* (MSE) of two correlative channel estimators in time-variant channels. Both estimators make use of orthogonal basis functions that approximate the time-variant channel. The first approximation is based on a set of time shifted *rectangular basis* (RB) functions. The second approximation makes use of *Slepian basis* (SB) functions. We calculate the number of basis functions that minimize the MSE for the RB- and the SB-approach. Both estimators are designed for the full velocity range using the maximum velocity as design parameter only. Further, we apply the two estimators to the *high-speed downlink packet access* (HSDPA) mode of UMTS and illustrate the impact of the MSE on the throughput. With the SB-approach we observe a gain of roughly $0.4\,\mathrm{dB}$ over the RB-functions in case of an i.i.d. Rayleigh multipath channel with five taps.

1. INTRODUCTION

In [1] we have observed that the quality of channel knowledge available at the receiver severely influences the single-user throughput of the UMTS-HSDPA mode. There, we have determined, by means of simulations, the number of rectangular basis functions used for channel estimation that leads to the highest throughput for constant pilot power in different time-variant ITU-channels.

In the present contribution we calculate the optimal number of basis functions for the special case of an i.i.d. Rayleigh fading channel. Furthermore, we propose an alternative channel estimator, based on Slepian sequences, that was introduced in the context of OFDM [2]. Only few coefficients are required to obtain a good approximation of the time-variant fading channel. Both time-variant channel estimators are designed for the full velocity range using the maximum velocity $v_{\max}$ as design parameter only. Thus, we avoid the necessity of detailed information about the second order channel statistics at the receiver side which are costly to obtain in mobile communication systems. The quality of the approximations is determined by a bias and a variance term and the minimum MSE is a function of the subspace-dimension D. We present an analysis for the calculation of the optimal number of basis functions. The analytical results of the associated MSE for the RB- as well as the SB-channel estimator show a good match to the simulation results. We also illustrate the impact of the two channel estimators on the achievable throughputs of a UMTS-HSDPA system.

The paper is organized as follows: In Section 2 we describe UMTS-HSDPA and present the signal model. The channel estimators and the analytical expression of the MSE are presented in Section 3. Numerical results on the MSE of the channel estimators and the associated throughputs are displayed in Section 4 before we conclude in Section 5.

2. SIGNAL MODEL

For the analysis of the channel estimators we consider the HSDPA sub-system of UMTS [3]. HSDPA was introduced in Release 5 of the UMTS-standard and aims at providing data rates up to $14.4\,\mathrm{Mbps}$. This rate can be shared among up to four simultaneous users via 15 *high-speed physical downlink shared channels* (HS-PDSCHs) that are QPSK or 16-QAM modulated. The spreading factor is fixed to $\mathrm{SF}_d = 16$ on all HS-PDSCHs. All information is Turbo-coded and different rates are accomplished by applying repetition and puncturing. HSDPA employs a *transmission time interval* (TTI) of only $2\,\mathrm{ms}$ that corresponds to a so called subframe. This short TTI allows for fast user scheduling executed in the Node-B. The selection of users is based on the particular scheduling algorithm that utilizes the *channel quality indicators* (CQIs) that are signaled from all user-terminals to the base station. The fading behavior of the time-variant channel is handled by *adaptive modulation and coding* (AMC) instead of fast power control. The *high-speed shared control channels* (HS-SCCHs) inform the scheduled users on the currently used modulation and coding scheme and the *redundancy version* (RV) of the (re-)transmitted packets. HSDPA deploys hybrid-ARQ where log-likelihood ratio values of falsely detected bits are combined with those of retransmitted bits at the receiver. The transmitted chip stream in the downlink reads

$$t_{\mathrm{ch}}[n] = \left\{ \sqrt{\frac{E_c}{T_c N}} \sum_{i=1}^{N} d_i(\lfloor n/\mathrm{SF}_d \rfloor)\, c_{i,\mathrm{SF}_d}[n \bmod \mathrm{SF}_d] + \sqrt{\frac{E_p}{T_c}}\, p(\lfloor n/\mathrm{SF}_p \rfloor)\, c_{1,\mathrm{SF}_p}[n \bmod \mathrm{SF}_p] + \sqrt{\frac{E_o}{T_c}}\, q[n] \right\} \times a[n \bmod 38400] + \sqrt{\frac{E_s}{T_c}} s[n] \tag{1}$$

with $\lfloor x \rfloor$ being the integer that is smaller or equal to x. Parentheses $(\cdot)$ denote timing on symbol level while brackets $[\cdot]$ indicate timing on chip level. The duration of a UMTS-chip is $1/3.84\,\mathrm{Mcps} = 260\,\mathrm{ns}$. The sequence $a[n]$ denotes the scrambling code that is uniquely assigned to a particular base station. It is a Gold-sequence

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that is terminated after 38,400 chips. The four terms in (1) specify the N equally strong HS-PDSCHs, the signal on the *primary common pilot channel* (P-CPICH), the parts that resemble the *orthogonal channel noise simulator* (OCNS), and the *synchronization signal* (SCH). The OCNS $q[n]$ accounts for other physical channels active in the cell. The normalized data d_i and pilot symbols p are separated via so called *orthogonal variable spreading factor* (OVSF) codes where the spreading factor of the data channel is $\mathrm{SF}_d = 16$ and the one of the pilot channel $\mathrm{SF}_p = 256$. Therefore, we transmit $P = 30$ pilot symbols per subframe. Four Turbo-coded bits $\boldsymbol{b}_i(m) = [b_{i,0}(m)\ b_{i,1}(m)\ b_{i,2}(m)\ b_{i,3}(m)] \in \{-1,+1\}^4$ of channel i are mapped to a 16-QAM symbol along $d_i(m) = f(\boldsymbol{b}_i(m))$ where $f(\cdot)$ denotes the Gray-mapping function. In total there are $M = 480$ symbols in a subframe per data channel i. Synchronization is time shared with the *primary common control physical channel* (P-CCPCH) and conducted on a slot base: The first 256 chips are used for synchronization where $s[n]$ is a Golay-sequence and the remaining 2304 chips per slot are used by the P-CCPCH. The sum of the energy of all channels equals the total transmit power spectral density $I_{or} = E_c + E_p + E_s + E_o$.

The chip stream t_{ch} is upsampled by the oversampling factor $\mathrm{OS} = 2$,

$$t_{\mathrm{up}}[z] = \begin{cases} t_{\mathrm{ch}}[z/2], & \text{if } z \text{ even} \\ 0, & \text{else} \end{cases} \tag{2}$$

and passed through the transmitter's and the receiver's pulse shaping filters (both are modeled as root raised cosine FIR filters with length $L_{\mathrm{RRC}} = 49$ and roll-off factor $\alpha = 0.22$). The received signal after chip matched filtering reads

$$r[z] = \sum_{u=0}^{2L_{\mathrm{RRC}}-1} h_{\mathrm{RC}}[u] \sum_{l=0}^{L-1} h[z-u-\tau_\ell, \tau_\ell] t_{\mathrm{up}}[z-u-\tau_\ell] + v[z] \tag{3}$$

where $r[z]$ is the received oversampled signal, $t_{\mathrm{up}}[z]$ is the upsampled transmitted baseband signal (2), and $v[z]$ is complex circularly symmetric zero-mean Gaussian noise with variance σ_v^2. The effective filter $h_{\mathrm{RC}}[u]$ combines the transmit and receive chip filters and has raised cosine characteristic. In our model we use a time-variant Rayleigh fading channel that is *wide sense stationary* (WSS). Furthermore, we assume *uncorrelated scattering* (US), thus the time-variation of the individual channel taps $h[z, \tau_\ell]$ is modeled independently. The index z denotes absolute time (in samples) and τ_ℓ the delay. The channel consists of L i.i.d. multipath components that are quantized to the sampling instances. The channel is normalized to unit gain

$$\mathbb{E}\left\{ \sum_{l=0}^{L-1} \left| h[z, \tau_\ell] \right|^2 \right\} = 1, \forall\, z \tag{4}$$

and the power of a single tap is $\sigma_{h,l}^2 = \mathbb{E}\{|h[z, \tau_\ell]|^2\} = \sigma_h^2 = 1/L$. Except from the L path positions all other channel coefficients at different time lags are set to zero. The L Rayleigh fading paths are generated as proposed in [2]. Latter model is a modification of [4] by correcting the fading statistics for low velocities.

3. CHANNEL ESTIMATION

We aim at delivering a channel estimate every $\Delta = \mathrm{OS} \times \mathrm{SF}_d$ samples for each path. This is less frequent than the actual variation of the channel but sufficiently accurate for analysis since the channel stays practically constant during this time. The rate coincides with the data symbol rate on the HS-PDSCH. We aim at a subspace-based channel description which approximates the temporal evolution of the ℓ-th tap $h_\ell(m) \triangleq h[(m+1/2)\Delta, \tau_\ell]$ in terms of D orthogonal basis functions $u_i(m), i \in \{0, \ldots, D-1\}$. Mathematically, this writes

$$\hat{h}_\ell(m) = \boldsymbol{f}^{\mathrm{T}}(m) \hat{\boldsymbol{\gamma}}_\ell,\ m \in \{0, \ldots, M-1\}. \tag{5}$$

We define $\boldsymbol{f}(m) \triangleq [u_0(m), \ldots, u_{D-1}(m)]^{\mathrm{T}} \in \mathbb{R}^{D\times 1}$. The vector $\hat{\boldsymbol{\gamma}}_\ell = [\hat{\gamma}_\ell(0), \ldots, \hat{\gamma}_\ell(D-1)]^{\mathrm{T}} \in \mathbb{C}^{D\times 1}$ contains the estimated coefficients for the subspace description.[1] The vector of estimates is computed as

$$\hat{\boldsymbol{\gamma}}_\ell = \sum_{m=0}^{M-1} \boldsymbol{f}^*(m) \tilde{h}_\ell(m)\,. \tag{6}$$

The M average values $\tilde{h}_\ell(m)$ are obtained by means of correlation of the P-CPICH signal $p[z']$ and the received signal $r[z']$ along

$$\tilde{h}_\ell(m) \triangleq \sum_{z'=m\Delta+\tau_\ell}^{(m+1)\Delta-1+\tau_\ell} r[z'-\tau_\ell] p^*[z']\,. \tag{7}$$

Due to the orthogonality of the basis functions the correlative estimator (6) coincides with the corresponding least-squares estimator. The (MSE) between the true channel $h_\ell(m)$ and its approximation (5) is defined as

$$\mathrm{MSE}_\ell(m) \triangleq \mathbb{E}\left\{ \left| h_\ell(m) - \hat{h}_\ell(m) \right|^2 \right\}. \tag{8}$$

The MSE per subframe and channel tap is similarly given in [5] as

$$\mathrm{MSE}_{M,\ell} \triangleq \frac{1}{M} \sum_{m=0}^{M-1} \mathrm{MSE}_\ell(m)\,. \tag{9}$$

The $\mathrm{MSE}_{M,\ell}$ can be described by the sum of a square bias and a variance term along

$$\mathrm{MSE}_{M,\ell} = \mathrm{bias}_{M,\ell}^2 + \mathrm{var}_{M,\ell}\,. \tag{10}$$

Practically, we are interested in basis functions $u_i(m)$ that minimize $\mathrm{MSE}_{M,\ell}$. In the following we present two suitable choices of such basis functions. Since we consider an i.i.d. channel we will omit the path index ℓ in the sequel. Note that we are interested in a set of basis functions that is calculated ones and kept fixed. We do not adapt the set of basis functions to the second order statistics of the channel since this information is costly to obtain in wireless communication systems. The basis function design uses the normalized maximum Doppler bandwidth ν_{Dmax} as design parameter only. The normalized maximum Doppler bandwidth is defined as

$$\nu_{\mathrm{Dmax}} = \mathrm{SF}_d T_{\mathrm{C}} f_{\mathrm{D}} = \mathrm{SF}_d T_{\mathrm{C}} \frac{f_{\mathrm{C}} v_{\max}}{c_0} \tag{11}$$

with f_{C} as carrier frequency, $v_{\max}$ as maximum relative velocity between transmitter and receiver, and c_0 the speed of light.

[1] In case of a non-uniform power-delay profile the optimal suspace for each tap ℓ can have different dimensions.

3.1. Approximation by Rectangular Basis (RB) Functions

A very simple approximation of a time-variant channel is based on a block-static assumption, meaning that the channel is considered quasi-static over some time. In order to reduce the variance of the channel estimates an average of M/D channel estimates $\hat{h}_\ell(m)$ is computed. This simple estimator can be represented in the framework of orthogonal basis functions (5). The corresponding rectangular orthonormal basis functions are expressed as

$$u_i(m) = \begin{cases} \sqrt{\dfrac{D}{M}} & \text{for } m \in \left[i\dfrac{M}{D}, (i+1)\dfrac{M}{D} - 1\right], \\ 0 & \text{else.} \end{cases} \quad (12)$$

In case of a time-variant channel we have observed that by increasing the length of the averaging interval M/D the variance term in (10) will be reduced. However at the same time bias_M^2 will be increased, thus there exists an optimal trade-off between bias_M^2 and var_M [1].

3.2. Approximation by Slepian Basis (SB) Functions

Slepian showed in [6] that *discrete prolate spheroidal* (DPS) sequences are bandlimited to the frequency range $[-\nu_{\mathrm{Dmax}}, \nu_{\mathrm{Dmax}}]$ and simultaneously show maximum energy concentration within a finite time interval of length M. In [2] DPS sequences are applied to time-variant channel estimation in the context of OFDM.

The results in [2, Fig. 4] demonstrate that the DPS sequences provide a consistent performance over the full Doppler range $\nu_{\mathrm{D}} \in [0, \nu_{\mathrm{Dmax}}]$ outperforming complex exponential basis functions [7, 8], polynomial basis functions [9], and nonorthogonal complex exponential basis functions [8]. For more details see [2] and the references therein.

The DPS sequence $u_0(m)$ is the unique sequence that is bandlimited and most time-concentrated in a given interval of length M and serves as first basis function. The second basis function $u_1(m)$ exhibits maximum energy concentration among the DPS sequences that are orthogonal to $u_0(m)$, and so on. Thus, DPS sequences form a set of orthogonal sequences that are exactly bandlimited and simultaneously possess a high (though not complete) time-concentration in a certain interval of length M. In our application we are interested in $u_i(m)$ for the time index set $m \in \{0, \ldots, M-1\}$ only. We use the term *Slepian basis* (SB) function for the index limited DPS sequences and define the vector $\boldsymbol{u}_i \in \mathbb{R}^{M\times 1}$ with elements $u_i(m)$ for $m \in \{0, \ldots, M-1\}$. The SB-functions $\boldsymbol{u}_i$ are eigenvectors of the matrix $\boldsymbol{C} \in \mathbb{R}^{M\times M}$ satisfying the matrix pencil $\boldsymbol{C}\boldsymbol{u}_i = \lambda_i \boldsymbol{u}_i$. The matrix $\boldsymbol{C}$ is defined as [6]

$$[\boldsymbol{C}]_{i,\ell} = \frac{\sin\left(2\pi(i-\ell)\nu_{\mathrm{Dmax}}\right)}{\pi(i-\ell)} \quad i, \ell \in \{0, \ldots, M-1\}. \quad (13)$$

The eigenvalues λ_i are clustered near one for $i \leq \lceil 2\nu_{\mathrm{Dmax}} M \rceil$ and decay exponentially for $i > \lceil 2\nu_{\mathrm{Dmax}} M \rceil$. Therefore, the signal space dimension [6, Sec. 3.3] of time-limited snapshots of a bandlimited signal is approximately given by $D' = \lceil 2\nu_{\mathrm{Dmax}} M \rceil + 1$ [10]. In case of a wireless UMTS channel, three to five Slepian basis functions are already sufficient to represent the time-variant channel taps for a subframe length of $M = 480$ data symbols and maximum relative velocities of several hundred kmph.

Table 1. Simulation Parameters.

Parameter	Value
Chip duration T_{c}	$260\,\mathrm{ns}$
No. of chips per subframe	7680
Max. no. of retransmissions	3
RV coding sequence	$\{6, 2, 1, 5\}$
HS-PDSCH E_c/I_{or}	$-6\,\mathrm{dB}$
P-CPICH E_p/I_{or}	$\{-23, \ldots, -3\}\,\mathrm{dB}$
SCH and P-CCPCH E_s/I_{or}	$-12\,\mathrm{dB}$
I_{or}/I_{oc}	$10\,\mathrm{dB}$
Channel model	i.i.d. multipath Rayleigh, $L = 5$, $v_{\max} = 100\,\mathrm{kmph}$
Multipath positions	chip spaced
Path delay knowledge	perfect
Channel coefficient estimators	{ Rectangular base, Slepian base }
No. of pilots per subframe P	30
Chip equalizer	LMMSE, length $W = 20 \times$ OS
Demapper	max-log
Turbo-decoding	max-log-MAP - 8 iterations

3.3. MSE-Analysis of Orthogonal Basis Functions

We use here the expression for square bias and variance that were developed in [5, Sec. 6] and [2] to express the average MSE_M (10). We define the instantaneous frequency response of the basis expansion

$$H(m,\nu) = \boldsymbol{f}^{\mathrm{T}}(m) \sum_{k=0}^{M-1} \boldsymbol{f}^*(k) \mathrm{e}^{-\mathrm{j}2\pi\nu(m-k)}\,, \quad (14)$$

where $m \in \{0, \ldots, M-1\}$ and $|\nu| < 1/2$. The instantaneous error characteristic of the basis expansion is defined as [5, Sec. 6.1.4]

$$E(m,\nu) = \left| 1 - H(m,\nu) \right|^2. \quad (15)$$

The square bias per subframe bias_M^2 can be expressed as the integral over the instantaneous error characteristic $E(m,\nu)$ multiplied by the power spectral density $S_{hh}(\nu)$

$$\mathrm{bias}_M^2 = \sum_{m=0}^{M-1} \int_{-\frac{1}{2}}^{\frac{1}{2}} E(m,\nu) S_{hh}(\nu)\,\mathrm{d}\nu\,. \quad (16)$$

The power spectral density is assumed identical for all paths ℓ. The variance can be expressed as

$$\mathrm{var}_M \approx \frac{1}{\mathrm{SNR}} \frac{D}{M} \quad (17)$$

where SNR describes the effective *signal to noise ratio* (SNR) of path ℓ. We use only segments of SF_d chips to obtain an estimate $\hat{h}_\ell(m)$. During this segment the interference consists of contributions from the CPICH and the HS-PDSCH at time lags different than τ_ℓ. For τ_ℓ itself those channels remain orthogonal. The SCH, the OCNS, and AWGN are not orthogonal to the segment of the CPICH and hence need to be taken fully into account. We model all interference as one single joint Gaussian process. Hence, the corresponding SNR is given by

$$\mathrm{SNR} = \frac{\mathrm{SF}_d\, \sigma_h^2 \frac{E_p}{I_{or}}}{\frac{I_{or}}{I_{oc}} + (1-\sigma_h^2)\left(\frac{E_c}{I_{or}} + \frac{E_p}{I_{or}}\right) + \frac{E_o}{I_{or}} + \frac{E_s}{I_{or}}}. \quad (18)$$

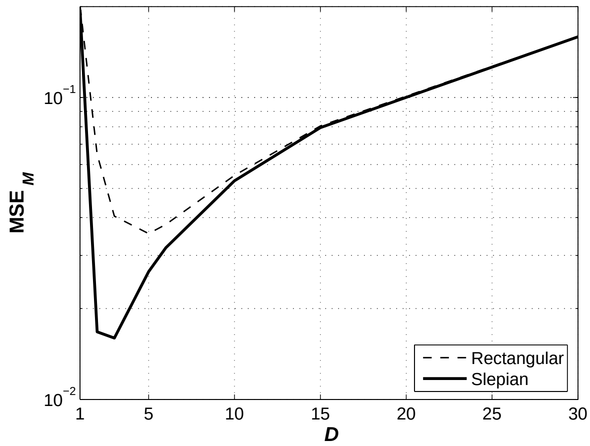

Fig. 1. Analytical evaluation of the MSE_M given in Eq. (10) as a function of the number of basis functions D for a multipath Rayleigh fading channel with $L = 5$ and $v_{\max} = 100\,\mathrm{kmph}$. E_p/I_{or} is set to $-9\,\mathrm{dB}$.

The subspace dimension minimizing the MSE_M (9) for a given SNR is found to be

$$D = \underset{D' \in \{1,\ldots,Q\}}{\operatorname{argmin}} \left(\mathrm{bias}_M^2 + \frac{1}{\mathrm{SNR}} \frac{D'}{M} \right) . \tag{19}$$

The Slepian basis functions coincide with the basis functions of the Karhunen-Loève expansion for a fading process with flat Doppler spectrum and Doppler bandwidth ν_{Dmax} [6]. Thus, in this case, we can approximate the bias by $\mathrm{bias}_M^2 \approx 1/M \sum_{i=D}^{M-1} \lambda_i/(2\nu_{\mathrm{Dmax}})$ so that (19) can be easily evaluated [11]. On the other hand, to minimize D for rectangular basis functions we need to compute (16).

4. NUMERICAL RESULTS

In order to validate our theoretical findings we perform Monte-Carlo simulations with the UMTS-HSDPA simulator described in [12]. The ratio of the spectral densities of totally radiated transmit power and additive white Gaussian noise is denoted as I_{or}/I_{oc} and fixed to $10\,\mathrm{dB}$. We discuss the optimal number of dimensions D used for the channel estimation first. Then, we compute the MSE of the RB- and SB-channel estimators for an i.i.d. multipath Rayleigh fading channel with $L = 5$ chip-spaced taps and $v_{\max} = 100\,\mathrm{kmph}$. We also illustrate the MSE_M as a function of v for sequences that were designed for $v_{\max} = 100\,\mathrm{kmph}$. Last, we illustrate throughput results for the different channel estimators. The results for the MSE and the throughput are averaged over 200 UMTS frames.

4.1. Mean-Square Error (MSE)

We vary the power of the P-CPICH, *i.e.*, E_p/I_{or} from $-30\,\mathrm{dB}$ to $-3\,\mathrm{dB}$ while keeping the power on the data channel HS-PDSCH E_c/I_{or} constant at $-6\,\mathrm{dB}$. The settings for the employed reference channel H-Set 6 are given in [13] and the common simulation parameters are listed in Table 1.

Eq. (10) indicates that for a particular choice of basis functions and a particular channel there exists a dimension D that minimizes the MSE_M, given E_p/I_{or}. We evaluate the MSE_M against $D \in \{1, 2, 3, 5, 6, 10, 15, 30\}$, the divisors of $P = 30$, for a multipath Rayleigh fading channel with $L = 5$ and RB/SB-functions. Fig. 1 shows that the dimension D minimizing the MSE_M is three in case of SB and five in case of RB. We also see that the MSE_M associated with SB lies at 1.7×10^{-2} and that of RB at 3.6×10^{-2}.

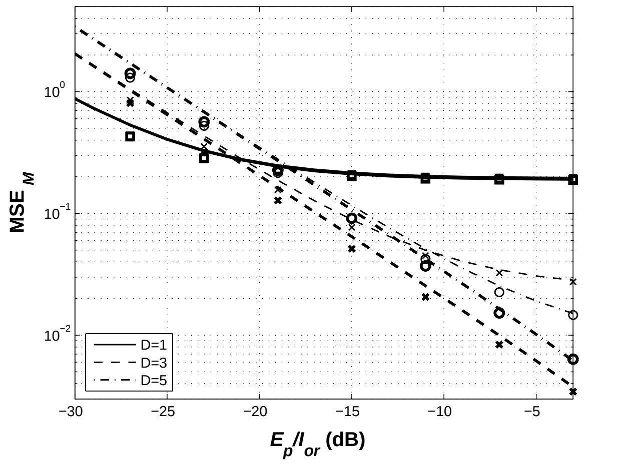

Fig. 2. Analytical (lines) and simulative (markers) MSE with different number of basis functions for (i) RB-approximation (thin line) and (ii) SB-approximation (bold line). We consider a multipath Rayleigh fading channel with $L = 5$ and $v_{\max} = 100\,\mathrm{kmph}$.

Fig. 2 shows the MSE_M from the analysis and the MSE_M that was observed in the UMTS-HSDPA simulator for a multipath Rayleigh fading channel with $L = 5$ and $v_{\max} = 100\,\mathrm{kmph}$. We consider the estimator using RB-functions (thin lines) and the one using SB-functions (bold lines). The MSE values obtained from the simulations are indicated through the markers and they are slightly better than the analysis. We interpret this mismatch as a consequence of the on-off signalling of the SCH. It is sometimes zero and hence reduces the interference in the corresponding segment.

In Fig. 3 we vary the velocity of the user, *i.e.*, the Doppler bandwidth, over the full range $v \in (0.01, v_{\max}]\,\mathrm{kmph}$. We compare the performance of the RB-functions and the SB-funcitons for $E_p/I_{or} = -9\,\mathrm{dB}$ in terms of MSE_M versus v. The results in Fig. 3 clearly demonstrate the constant performance of the SB-functions over the full velocity range.

4.2. Throughput

We determine the impact of the MSE of the channel estimates on the total throughput of the HSDPA chain. Since an analytical model of the HSDPA chain is not known to the authors we have devised simulations to study the performance. Fig. 4 shows the throughput versus E_c/I_{or} of the data channels.

The powers of the SCH and P-CPICH are given in Table 1. We compare the MSE-minimizing dimension for RB, *i.e.*, $D = 5$ with that one of the Slepian approach. The SB has dimension $D = 3$ and we illustrate the throughput that is achieved by updating the LMMSE chip-level equalizer coefficients $U = 5$ times per subframe and $U = 15$ times per subframe. With the Slepian basis we could compute a channel estimate for every data symbol, *i.e.*, 480 times per subframe. However, if perfect channel knowledge

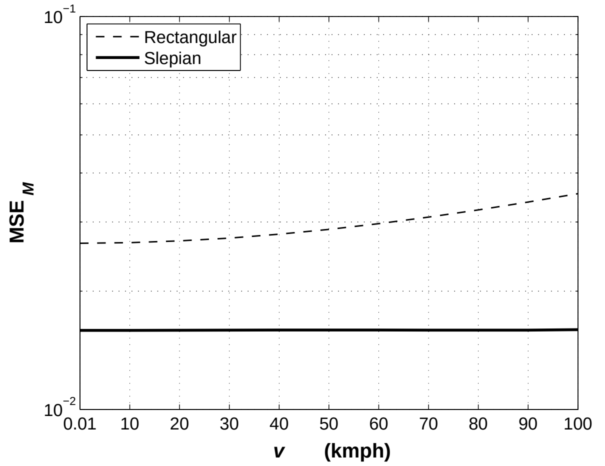

Fig. 3. Analytic evaluation of the mean square error MSE_M versus velocity $v \in [0.01, 100)\,\mathrm{kmph}$ for RB- and SB-functions at $E_p/I_{or} = -9\,\mathrm{dB}$.

is available, we have observed that update rates of the equalizer coefficients higher than $U = 15$ do not improve the throughput. At a throughput of $1.5\,\mathrm{Mbps}$ the improvement in E_c/I_{or} to the RB-estimator makes up $0.25\,\mathrm{dB}$ for SB with $U = 5$ and $0.4\,\mathrm{dB}$ for SB with $U = 15$.

5. CONCLUSIONS

We have analyzed the mean square error of two channel estimators that are based on a set of orthogonal basis functions. The first is generated from rectangular basis (RB) functions and the second is derived from Slepian basis (SB) functions. We have presented a theoretical framework that accurately describes the MSE in both cases. The theoretical outcome was applied to UMTS-HSDPA and shows an excellent match to the MSE that was observed during simulations. From the analytical expressions we derive the number of basis functions that minimizes the MSE for an i.i.d. multipath Rayleigh channel. The Slepian-based approach shows an improvement of roughly $0.4\,\mathrm{dB}$ over the rectangular based approach for a five tap i.i.d. Rayleigh multipath channel.

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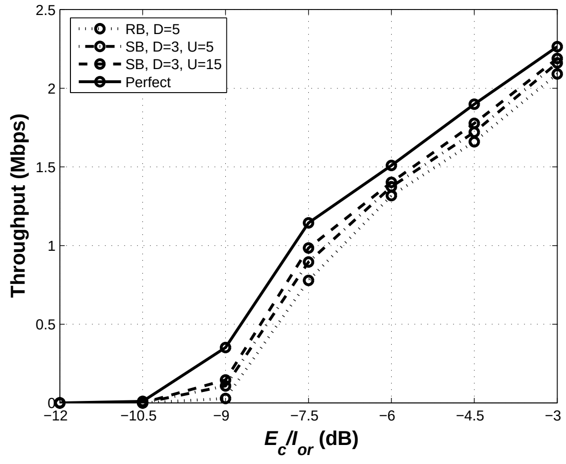

Fig. 4. Throughput for H-Set 6 in a Rayleigh multipath channel with $L = 5$ and $v_{\max} = 100\,\mathrm{kmph}$. The subspace has dimension (i) $D = 5$ for rectangular and (ii) $D = 3$ for Slepian basis functions. The update rates U indicate how often the LMMSE equalizer coefficients are updated per subframe.

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