Time-Variant Channel Equalization via Discrete Prolate Spheroidal Sequences

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***Abstract*— In this work we present an equalization scheme for a multi-carrier (MC) code division multiple access (CDMA) communication system that is operated in a frequency-selective time-variant (TV) channel. For block oriented data transmission we are interested to describe the TV channel for the duration of a data block with the smallest amount of parameters possible. Slepian showed that time-limited parts of band-limited sequences span a subspace with strongly reduced dimensionality. The discrete prolate spheroidal (DPS) sequences are the basis of this subspace. We exploit this property by using the Slepian basis expansion model (BEM) to describe a TV channel with a minimum amount of parameters. The mean squared error (MSE) of the Slepian BEM is 30 dB smaller than the MSE of the Fourier BEM for a frequency-flat TV channel. We present simulation results in terms of bit error rate (BER) versus E_b/N_0 for a multi-user MC-CDMA forward link in a frequency-selective TV channel comparing the Fourier BEM and the Slepian BEM for channel equalization.**

I. Introduction

This paper describes a solution for the problem of channel equalization in a wireless communication system based on multi-carrier (MC) code division multiple access (CDMA) when the users are moving at vehicular speed. In MC-CDMA a data symbol is spread by a user specific spreading code, the resulting chips are processed by an inverse discrete Fourier transform (DFT) to obtain an orthogonal frequency division multiplexing (OFDM) symbol. One data symbol corresponds to one transmitted OFDM symbol. The transmission scheme is block oriented.

We deal with a frequency-selective channel that varies significantly over the duration of a long block of OFDM symbols but for the duration of an OFDM symbol the channel time variation is small enough to be neglected. This is directly related to very small inter-carrier interference (ICI). Every OFDM symbol is preceded by a cyclic prefix to avoid inter-symbol interference (ISI). We assume perfect time and frequency synchronization.

At the receiver we perform block oriented processing for data detection and channel estimation. Therefore we are interested in a channel representation that models a time variant (TV) channel for the duration of a data block.

The time variation of a wireless channel over the duration of a data block is caused by Doppler effects. The Doppler frequency depends on the velocity, on the carrier frequency and on the angle of arrival. Therefore the maximum time variation of the wireless channel is upper bounded by the maximum (one sided) Doppler bandwidth B_{Dmax} when the carrier frequency f_C and the maximum supported velocity $v_{\max}$ of the user is given:

$$B_{\mathrm{Dmax}} = \frac{v_{\max} f_C}{c_0} \tag{1}$$

where c_0 denotes the speed of light.

We will use a basis expansion model (BEM) with an appropriately chosen set of basis functions to describe the time variation of a channel. The Fourier basis functions are applied in [1]–[4] to represent the TV channel through a Fourier BEM. The Fourier BEM has the following drawbacks: The rectangular windowing associated with the DFT introduces spectral leakage, see [5, Sec. 5.4] and [6, Sec. 3.7]. The energy from low frequency Fourier coefficients leaks to the full frequency range. When the DFT is truncated at B_{Dmax} the Gibbs effect [5, Sec. 2.4.2] together with spectral leakage leads to significant phase and amplitude errors at the beginning and the end of the data block [7]. This results in a bit error floor for channels with Doppler spread as was shown by [8] and [9].

To overcome the Fourier BEM deficiencies we will apply a BEM described by Slepian [10]. The Slepian BEM is able to represent band-limited sequences with a minimum amount of basis functions. Slepian showed that time-limited parts of a band-limited sequence span a subspace with strongly reduced dimensionality. The basis functions are the so-called discrete prolate spheroidal (DPS) sequences (see [11] for the continuous time counter part). These DPS sequences have a remarkable *double* orthogonality property over an infinite and a finite interval. This property enables parameter estimation without the drawbacks of windowing as in the case of the Fourier BEM [10, Sec. 3.1.4]. The Slepian basis functions are matched to the known maximum time variation of the channel B_{Dmax} and the length of the transmitted data block.

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Our contribution in this paper is the application of the Slepian BEM for the task of channel equalization in a MC-CDMA system. To our best knowledge this application was not explored until now.

The paper is organized as follows: We introduce the signal model for MC-CDMA in Section II. In Section III we present the Slepian BEM for TV channel modeling. The Slepian BEM can be approximated by the finite Slepian BEM, we show this in Section IV. The MSE of the Slepian BEM for a flat-fading TV channel is analyzed in Section V. The MC-CDMA forward link simulation results are given in Section VI. Finally, we conclude in Section VII.

II. MC-CDMA Signal Model

In MC-CDMA a data symbol is spread by a user specific spreading code. The resulting chips are processed by an inverse DFT to obtain an OFDM symbol. Our transmission is block oriented, a data block consists of M OFDM data symbols. Every OFDM symbol is preceded by a cyclic prefix to avoid ISI. The channel varies significantly over the duration of a long data block. For the duration of an OFDM symbol the channel time variation can be neglected.

The base station transmits quaternary phase shift keying (QPSK) modulated symbols $b_k[m]$ with symbol rate $R_S = 1/T_S$ drawn from the alphabet $\frac{1}{\sqrt{2}}\{\pm 1 \pm \mathrm{j}\}$ in blocks of length M. Discrete time is denoted by m, $t = mT_S$. There are K users in the system, the user index is denoted by k. Each symbol is spread by a random spreading sequence $\boldsymbol{s}_k \in \mathbb{C}^{N\times 1}$ with elements $\frac{1}{\sqrt{2N}}\{\pm 1 \pm \mathrm{j}\}$, see Fig. 1[1].

The data symbols result from the binary information sequence $\chi_k[m'']$ of length $2MR_C$ by convolutional encoding with code rate R_C, random interleaving and QPSK modulation with Gray labeling. The spread signals of all users are added together. Then, an N point inverse DFT is performed and a cyclic prefix with length G is inserted. One OFDM symbol together with the cyclic prefix has length of $P = N+G$ chips. The resulting chip stream $\mu[n]$ with chip rate $1/T_C = P/T_S$ is transmitted over a TV multipath fading channel with a delay spread of L chips, $t_D = LT_C$.

The transmit filter, the TV channel and the matched receive filter together are represented by $h(t,\tau)$. We denote the sampled TV impulse response by $h'[n',n] = h(n'T_C, nT_C)$. For processing at the receiver the channel time variation over the duration of an OFDM symbol is small enough to be neglected. Therefore we represent the channel through $h[m,n] = h'[mP,n]$,

$$\boldsymbol{h}[m] = [h(mPT_C,0),\ldots,h(mPT_C,(L-1)T_C)]^T \in \mathbb{C}^{L\times 1} \tag{2}$$

in vector notation.

The receiver removes the cyclic prefix and performs a DFT. The received signal vector after these two operations is given by

$$\boldsymbol{y}[m] = \mathrm{diag}\left(\boldsymbol{g}[m]\right)\boldsymbol{S}\boldsymbol{b}[m] + \boldsymbol{v}[m] \in \mathbb{C}^{N\times 1} \tag{3}$$

where $\boldsymbol{S} = [\boldsymbol{s}_1,\ldots,\boldsymbol{s}_K] \in \mathbb{C}^{N\times K}$, and $\boldsymbol{b}[m] = [b_1[m],\ldots,b_K[m]]^T \in \mathbb{C}^{K\times 1}$ contains the stacked data symbols for K users. Complex additive white Gaussian noise with zero mean and covariance $\sigma_v^2\boldsymbol{I}_N$ is denoted by $\boldsymbol{v}[m] \in \mathbb{C}^{N\times 1}$. The frequency response $\boldsymbol{g}[m] \in \mathbb{C}^{N\times 1}$ is related to the impulse response via

$$\boldsymbol{g}[m] = \sqrt{N}\boldsymbol{F}_{N\times L}\boldsymbol{h}[m] \tag{4}$$

where $\boldsymbol{F}_N \in \mathbb{C}^{N\times N}$ is the unitary DFT matrix with elements

$$F_{i,j} = \frac{1}{\sqrt{N}}\,\mathrm{e}^{\frac{-\mathrm{j}2\pi ij}{N}}, \qquad i,j = 0,\ldots,N-1.$$

The signal model (3) is valid for TV channels when the ICI is small [12]. This is true when the (one sided) Doppler bandwidth B_D is much smaller than the subcarrier bandwidth $\Delta f = 1/(NT_C)$: $B_D = \varepsilon\Delta f$ for $0.01 > \varepsilon > 0$.

The receiver is a TV unbiased LMMSE filter [13], [14]

$$z_k[m] = (\boldsymbol{f}_k[m])^H\boldsymbol{y}[m]. \tag{5}$$

We define the TV effective spreading sequences

$$\tilde{\boldsymbol{s}}_k[m] = \mathrm{diag}\left(\boldsymbol{g}[m]\right)\boldsymbol{s}_k \in \mathbb{C}^{N\times 1} \tag{6}$$

to express the TV unbiased LMMSE filter

$$\boldsymbol{f}_k^H[m] = \frac{\tilde{\boldsymbol{s}}_k[m]^H(\sigma_v^2\boldsymbol{I} + \tilde{\boldsymbol{S}}[m]\tilde{\boldsymbol{S}}[m]^H)^{-1}}{\tilde{\boldsymbol{s}}_k[m]^H(\sigma_v^2\boldsymbol{I} + \tilde{\boldsymbol{S}}[m]\tilde{\boldsymbol{S}}[m]^H)^{-1}\tilde{\boldsymbol{s}}_k[m]}. \tag{7}$$

The resulting code symbol estimates are demapped, deinterleaved and decoded by a BCJR decoder [15] to obtain soft values for the transmitted data bits $\hat{\chi}_k[m'']$.

The performance of the receiver depends crucially on the model assumptions for $g[m]$.

III. Slepian Basis Expansion Model

Slepian [10] described a complete set of discrete prolate spheroidal (DPS) sequences that are most concentrated within a discrete time interval of length M and most band-limited to bandwidth W, $|W| < 1/2$. The DPS sequences $\zeta_i[m]$ are doubly orthogonal on the intervals $[-\infty,\infty]$ and $[0, M-1]$:

$$\sum_{m=0}^{M-1}\zeta_i[m]\zeta_j[m] = \frac{1}{\lambda_i}\sum_{m=-\infty}^{\infty}\zeta_i[m]\zeta_j[m] = \delta_{ij}, \tag{8}$$

for $i,j = 0,1,\ldots,M-1$. The vector $\boldsymbol{\zeta}_i \in \mathbb{R}^{M\times 1}$ obtained by index limiting the DPS sequences $\zeta_i[m]$ to $[0, M-1]$ is an eigenvector of matrix $\boldsymbol{C} \in \mathbb{R}^{M\times M}$ fulfilling

$$\boldsymbol{C}\boldsymbol{\zeta}_i = \lambda_i\boldsymbol{\zeta}_i.$$

The eigenvalues are denoted by λ_i and matrix $\boldsymbol{C}$ is defined as

$$[\boldsymbol{C}]_{i,j} = \frac{\sin 2\pi W(i-j)}{\pi(i-j)}, \quad i,j = 0,1,\ldots,M-1. \tag{9}$$

[1]We will use the following notation: All vectors are defined as columns vectors and denoted with bold lower case letters. Matrices are given in bold upper case, $(\cdot)^T$ denotes transpose, $(\cdot)^*$ denotes complex conjugate, $(\cdot)^H$ denotes Hermitian (i.e. complex conjugate) transpose and $\boldsymbol{I}_Q$ denotes the $Q\times Q$ identity matrix. The $Q\times R$ upper left part of matrix $\boldsymbol{A}$ is referenced as $\boldsymbol{A}_{Q\times R}$ and the element on ith row and jth column of matrix $\boldsymbol{A}$ is referenced by $[\boldsymbol{A}]_{i,j}$. The result of $\mathrm{diag}(\boldsymbol{a})$ is a diagonal matrix with the elements $[\boldsymbol{a}]_i$ on its main diagonal.

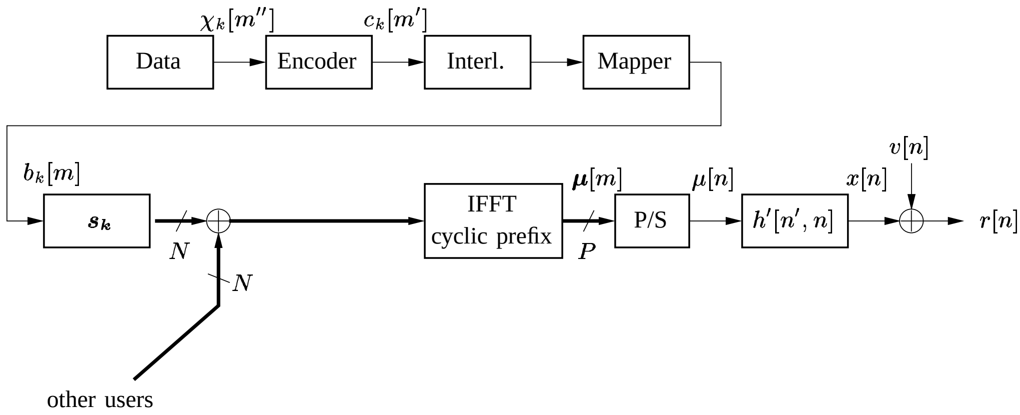

Fig. 1. Model for the MC-CDMA transmitter in the forward link.

We will use the term Slepian sequence to refer to the index limited sequence $\boldsymbol{\zeta}_i$ and DPS sequence to refer to the unlimited sequence $\zeta_i[m]$.

In our application we set

$$W = B_{\mathrm{Dmax}} T_S = \nu_{\mathrm{Dmax}} \tag{10}$$

and identify M with the number of symbols transmitted in a data block. The set of Slepian sequences corresponding to the system parameters M and W are the basis functions for the Slepian BEM. The eigenvalues λ_i decay to zero for index values greater than two times the time bandwidth product $2WM$. This is an indication that the approximate dimension of the signal space is limited to $2WM$ [10, Sec. 3.3].

The maximum Doppler frequency is much smaller than the symbol rate $B_{\mathrm{Dmax}} \ll R_S$. For a system with block length $M = 256$, symbol rate $R_S = 48\,\mathrm{kBaud/s}$, carrier frequency $f_C = 2\,\mathrm{GHz}$ and $v_{\max} = 100\,\mathrm{km/h}$ the maximum Doppler frequency $B_{\mathrm{Dmax}} = 185\,\mathrm{Hz}$ ($\nu_{\mathrm{Dmax}} = 0.004$) which gives $2WM + 1 = 3$. This is a very considerable saving in the number of parameters to describe a TV channel for a block length of $M = 256$.

We describe the TV impulse response $h[m, n]$ through the Slepian BEM

$$\tilde{h}[m, n] = \sum_{i=0}^{D'-1} a_i[n] \zeta_i[m], \tag{11}$$

where $D' = \lceil 2\kappa WM \rceil + 1$, $m = 0, \ldots, M-1$ and $n = 0, \ldots, L-1$. By choosing κ we can control the MSE of the Slepian BEM defined as

$$e = \frac{1}{LM} \sum_{m=0}^{M-1} \sum_{n=0}^{L-1} \left| h[m, n] - \tilde{h}[m, n] \right|^2 . \tag{12}$$

For a perfectly known channel $h[m, n]$ the coefficients in (11) are calculated through

$$a_i[n] = \sum_{m=0}^{M-1} h[m, n] \zeta_i[m], \qquad i = 0, 1, \ldots, D-1. \tag{13}$$

IV. Finite Slepian Basis Expansion Model

The Slepian sequences $\boldsymbol{\zeta}_i$ can be approximated by finite Slepian sequences $\boldsymbol{u}_i \in \mathbb{R}^{M \times 1}$. The eigenvalue problem for the finite Slepian sequences is numerically easier to calculate than for the Slepian sequences. In analogy to the previous section we define the finite Slepian sequences as the vector $\boldsymbol{u}_i$ obtained by index limiting the finite discrete prolate spheroidal (FDPS) sequences $u_i[m]$ to $[0, M-1]$.

The FDPS sequences $u_i[m]$ are orthonormal over the finite interval $[0, \eta M - 1]$

$$\sum_{m=0}^{\eta M - 1} u_i[m] u_j[m] = \delta_{ij}\,, \qquad i, j = 0, 1, \ldots, D-1,$$

and orthogonal over the interval $[0, M-1]$

$$\sum_{m=0}^{M-1} u_i[m] u_j[m] = \sigma_d^2 \delta_{ij}\,, \qquad i, j = 0, 1, \ldots, D-1. \tag{14}$$

For our application we link these two intervals by the integer parameter $\eta > 1$, $\eta \in \mathbb{Z}^+$.

The FDPS sequences $\tilde{\boldsymbol{u}}_i \in \mathbb{R}^{\eta M \times 1}$ with elements $u_i[m]$ are the left singular vectors of matrix $\boldsymbol{\mathcal{C}} \in \mathbb{C}^{\eta M \times M}$ with elements

$$[\boldsymbol{\mathcal{C}}]_{i,j} = \frac{1}{\eta M} \frac{\sin\left[\pi(2\lceil \eta WM \rceil + 1)(i-j)/(\eta M)\right]}{\sin\left[\pi(i-j)/(\eta M)\right]} \tag{15}$$

fulfilling

$$\boldsymbol{\mathcal{C}}\boldsymbol{\mathcal{C}}^H \tilde{\boldsymbol{u}}_i = \sigma_i^2 \tilde{\boldsymbol{u}}_i\,, \quad i = 0, \ldots, D-1. \tag{16}$$

The block length M and the bandwidth W are defined as in Section III. The singular values are denoted by σ_i. The rank of $\boldsymbol{\mathcal{C}}$ is $D = 2\lceil \eta WM \rceil + 1$.

The finite Slepian BEM is given by

$$\tilde{h}[m, n] = \sum_{i=0}^{D-1} \gamma_i[n] u_i[m], \quad m = 0, 1, \ldots, M-1. \tag{17}$$

For a perfectly known channel the parameters $\gamma_i[n]$ are calculated through

$$\gamma_i[n] = \sum_{m=0}^{M-1} h[m, n] u_i[m], \quad i = 0, 1, \ldots, D-1. \tag{18}$$

The finite Slepian sequences converge to the Slepian sequences for $\eta \to \infty$ [16]. This is also indicated by the convergence of the elements of the defining matrix: $\lim_{\eta\to\infty} [\mathcal{C}]_{i,j} = [\boldsymbol{C}]_{i,j}$. The finite Slepian sequences approximate the Slepian sequences for values $\eta \geq 2$ which we will show by numerical simulation in Section V.

We define the zero padded matrix

$$\tilde{\mathcal{C}} = \left[\mathcal{C}, \mathbf{0}_{\eta M\times(\eta-1)M}\right] \in \mathbb{R}^{\eta M\times\eta M}. \tag{19}$$

This allows us to explain the matrix that defines the finite Slepian sequences as a concatenation of a time-limiting operator to the discrete time interval $[0, M-1]$ and a band-limiting operator to the discrete frequency range $[-W, W]$ [16], [17].

To construct matrix $\tilde{\mathcal{C}}$ we define the time-limiting operator $\mathcal{D} \in \mathbb{R}^{\eta M\times\eta M}$ as the diagonal matrix

$$\mathcal{D} = \operatorname{diag}\left(\begin{bmatrix} \mathbf{1}_M \\ \mathbf{0}_{\eta M-M} \end{bmatrix}\right) \tag{20}$$

and the band-limiting operator $\mathcal{B} \in \mathbb{R}^{\eta M\times\eta M}$ in the frequency domain as diagonal matrix according to

$$\mathcal{B} = \operatorname{diag}\left(\begin{bmatrix} \mathbf{1}_{\lceil\eta WM\rceil+1} \\ \mathbf{0}_{\eta M-2\lceil\eta WM\rceil-1} \\ \mathbf{1}_{\lceil\eta WM\rceil} \end{bmatrix}\right).$$

Using this definition matrix $\tilde{\mathcal{C}}$ can be expressed by

$$\tilde{\mathcal{C}} = \boldsymbol{F}_{\eta M}^{-1}\mathcal{B}\boldsymbol{F}_{\eta M}\mathcal{D} \tag{21}$$

where $\boldsymbol{F}_{\eta M} \in \mathbb{C}^{\eta M\times\eta M}$ is the DFT matrix with

$$\{\boldsymbol{F}\}_{i,j} = \frac{1}{\sqrt{\eta M}}\,\mathrm{e}^{\frac{-\mathrm{j}2\pi ij}{\eta M}}, \qquad i,j = 0,\ldots,\eta M-1.$$

V. Mean Squared Error of the Slepian BEM for a Flat-Fading Time-Variant Channel

To demonstrate the merits of the finite Slepian BEM we compare the MSE of the Fourier BEM, the Slepian BEM and the finite Slepian BEM when used to approximate a flat-fading TV channel $h[m]$.

We model the sampled flat-fading TV channel $h[m]$ as superposition of 20 discrete paths according to the model defined in [18].

The averaged autocorrelation of $h[m]$ over different realizations converges to $R_{hh}(\tilde{m}) = J_0(2\pi\nu_D\tilde{m})$, where ν_D is the (one sided) Doppler bandwidth normalized to the symbol rate given by $\nu_D = \frac{vf_C}{c_0}T_S$ in the range $0 \leq \nu_D < \nu_{\mathrm{Dmax}} = B_{\mathrm{Dmax}}T_S$. The actual speed of the user is denoted by v, with $|v| < v_{\max}$.

It is important to note that the Slepian BEM does *not* depend on the knowledge of the autocorrelation of the TV channel. Usually the autocorrelation is only known for a dense scatterer model in the limit of an infinite number of scatterers. This assumption is not fulfilled in practical channels [19]. The Slepian BEM does only exploit the strict band limitation of $h[m]$ when the maximum allowed speed of the users v_{max} is defined as system parameter. The performance of the Slepian BEM is independent of the actual autocorrelation. Because of space limitation we show this in [20].

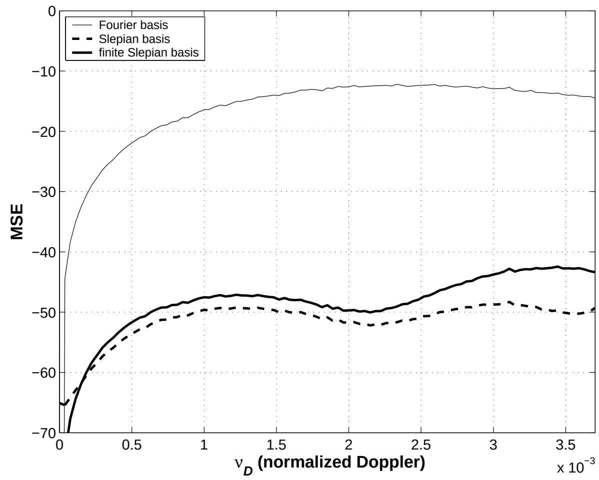

Fig. 2. Mean squared error (MSE) for the Fourier BEM, the Slepian BEM, and the finite Slepian BEM with $D = 5$.

The symbol duration is $T_S = (64 + 15)/3.84\,\mathrm{Mcps} = 20.57\,\mu\mathrm{s}$, the speed of the user is varied in the range $0 < v < v_{max} = 100\,\mathrm{km/h} = 27.7\,\mathrm{m/s}$. This results in a range of the doppler bandwidth of $0 \leq B_D \leq 180\,\mathrm{Hz}$ or normalized to the symbol duration: $0 \leq \nu_D \leq 0.0037$. The length of the data block is $M = 256$ symbols, the time index is restricted to $0 \leq m \leq M-1$. The parameters for the Slepian BEM are $\kappa = 2$ which results in $D' = 5$, for the finite BEM we choose $\eta = 2$, this gives $D = 5$. For the Fourier BEM (see Appendix) we choose $\kappa' = 2$ to get $D'' = 5$. All BEM use the same amount of parameters to model the channel $D = D' = D'' = 5$.

Figure 2 shows that the MSE for the Slepian BEM is 30dB smaller than the MSE of the Fourier BEM for the full range of ν_D.

VI. MC-CDMA Forward Link

To generate the TV channel realization for the frequency-selective TV channel $h'[n', n]$ we use the model defined in the previous section for every channel tap of the exponentially decaying typical urban (TU) power-delay profile (PDP) from COST 259 [21] with a delay spread $L = 15$ corresponding to $T_D = 3.9\mu\mathrm{s}$.

The system is operated at carrier frequency $f_C = 2\,\mathrm{GHz}$, the users move with velocity $v = 70\,\mathrm{km/h}$, this gives $B_D = 126\,\mathrm{Hz}$. The number of subcarriers $N = 64$ and the OFDM symbol with cyclic prefix has length of $P = G + N = 79$. The chip rate $1/T_C = P/T_S = 3.84\,\mathrm{Mcps}$. The data block length $M = 256$. For the finite Slepian BEM with $\eta = 2$ this results in $D = 5$.

For data transmission the convolutional code used, is a non-systematic, non-recursive, 4 state, rate $R_C = 1/2$ code with generator polynomial $(5, 7)_8$. We average all simulations over 100 independent channel realizations. The QPSK symbol

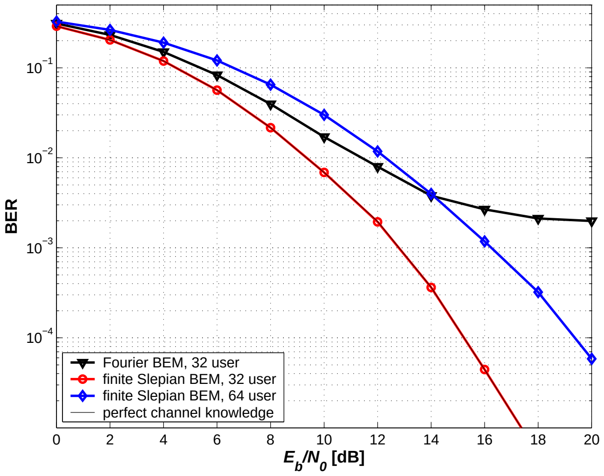

Fig. 3. Forward link MC-CDMA receiver performance in terms of BER versus SNR. We compare the finite Slepian BEM and the Fourier BEM, both with $D = 5$. The users are moving with $v = 70$ km/h.

energy is normalized to 1, the E_b/N_0 is therefore defined as

$$\frac{E_b}{N_0} = \frac{1}{2R_C\sigma_v^2}\frac{P}{N}.$$

We give the forward link MC-CDMA receiver performance with $K = \{32, 64\}$ users and perfectly known *model parameters* $\gamma_i[n]$ for the finite Slepian BEM and compare it with the Fourier BEM. Additionally we also give the performance for the perfectly known channel $\boldsymbol{g}[m]$, see Fig. 3. The performance of the finite Slepian BEM is equal to the one for a perfectly known channel.

VII. Conclusion

We showed that the finite Slepian BEM is very suitable to model a TV frequency-selective channel for the duration of a data block. The finite Slepian BEM is designed according to two system parameters: the maximum Doppler bandwidth B_{Dmax} and the block length M. The MSE of the finite Slepian BEM is 30dB smaller than the MSE of the Fourier BEM. We apply the finite Slepian BEM for channel equalization in a multi-user MC-CDMA forward link for a frequency-selective TV channel. For the finite Slepian BEM the necessary $E_b/N_0 = 10$ dB for a BER$= 7\cdot 10^{-3}$ which is 2dB better than for the Fourier BEM. There is no error floor behavior for the finite Slepian BEM compared to the Fourier BEM.

Appendix

The Fourier BEM is defined as

$$\tilde{h}'[m,n] = \sum_{i=0}^{D''-1} \beta_i[n] f_i[m], \quad m = 0, \ldots, M-1 \qquad (22)$$

where

$$f_i[m] = \mathrm{e}^{\frac{\mathrm{j}2\pi(i-(D''-1)/2)m}{M}} \qquad (23)$$

and $D'' = 2\lceil \kappa' W M \rceil + 1$. By choosing κ' we can control the MSE of the Fourier BEM. We calculate the Fourier BEM parameters according to

$$\beta_i[n] = \sum_{m=0}^{M-1} h[m,n] f_i[m], \quad i = 0, 1, \ldots, D''-1. \qquad (24)$$

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