Improved Channel Estimation for Iterative Receivers

Thomas Zemen
Siemens Austria, PSE PRO RCD
Erdbergerlände 26
A-1031 Vienna, Austria
E-mail: thomas.zemen@siemens.com

Maja Lončar[1]
Dept. of Information Technology
Lund University
P.O.Box 118,
SE-22100 Lund, Sweden
Email: maja@it.lth.se

J. Wehinger, C. Mecklenbräuker and R. Müller
Telecommunication Research Center Vienna (ftw.)
Tech Gate Vienna, Donau-City Str. 1/III
A-1220 Vienna, Austria
Email: {wehinger, cfm, mueller}@ftw.at

***Abstract*— In iterative receiver structures, soft information becomes available after the decoding stage. This information is used to enhance the quality of the channel estimates for the next iteration. We derive a generalized estimator based on the linear minimum mean square error (LMMSE) principle for deterministic pilot information combined with soft information. We present the special case of multi-carrier code division multiple access (MC-CDMA) in detail and provide simulation results. The presented channel estimation algorithm can be also applied to direct sequence (DS)-CDMA and multiple-input multiple-output (MIMO) systems.**

I. Introduction

An iterative receiver feeds back soft values on code bits to get better detection results and enhanced channel estimates. The soft feedback values are computed from a-posteriori probabilities (APP) and extrinsic probabilities (EXT) [1]–[4]. A soft-input soft-output (SISO) decoder for binary convolutional codes, implemented using the BCJR algorithm [5], supplies these measures. The input values to the decoder are derived from the minimum mean square error (MMSE)-filter output after demapping and deinterleaving.

Channel estimation is a crucial part of a multi user receiver. The least-squares algorithm (LS) suffers from slow convergence with increasing load $\beta = K/N$. The number of users in the system is denoted by K, the length of the spreading sequence by N. Our contribution is the derivation of an improved channel estimator based on the linear minimum mean square error (LMMSE) criterion, which also takes into account the variances in the estimated data symbols. See also [6] for a related approach in a single-user scenario.

In this work we present an iterative receiver for the uplink of a multi-carrier code division multiple access (MC-CDMA) system. MC-CDMA is based on orthogonal frequency division multiplexing (OFDM) [7], where the spreading codes to distinguish each user are applied in the frequency domain; the chips are therefore transmitted over different subcarriers.

We present the multi-user signal model in section II. A short description of the data detection and decoding algorithm is given in section III. We present the LS channel estimation algorithm in section IV. In section V the new LMMSE channel estimation algorithm is derived and presented together with simulation results. Finally, we give hints on how to apply this new algorithm to other communication systems in section VI and summarize the work with concluding remarks in section VII.

The work is funded by the Radio Communication Devices department (RCD), part of the Siemens AG Austria, Program and System Engineering (PSE) and the Telecommunications Research Center Vienna (ftw.) in the C0 project.

[1]Maja Lončar was with Telecommunication Research Center Vienna (ftw.) when she performed the work for this paper.

II. Signal Model

We will give a short overview of MC-CDMA, details can be found in [1]. The block structure of the individual MC-CDMA transmitter is shown in Fig. 1. Each user transmits quadrature phase shift keying (QPSK) modulated symbols $b_k(m)$ in blocks of length M. Each symbol is spread by a random spreading sequence $\boldsymbol{s}_k$ of length N and each chip is transmitted over an individual subcarrier. The number of subcarriers is equal to the length of the spreading sequence. The elements of the spreading sequence are randomly chosen from the QPSK constellation set $(\{\pm 1 \pm \mathrm{j}\})/(\sqrt{2N})$ satisfying

$$\sum_{n=0}^{N-1} |s_k(n)|^2 = 1 \ \forall k.$$

We will use the following notation: All vectors are defined as columns vectors and denoted with bold lower case letters. Matrices are given in bold upper case, $(\cdot)^T$ denotes transpose, $(\cdot)^*$ denotes complex conjugate, $(\cdot)^H$ denotes Hermitian (i.e. complex conjugate) transpose and $\boldsymbol{I}_N$ denotes the $[N \times N]$ identity matrix. The $[M \times N]$ upper left part of matrix $\boldsymbol{A}$ is referenced as $\boldsymbol{A}_{M\times N}$ and the element in ith row and ℓth column of matrix $\boldsymbol{A}$ is referenced by $[\boldsymbol{A}]_{i,\ell}$. The result of $\mathrm{diag}(\boldsymbol{a})$ is a diagonal matrix with the elements of $\boldsymbol{a}$ on its main diagonal.

The first J QPSK symbols in each block are pilot symbols. The remaining $M - J$ data symbols result from the convolutionally encoded, randomly interleaved and QPSK modulated binary information sequence $\chi_k(m'')$ of length $2(M - J)R_C$ by applying Gray labeling. The code rate is denoted by R_C.

After the spreading operation $\boldsymbol{s}_k b_k(m) a_k$, an N point inverse discrete Fourier transform (IDFT) is performed and a cyclic prefix (CP) with length G is inserted [8]–[10]. The amplitude of user k is denoted by a_k. Here all a_k are equal.

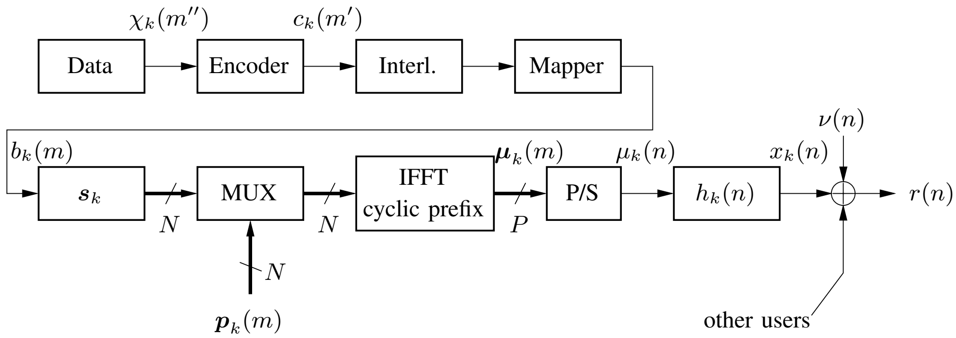

Fig. 1. Model for the MC-CDMA transmitter.

The resulting signal is transmitted over a multipath Rayleigh fading channel with block fading characteristic, i.e. we assume the channel to remain constant over M symbols. The multipath fading channel $h_k(n)$ has a delay spread of L chips, $\boldsymbol{h}_k = [h_k(0), h_k(1), \cdots, h_k(L-1)]$ in vector notation. To completely remove the resulting intersymbol interference (ISI) it is required that $G > L$. The length of an OFDM symbol in chip-time after insertion of the cyclic prefix is denoted by $P = N + G$.

The receiver removes the CP and performs a discrete Fourier transform (DFT). The received signal vector in a multi-user system is given by

$$\boldsymbol{y}(m) = \tilde{\boldsymbol{S}}\boldsymbol{b}(m) + \boldsymbol{\nu}(m) \tag{1}$$

where $\boldsymbol{\nu}(m)$ denotes complex additive white Gaussian noise with zero mean and covariance $\sigma_\nu^2 \boldsymbol{I}_N$. Vector $\boldsymbol{b}(m) = [b_1(m), b_2(m), \cdots, b_K(m)]^T$ contains the stacked data symbols for K users, and matrix $\tilde{\boldsymbol{S}} = [\tilde{\boldsymbol{s}}_1, \cdots, \tilde{\boldsymbol{s}}_K]$ is the effective spreading matrix. The effective spreading sequences are defined as

$$\tilde{\boldsymbol{s}}_k = \operatorname{diag}(\boldsymbol{g}_k)\, \boldsymbol{s}_k, \tag{2}$$

where

$$\boldsymbol{g}_k = \sqrt{N}\boldsymbol{F}_{N\times L}\boldsymbol{h}_k. \tag{3}$$

The unitary DFT matrix $\boldsymbol{F}$ has elements

$$[\boldsymbol{F}]_{i,\ell} = \frac{1}{\sqrt{N}}\,\mathrm{e}^{\frac{-\mathrm{j}2\pi i\ell}{N}}, \quad i,\ell = 0\ldots N-1.$$

For further details see [1].

III. Data Detection and Decoding

Our receiver detects the data $\boldsymbol{b}(m)$ using the received chip sequence $\boldsymbol{y}(m)$, the estimated effective spreading matrix $\tilde{\boldsymbol{S}}^{(i)}$, and the fed back extrinsic probabilities $\mathrm{EXT}(c_k^{(i)}(m'))$ of the code symbols at iteration step i as shown in Fig. 2. The multiuser detection is based on parallel interference cancellation (PIC) and MMSE filtering. The output of the MMSE filter is used as input to a soft-input soft-output (SISO) decoder for binary convolutional codes, implemented using the BCJR algorithm [5]. The soft feedback values used to enhance the channel estimation are computed from the a-posteriori probabilities (APP) that the decoder supplies. For more details see [1].

IV. Least Squares Channel Estimation

The importance of having a good channel estimate is made obvious by (2), since the filter matched to the effective spreading sequence depends directly on the quality of the channel estimate.

The J resulting chip sequences $\boldsymbol{p}_k(m)$ are randomly chosen in the same way as the spreading sequences $\boldsymbol{s}_k$. They construct the OFDM pilot symbols for user k (see Fig. 1).

A least squares estimate of the channels $\hat{\boldsymbol{g}}_k$ in the frequency domain can be obtained jointly for all K users but individually for every subcarrier q. We define the vector of channel coefficients $\hat{\boldsymbol{g}}_q = [\hat{g}_1(q), \hat{g}_2(q), \cdots, \hat{g}_K(q)]^T$ where $\hat{g}_k(q)$ denotes the channel coefficient for user k and subcarrier q. The same notation is introduced for $\boldsymbol{y}_q = [y_q(0), y_q(1), \cdots, y_q(M-1)]^T$. The received signal on subcarrier q and at discrete time m is given by $y_q(m)$.

We define the matrix

$$\boldsymbol{P}_q = \begin{pmatrix} p_{1,q}(0) & \cdots & p_{K,q}(0) \\ \vdots & \ddots & \vdots \\ p_{1,q}(J-1) & \cdots & p_{K,q}(J-1) \end{pmatrix}$$

that contains the pilot spreading coefficients $p_{k,q}(m)$ for user k at discrete time m and subcarrier q.

The APP

$$\tilde{b'}_k(m) = \varphi\big(c_k(2m)\big) + \mathrm{j}\varphi\big(c_k(2m+1)\big) \tag{4}$$

from the decoder is mapped to QPSK symbols and used as additional pilots to further refine the channel estimates [11] where $\varphi(\cdot) = 2\,\mathrm{APP}(\cdot) - 1$. For the first iteration these values are set to zero. Matrix $\boldsymbol{Q}_q$ is defined as

$$\boldsymbol{Q}_q = \begin{pmatrix} s_1(q)\tilde{b'}_1(J) & \cdots & s_K(q)\tilde{b'}_K(J) \\ \vdots & \ddots & \vdots \\ s_1(q)\tilde{b'}_1(M-1) & \cdots & s_K(q)\tilde{b'}_K(M-1) \end{pmatrix}$$

and combined with the pilots to the matrix

$$\tilde{\boldsymbol{B}}_q = \begin{pmatrix} \boldsymbol{P}_q \\ \boldsymbol{Q}_q \end{pmatrix}. \tag{5}$$

The least squares estimator is given by

$$\hat{\boldsymbol{g}}_{q,\mathrm{LS}} = \left(\tilde{\boldsymbol{B}}_q^H \tilde{\boldsymbol{B}}_q\right)^{-1} \tilde{\boldsymbol{B}}_q^H \boldsymbol{y}_q. \tag{6}$$

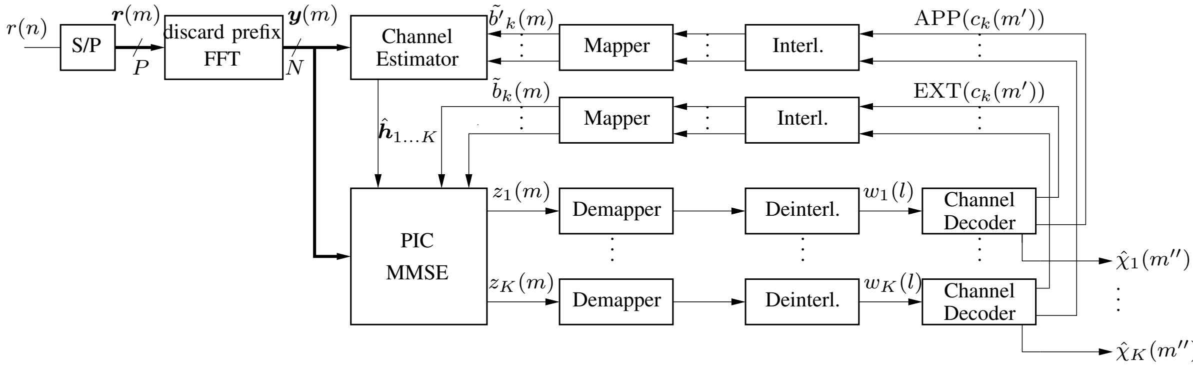

Fig. 2. Model for the MC-CDMA joint channel estimation and decoding multi-user receiver.

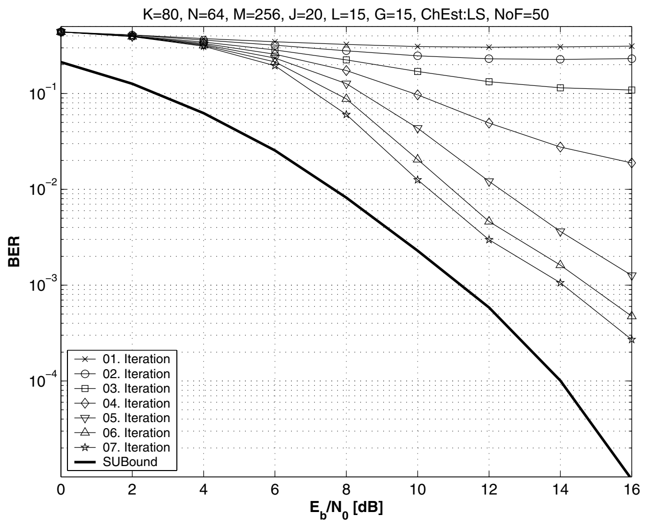

Fig. 3. Receiver performance in terms of BER versus SNR for $K = 80$ users after iteration 1 to 7 with LS channel estimation (6) [1], spreading length and number of subcarriers $N = 64$, delay spread $L = 15$, length of cyclic prefix $G = 15$ and $J/M = 20/256 = 7.8\%$ pilot symbols.

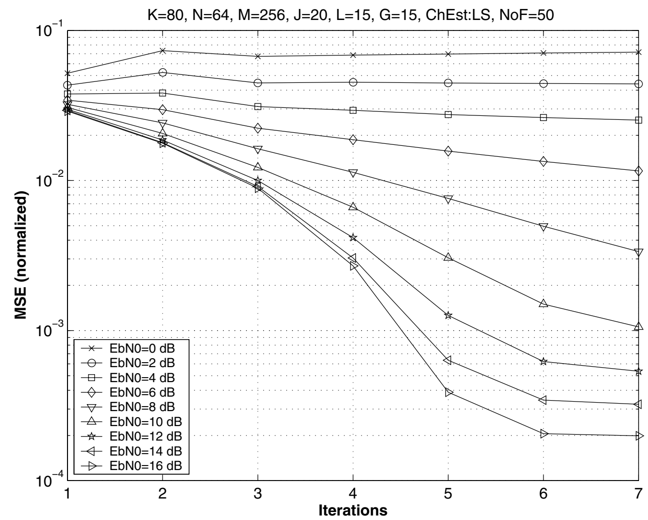

Fig. 4. Mean square error (MSE) of the channel estimates after iteration 1 to 7 and different SNR for $K = 80$ users with LS channel estimation (6) [1], spreading length and number of subcarriers $N = 64$, delay spread $L = 15$, length of cyclic prefix $G = 15$ and $J/M = 20/256 = 7.8\%$ pilot symbols.

The channel impulse response in the time domain $\boldsymbol{h}_k$ has L taps and can be estimated by

$$\hat{\boldsymbol{h}}_k = \frac{1}{\sqrt{N}} \boldsymbol{F}^H_{N \times L}\, \hat{\boldsymbol{g}}_k$$

which also reduces the noise in the channel estimates. The estimates $\hat{\boldsymbol{h}}_k$ are supplied to the PIC and MMSE detector. In the first iteration, the missing energy from $\hat{h}_k(n), n = L+1 \dots N-1$ (assumed to be zero) is compensated by scaling with $\sqrt{N/L}$, i.e.

$$\hat{\boldsymbol{h}}^{(1)}_k = \frac{1}{\sqrt{N}} \boldsymbol{F}^H_{N \times L}\, \hat{\boldsymbol{g}}^{(1)}_k \sqrt{\frac{N}{L}}.$$

The absolute values of the soft symbols in matrix $\boldsymbol{Q}_q$ can be very small, particularly during the first iteration and in overloaded systems ($\beta = K/N > 1$) due to strong interference. This leads to slow convergence. A partial heuristic solution is the normalization for each column of $\boldsymbol{B}_q$ to $\sqrt{M/N}$ which was introduced in [1].

The simulation results in terms of BER vs. SNR for the least squares channel estimator are given in Fig. 3. The mean-squared error (MSE) of the channel estimate is given in Fig. 4. Here it can be clearly seen, that the MSE has a slight increase after the second iteration particularly in the low E_b/N_0 region for a system with load $\beta = 1.25$ and $K = 80$ users.

To demonstrate the performance of the system we use the typical urban (TU) power-delay-profile (PDP) from COST 259 [12], the chip rate is 3.84 Mcps as in UMTS and the delay spread $L = 15$ corresponds to 3.9μs. The channel impulse response is normalized so that

$$\sum_{n=0}^{L-1} \mathrm{E}\left\{ |h_k(n)|^2 \right\} = 1 \;\; \forall k.$$

The number of iterations is limited to 7. The single-user bound (SUB) is taken as a reference for the multi-user receiver performance. In this context it is defined as the receiver performance with $K = 1$ and perfect channel knowledge. The spreading sequence has length $N = 64$ equal to the number of subcarriers. The complete OFDM symbol with cyclic prefix has length $P = G + N = 79$. The convolutional code used is a non-systematic, non-recursive, 4 state, rate $R_C = 1/2$ code with generator polynomials $(5, 7)_8$. All simulations are averaged over 50 independent channel realizations. The energy of the transmitted QPSK symbols is normalized to 1, the E_b/N_0 is therefore defined as

$$\frac{E_b}{N_0} = \frac{1}{2R_C\sigma_\nu^2}\frac{P}{N}\frac{M}{M-J}.$$

To further enhance the channel estimation quality we will derive a LMMSE channel estimator in the next section that also takes into account the statistical information about the estimated data symbols.

V. Linear MMSE Channel Estimation

Minimization of the mean square estimation error (MSE) is a widely used optimization criterium in parameter estimation. We use this approach to estimate the channel coefficients. In general, for a linear data model, if the observed data and the unknown parameters are jointly Gaussian distributed, the resulting MMSE estimator is a linear function of the data. In our model (1), we assume that channel coefficients in the vector $\boldsymbol{g}_q$ are independent, complex Gaussian distributed random variables, with zero mean and unit variance (which corresponds to Rayleigh fading), while symbols in the matrix $\boldsymbol{B}_q$ have discrete binary distribution determined by the probabilities $\Pr\left(b_k(m)=+1\right)$. The resulting distribution of the observed vector $\boldsymbol{y}_q$ is not Gaussian, thus the MMSE estimator is not linear. Due to the shape of the pdf $f(\boldsymbol{y}_q)$, the derivation of the exact MMSE estimator is a complicated task. Therefore, we constrain the channel estimator to be linear in $\boldsymbol{y}_q$ (the index $(\cdot)_q$ will be omitted in the following derivations for the sake of a simplified notation):

$$\hat{\boldsymbol{g}}_{\text{LMMSE}} = \boldsymbol{A}\boldsymbol{y},$$

then matrix $\boldsymbol{A}$ satisfies the Wiener-Hopf equation:

$$\boldsymbol{C}_{\boldsymbol{yy}}\boldsymbol{A}^H = \boldsymbol{C}_{\boldsymbol{yg}},$$

where the covariance matrices are given by ($\boldsymbol{y}$, $\boldsymbol{g}$ and $\boldsymbol{\nu}$ are zero-mean):

$$\begin{aligned}\boldsymbol{C}_{\boldsymbol{yy}} = \underset{\boldsymbol{B}}{\mathrm{E}}\,\underset{\boldsymbol{g}}{\mathrm{E}}\,\underset{\boldsymbol{\nu}}{\mathrm{E}}\{\boldsymbol{y}\boldsymbol{y}^H\} &= \underset{\boldsymbol{B}}{\mathrm{E}}\{\boldsymbol{B}\boldsymbol{B}^H\} + \sigma_\nu^2\boldsymbol{I}_M;\\ \boldsymbol{C}_{\boldsymbol{yg}} = \underset{\boldsymbol{B}}{\mathrm{E}}\,\underset{\boldsymbol{g}}{\mathrm{E}}\,\underset{\boldsymbol{\nu}}{\mathrm{E}}\{\boldsymbol{y}\boldsymbol{g}^H\} &= \underset{\boldsymbol{B}}{\mathrm{E}}\{\boldsymbol{B}\} \triangleq \tilde{\boldsymbol{B}},\end{aligned} \quad (7)$$

where the index under the expectation operator denotes the random variable with respect to which the expectation is taken. Expectations with respect to $\boldsymbol{B}$ are computed using APPs of data symbols, see (4) and (5). Unless stated otherwise, all expectations in the subsequent expressions are taken with respect to $\boldsymbol{B}$, therefore we omit the index to simplify the notation. The linear MMSE (LMMSE) estimator is then:

$$\begin{aligned}\hat{\boldsymbol{g}}_{\text{LMMSE}} &= {\boldsymbol{C}_{\boldsymbol{yg}}}^H{\boldsymbol{C}_{\boldsymbol{yy}}}^{-1}\boldsymbol{y} =\\ &= \tilde{\boldsymbol{B}}^H\left(\mathrm{E}\{\boldsymbol{B}\boldsymbol{B}^H\} + \sigma_\nu^2\boldsymbol{I}_M\right)^{-1}\boldsymbol{y}.\end{aligned} \quad (8)$$

For evaluation of this estimator it is necessary to invert an M-dimensional matrix, which is computationally expensive. Therefore we will reformulate (8) in order to reduce the dimensionality. To this end we first note that due to the independence of the users and the data symbols within one block, it holds:

$$\begin{gathered}\forall\ i,j \in \{1,...,K\},\quad \forall\ m,n \in \{0,1,...,M-1\}\\ \mathrm{E}\{b_i(m)b_j(n)\} = \begin{cases}\tilde{b}_i(m)\tilde{b}_j(n), & i \neq j, m \neq n\\ 1, & i = j, m = n\end{cases}\end{gathered} \quad (9)$$

Hence the elements of the covariance matrix $\mathrm{E}\{\boldsymbol{B}\boldsymbol{B}^H\}$ are equal to the elements of the matrix $\mathrm{E}\{\boldsymbol{B}\}\mathrm{E}\{\boldsymbol{B}^H\} = \tilde{\boldsymbol{B}}\tilde{\boldsymbol{B}}^H$, *except* for the main diagonal elements. In other words,

$$\mathrm{E}\{\boldsymbol{B}\boldsymbol{B}^H\} = \tilde{\boldsymbol{B}}\tilde{\boldsymbol{B}}^H + \boldsymbol{\Lambda}. \quad (10)$$

The elements of the diagonal matrix $\boldsymbol{\Lambda}$ are defined as:

$$\boldsymbol{\Lambda}_{mm} = \sum_{k=1}^{K}\mathrm{var}\{b_k(m)\}, \quad (11)$$

where the symbol variance is

$$\mathrm{var}\{b_k(m)\} = \mathrm{E}\left\{\left(b_k(m) - \mathrm{E}\{b_k(m)\}\right)^2\right\} = 1 - \tilde{b}_k^2(m).$$

Inserting (10) into (8) we obtain:

$$\hat{\boldsymbol{g}}_{\text{LMMSE}} = \tilde{\boldsymbol{B}}^H\left(\tilde{\boldsymbol{B}}\tilde{\boldsymbol{B}}^H + \underbrace{\boldsymbol{\Lambda} + \sigma_\nu^2\boldsymbol{I}_M}_{\triangleq\boldsymbol{\Delta}}\right)^{-1}\boldsymbol{y}. \quad (12)$$

After applying the matrix inversion lemma to (12), the final expression yields:

$$\hat{\boldsymbol{g}}_{\text{LMMSE}} = \left(\tilde{\boldsymbol{B}}^H\boldsymbol{\Delta}^{-1}\tilde{\boldsymbol{B}} + \boldsymbol{I}_K\right)^{-1}\tilde{\boldsymbol{B}}^H\boldsymbol{\Delta}^{-1}\boldsymbol{y}. \quad (13)$$

The matrix to be inverted in (13) is K-dimensional, which is less than (8) that is M-dimensional. The rows of matrix $\boldsymbol{B}$ are scaled by the diagonal matrix $\boldsymbol{\Delta}$, taking into account the variances of the noise and of the soft symbol estimates.

If the data symbols are known, then $\tilde{b}_k(m) = b_k(m)$, thus $\boldsymbol{\Lambda} = \boldsymbol{0}$, $\boldsymbol{\Delta} = \sigma_\nu^2\boldsymbol{I}_K$ and the estimator (13) becomes the exact MMSE estimator (conditioned on the given $\boldsymbol{B}$, $\boldsymbol{g}$ and $\boldsymbol{y}$ are jointly Gaussian, thus the LMMSE coincides with the true MMSE estimator):

$$\hat{\boldsymbol{g}}_{\text{MMSE}|\boldsymbol{B}} = \left(\boldsymbol{B}^H\boldsymbol{B} + \sigma_\nu^2\boldsymbol{I}_K\right)^{-1}\boldsymbol{B}^H\boldsymbol{y}. \quad (14)$$

For the training part of the data burst (first J symbols), the estimator will be equal to the exact MMSE estimator for the given pilot symbols. For the data part the variances of the estimates of the unknown data symbols are taken into account by matrix $\boldsymbol{\Delta}$.

The simulation results for this estimator are given in Fig. 5 and Fig. 6. The parameters are the same as in the LS case (see section IV). When Fig. 6 is compared with the results obtained with the LS estimator in Fig. 4, it can be clearly seen that now the MSE is a monotonic decreasing function of the iteration number and the SNR. The BER performance in Fig. 5 now shows no error floor behavior.

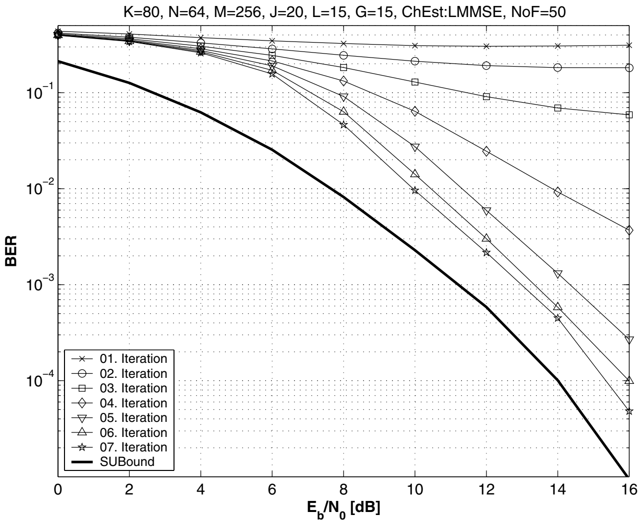

Fig. 5. Receiver performance in terms of BER versus SNR for $K = 80$ users after iteration 1 to 7 with LMMSE channel estimation (13), spreading length and number of subcarriers $N = 64$, delay spread $L = 15$, length of cyclic prefix $G = 15$ and $J/M = 20/256 = 7.8\%$ pilot symbols.

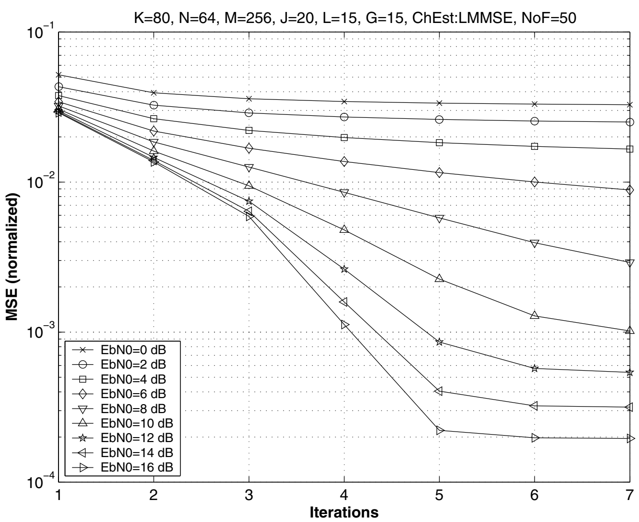

Fig. 6. Mean square error (MSE) of the channel estimates after iteration 1 to 7 and different SNR for $K = 80$ users with LMMSE channel estimation (13), spreading length and number of subcarriers $N = 64$, delay spread $L = 15$, length of cyclic prefix $G = 15$ and $J/M = 20/256 = 7.8\%$ pilot symbols.

VI. Other Communication Systems

The multi-user signal model given in (1) can be used accordingly for DS-CDMA after redefining the effective spreading matrix, as shown in [2]. For multiple-input multiple-output (MIMO) systems a similar approach can be taken, the full details are given in [3].

The estimator we developed in this paper for the MC-CDMA case can also be used for iterative receivers for DS-CDMA and MIMO systems. The modifications that need to be made is to take into account the different structure of the matrix $\boldsymbol{B}$, and to properly apply the rule in (9) for the soft symbol estimates. For further details see [13] and [3].

VII. Conclusions

In this work we presented an improved channel estimation algorithm for an iterative receiver structure in the uplink of a MC-CDMA system. We compared the performance in terms of MSE of the new LMMSE based algorithm with an LS estimator using simulation results. The LMMSE based channel estimation scheme improves the convergence speed of the iterative receiver. Furthermore, the performance in terms of BER is also enhanced especially in the high SNR region where the error floor of the LS based estimator is removed. A BER of 10^{-3} is reached after 7 iterations at $E_b/N_0 = 13\,\text{dB}$ with the LMMSE based channel estimation, compared to the LS estimator where $E_b/N_0 = 14\,\text{dB}$ is necessary for the same BER.

References

[1] T. Zemen, J. Wehinger, C. Mecklenbräuker, and R. Müller, "Iterative detection and channel estimation for MC-CDMA," in *IEEE International Conference on Communications, ICC, Anchorage (Alaska), USA*, May 2003.

[2] J. Wehinger, R. Müller, M. Lončar, and C. Mecklenbräuker, "Performance of iterative CDMA receivers with channel estimation in multipath environments," in *Asilomar Conference on Signals, Systems and Computers, Pacific Grove (CA), USA*, 2002.

[3] M. Lončar, R. Müller, J. Wehinger, C. Mecklenbräuker, and T. Abe, "Iterative channel estimation and data detection in frequency selective fading MIMO channels," submitted to *European Transcations on Telecommunications*, 2003.

[4] M. Kobayashi, J. Boutros, and G. Caire, "Successive interference cancellation with SISO decoding and EM channel estimation," *IEEE J. Select. Areas Commun.*, vol. 19, no. 8, Aug. 2001.

[5] L. Bahl, J. Cocke, J. F., and R. J., "Optimal decoding of linear codes for minimizing symbol error rate," *IEEE Trans. Inform. Theory*, vol. 20, pp. 284–287, Mar. 1974.

[6] M. Tüchler, R. Koetter, and A. Singer, "Turbo equalization: Principles and new results," *IEEE Trans. Commun.*, vol. 50, no. 5, pp. 754–767, May 2002.

[7] H. Rohling, T. May, K. Brüninghaus, and R. Grünheid, "Broad-band OFDM radio transmission for multimedia applications," *Proc. IEEE*, vol. 87, no. 10, pp. 1778–1789, October 1999.

[8] J. A. C. Bingham, "Multicarrier modulation for data transmission: An idea whose time has come," *IEEE Trans. Commun.*, pp. 5–14, 1990.

[9] Z. Wang and G. B. Giannakis, "Wireless multicarrier communications," *IEEE Signal Processing Mag.*, vol. 17, no. 3, pp. 29–48, May 2000.

[10] S. Zhou, G. B. Giannakis, and A. Swami, "Digital multi-carrier spread spectrum versus direct sequence spread spectrum for resistance to jamming and multipath," *IEEE Trans. Commun.*, vol. 50, no. 4, pp. 643–655, April 2002.

[11] P. Alexander and A. Grant, "Iterative channel and information sequence estimation in CDMA," in *IEEE Int. Symp. on Spread-Spectrum Tech. & Appli., NJIT*, 2000, pp. 593–597.

[12] L. M. Correia, *Wireless Flexible Personalised Communications*. Wiley, 2001.

[13] J. Wehinger, C. Mecklenbräuker, R. Müller, T. Zemen, and M. Lončar, "On channel estimators for iterative CDMA multi-user receivers in flat fading," submitted to *IEEE International Conference on Communications, ICC*, 2004.