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TIME VARIANT CHANNEL EQUALIZATION FOR MC-CDMA VIA FOURIER BASIS FUNCTIONS

Abstract. When users move at vehicular speed the radio channel is time variant. Sayeed et al. proposed a Fourier basis expansion model for time variant channels. We apply this concept to MC-CDMA for channel equalization and give simulation results for the forward link. To gain further insights we give a detailed discussion of the benefits and weaknesses of the Fourier basis expansion channel model.

1. INTRODUCTION

When users move at vehicular speed the radio channel is time variant. For channel equalization an accurate channel model is required otherwise the time variation is a significant cause of symbol errors.

The Doppler effect in multipath propagation is the starting point for the channel description given by Höher in [4]. This description depends in nonlinear manner on the Doppler frequencies for each individual path which is a major obstacle when used for channel equalization.

The representation of the time variant radio channel by means of the scattering function was used by Bello [2]. The continuous time equation that links the impulse response $h(t,\tau)$ to the scattering function $S_{\boldsymbol{H}}(\omega,\tau)$ is subsequently discretized in both time and frequency. The discretization is implemented by replacing the Fourier integral with a finite sum [8, 7]. Therefore the time variant impulse response is expanded linearly in terms of Fourier basis functions.

In this paper our contribution is to analyze the performance of a multi carrier (MC) code division multiple access (CDMA) forward link when the time variant channel is modeled by the Fourier Basis Expansion.

The paper is organized as follows: We introduce the signal model for MC-CDMA in Sec. 2 and describe the Fourier basis expansion model in Sec. 3. We present simulation results and analyze the approximation properties of the Fourier basis expansion model in Sec. 4. We conclude with some remarks in Sec. 5.

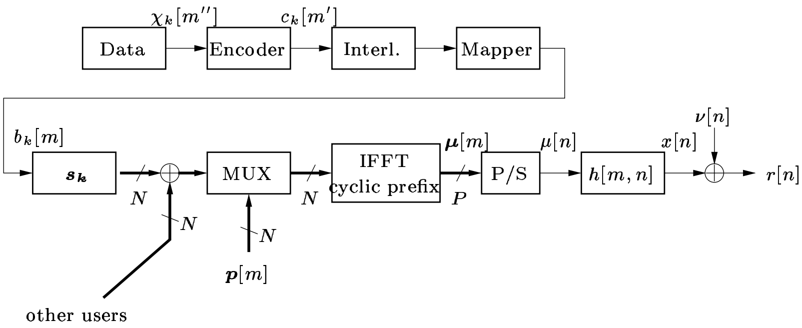

Figure 1. Model for the MC-CDMA transmitter in the forward link.

2. SIGNAL MODEL

In [11] we presented a MC-CDMA multi user receiver for *block fading* channels. MC-CDMA is based on orthogonal frequency division multiplexing (OFDM). Spreading codes are applied in the frequency domain to distinguish each user. Therefore each chip is transmitted over a different subcarrier.

In this paper we will deal with channels which vary significantly over the duration of a data block. To focus the presentation we will limit ourselves to the forward link. The base station transmits quaternary phase shift keying (QPSK) modulated symbols $b_k[m]$ drawn from the alphabet $\frac{1}{\sqrt{2}}\{\pm 1 \pm \mathrm{j}\}$ in blocks of length M. Discrete time is denoted by m. There are K users in the system, the user index is denoted by k. Each symbol is spread by a unique Walsh-Hadamard sequence $\boldsymbol{s}_k$ of length N with elements $\frac{1}{\sqrt{N}}\{\pm 1\}$, see Fig. 1.

We will use the following notation: All vectors are defined as columns vectors and denoted with bold lower case letters. Matrices are given in bold upper case, $(\cdot)^T$ denotes transpose, $(\cdot)^*$ denotes complex conjugate, $(\cdot)^H$ denotes Hermitian (i.e. complex conjugate) transpose and $\boldsymbol{I}_N$ denotes the $N \times N$ identity matrix. The $M \times N$ upper left part of matrix $\boldsymbol{A}$ is referenced as $\boldsymbol{A}_{M\times N}$ and the element on ith row and ℓth column of matrix $\boldsymbol{A}$ is referenced by $[\boldsymbol{A}]_{i,\ell}$. The result of $\mathrm{diag}(\boldsymbol{a})$ is a diagonal matrix with the elements of $\boldsymbol{a}$ on its main diagonal.

In each data block there are M data symbols. They result from the binary information sequence $\chi_k[m'']$ of length $2MR_C$ by convolutional encoding, random interleaving and QPSK modulation with Gray labeling. The code rate is denoted by R_C. The spread signals of all users are added together and multiplexed with optional pilot symbols. Then, an N point inverse discrete Fourier transform (IDFT) is performed and a CP (cyclic prefix) with length G is inserted. The resulting signal is transmitted over a time variant multipath fading channel $h(t,\tau)$ with a delay spread of L chips, $T_D = LT_C$. The chip duration is denoted by T_C. We denote the sampled time variant impulse response by $h[m,n] = h(mPT_C, nT_C)$, $\boldsymbol{h}[m] = [h(mPT_C, 0), \cdots, h(mPT_C, (L-1)T_C)]^T$ in vector notation, where $P = N + G$ gives the length of the OFDM symbol in chips.

The receiver removes the CP and performs a discrete Fourier transform (DFT). The received signal vector

$$\boldsymbol{y}[m] = \mathrm{diag}\,(\boldsymbol{g}[m])(\boldsymbol{S}\boldsymbol{b}[m] + \boldsymbol{p}[m]) + \boldsymbol{\nu}[m]$$

where $\boldsymbol{g}[m] = \sqrt{N}\boldsymbol{F}_{N\times L}\boldsymbol{h}[m]$, and $\boldsymbol{\nu}[m]$ denotes complex additive white Gaussian noise with zero mean and covariance $\sigma_\nu^2 \boldsymbol{I}_N$. The unitary DFT matrix $\boldsymbol{F}$ has elements $[\boldsymbol{F}]_{i,\ell} = \frac{1}{\sqrt{N}} \mathrm{e}^{\frac{-\mathrm{j}2\pi i\ell}{N}}$, $i, \ell = 0, \cdots, N-1$. Matrix $\boldsymbol{S}$ is defined as $\boldsymbol{S} = [\boldsymbol{s}_1, \cdots, \boldsymbol{s}_K]$, and $\boldsymbol{b}[m] = [b_1[m], \cdots, b_K[m]]^T$ contains the stacked data symbols for K users.

We define the effective spreading vector for user k as

$$\tilde{\boldsymbol{s}}_k[m] = \mathrm{diag}\,(\boldsymbol{g}[m])\boldsymbol{s}_k,$$

collect all users in matrix $\tilde{\boldsymbol{S}}[m] = [\tilde{\boldsymbol{s}}_1[m], \cdots, \tilde{\boldsymbol{s}}_K[m]]$ and finally get $\boldsymbol{y}[m] = \tilde{\boldsymbol{S}}[m]\boldsymbol{b}[m] + \mathrm{diag}\,(\boldsymbol{g}[m])\boldsymbol{p}[m] + \boldsymbol{\nu}[m]$.

This signal model is valid for time variant channels when the intercarrier interference (ICI) is small. This is true when the (one sided) Doppler bandwidth $B_D = v/\lambda$ is smaller than $\varepsilon = 1\%$ of the subcarrier bandwidth $\Delta f = 1/(NT_C)$ [5]: $B_D < \varepsilon \Delta f$. The speed of the mobile station is denoted by v and the wave length by λ.

The single user receiver is a time variant matched filter $z_k[m] = \frac{\tilde{\boldsymbol{s}}_k[m]^H}{\|\tilde{\boldsymbol{s}}_k[m]\|^2}\boldsymbol{y}[m]$. To calculate the effective spreading sequence we need a model to describe the time variant channel. A possible solution is a Fourier basis expansion model which we describe in the next section.

3. FOURIER BASIS EXPANSION MODEL

The sampled time variant channel impulse response $h[m,n]$ is represented by the scattering function in the Doppler delay domain

$$S_{\boldsymbol{H}}(f,n) = \sum_{m=-\infty}^{\infty} h[m,n]\mathrm{e}^{-\mathrm{j}2\pi mf}.$$

For a wireless system the Doppler bandwidth B_D is generally known, $S_{\boldsymbol{H}}(f,n)$ is therefore band limited and vanishes for $|f| > W$ $h[m,n] = \int_{-W}^{W} S_{\boldsymbol{H}}(f,n)\mathrm{e}^{\mathrm{j}2\pi mf}\mathrm{d}\,f$, where $W = B_D PT_C$ and $0 < W < 1/2$.

By limiting the time interval to $[0, M-1]$ the scattering function is discretized in the frequency domain, $S_{\boldsymbol{H}}[d,n] = \sum_{m=0}^{M-1} h[m,n]\mathrm{e}^{-\mathrm{j}2\pi md/M}$, which is equivalent to the discrete Fourier transform (DFT). The rectangular windowing results in spectral leakage (see [6] Sec. 5.4). Therefore $S_{\boldsymbol{H}}[d,n]$ does not vanish for $|d| > \lceil WM \rceil$. But $S_{\boldsymbol{H}}[d,n]$ will decay with increasing $|d|$.

The Fourier basis expansion model for the time variant channel is given by $\hat{h}[m,n] = \sum_{d=0}^{D-1} \gamma_d[n] u_d[m]$ with $D = 2\kappa\lceil WM \rceil + 1$ and $u_d[m] = \frac{1}{\sqrt{M}}\mathrm{e}^{\frac{\mathrm{j}2\pi m(d-(D-1)/2)}{M}}$. The parameters $\gamma_d[n]$ are calculated by $\gamma_d[n] = \sum_{m=0}^{M-1} h[m,n]u_d^*[m]$. The mean square approximation error $e' = \sum_{m=0}^{M-1}\sum_{n=0}^{L-1}\left|h[m,n] - \hat{h}[m,n]\right|^2$ is controlled by κ.

To express the basis expansion model in vector matrix notation we introduce the delay time channel matrix with dimension $L \times M$ $\boldsymbol{\mathcal{H}} = \left[\boldsymbol{h}[0], \boldsymbol{h}[1], \ldots, \boldsymbol{h}[M-1]\right]$ where each column in $\boldsymbol{\mathcal{H}}$ represents the time variant impulse response at discrete time m. The frequency time channel matrix $\boldsymbol{\mathcal{G}} = \left[\boldsymbol{g}[0], \boldsymbol{g}[1], \ldots, \boldsymbol{g}[M-1]\right]$ is related to the delay time matrix via $\boldsymbol{\mathcal{G}} = \sqrt{N}\boldsymbol{F}_{N\times L}\boldsymbol{\mathcal{H}}$.

We define the $D \times M$ synthesis matrix $\boldsymbol{U} = \left[\boldsymbol{u}[0], \boldsymbol{u}[1], \cdots, \boldsymbol{u}[M-1]\right]$, and the $D \times M$ analysis matrix $\boldsymbol{V} = \boldsymbol{U}^*$.

The Fourier basis expansion model in matrix vector notation is given by $\hat{\boldsymbol{h}}[m] = \boldsymbol{\Gamma}^T\boldsymbol{u}[m]$. To obtain the delay basis expansion parameter matrix $\boldsymbol{\Gamma} = \boldsymbol{V}\boldsymbol{\mathcal{H}}^T$, we apply the analysis matrix $\boldsymbol{V}$ on $\boldsymbol{\mathcal{H}}$. The elements of the $D \times L$ matrix $\boldsymbol{\Gamma}$ are defined as $[\boldsymbol{\Gamma}]_{d,n} = \gamma_d[n]$.

In the context of MC-CDMA the channel is naturally represented in the frequency time domain. We introduce the frequency basis expansion parameter matrix $\boldsymbol{\Phi} = \boldsymbol{V}\boldsymbol{\mathcal{G}}^T$, and express the time variant frequency characteristic $\hat{\boldsymbol{g}}[m] = \boldsymbol{\Phi}^T\boldsymbol{u}[m]$ through the Fourier basis expansion model.

This gives the time variant effective spreading sequence as $\tilde{\boldsymbol{s}}_k[m] \approx \mathrm{diag}\left(\boldsymbol{\Phi}^T\boldsymbol{u}[m]\right)\boldsymbol{s}_k$, and the received data signal

$$\boldsymbol{y}[m] = \mathrm{diag}\left(\boldsymbol{\Phi}^T\boldsymbol{u}[m]\right)\left(\boldsymbol{S}\boldsymbol{b}[m] + \boldsymbol{p}[m]\right) + \boldsymbol{\nu}[m].$$

4. PERFORMANCE ANALYSIS

To generate the time variant channel realization for $\boldsymbol{\mathcal{H}}$ we use the model

$$\boldsymbol{h}[m] = \mathrm{diag}(\boldsymbol{h}) \left[\mathrm{e}^{\frac{\mathrm{j}2\pi f_0 m}{M}}, \cdots, \mathrm{e}^{\frac{\mathrm{j}2\pi f_{(L-1)} m}{M}} \right]^T e^{\frac{\mathrm{j}2\pi \Delta f_C m}{M}}.$$

Vector $\boldsymbol{h}$ models the Rayleigh fading large scale channel statistics and is normalized so that $\mathrm{E}\{|\boldsymbol{h}|^2\} = 1$. We use the exponentially decaying typical urban (TU) power-delay-profile (PDP) from COST 259 [3], the chip rate $1/T_C = 3.84$Mcps as in UMTS and the delay spread $L = 15$ corresponding to $T_D = 3.9\mu$s.

The time variant characteristic is modeled as random Doppler component f_ℓ for every channel tap. To achieve a Jakes spectrum [4] $f_\ell = \sin(2\pi\xi_\ell) B_D M P T_C$, $\ell = 0, \cdots, L-1$, where ξ_ℓ is a random variable, uniformly distributed in the interval $(0, 1)$. The normalized carrier frequency offset at the receiver Δf_C is incorporated in the time variant channel model as common additional Doppler component for all channel taps. For every channel realization Δf_C is randomly sampled from a uniform distribution on the interval $(-\Delta f_{C,max}, +\Delta f_{C,max})$.

The system is operated at carrier frequency $f_C = 2\,$GHz, the users move with velocity $v = 50\,$km/h, this gives $B'_D = 90\,$Hz. The complete OFDM symbol with cyclic prefix has length of $P = G + N = 79$. The number of subcarriers $N = 64$ and the data block length $M = 256$. The maximum carrier frequency offset was chosen to be $\Delta f_{C,max} = 90\,\mathrm{Hz} M P T_C = 0.47$. For the Fourier basis expansion model this results in $\lceil WM \rceil = 1$ with $B_D = B'_D + \Delta f_{C,max}/(M P T_C)$.

The Fourier basis expansion model approximates for every channel tap a complex exponential with a rational frequency f in the range $|f| < WM$. We analyze the MSE of the Fourier basis expansion model for a single path channel with normalized Doppler frequency f varied over the interval $(0, 1)$, $h[m, 1] = \mathrm{e}^{\frac{\mathrm{j}2\pi m f}{M}}$, and define $\boldsymbol{h}_1 = \Big[h[0, 1], \cdots, h[M-1, 1]\Big]^T$.

We evaluate the MSE according to $\hat{\boldsymbol{h}}_1 = \boldsymbol{U}^T \boldsymbol{V} \boldsymbol{h}_1$, $e = \left|\boldsymbol{h}_1 - \hat{\boldsymbol{h}}_1\right|^2$. The MSE for the Fourier basis expansion is given in Fig. 2. With increasing number of basis functions, controlled by κ the MSE is reduced. At $f = 0.5$ the error is maximum.

To get more insight in the detailed approximation behavior we plot $h[m, 1]$ and $\hat{h}[m, 1]$ for $f = 0.5$ and $\kappa = \{1, 4\}$ in the complex plane in Fig. 3. The amplitude and phase error at the beginning and end of the data block is significant for $\kappa = 1$, and decreases with increasing κ.

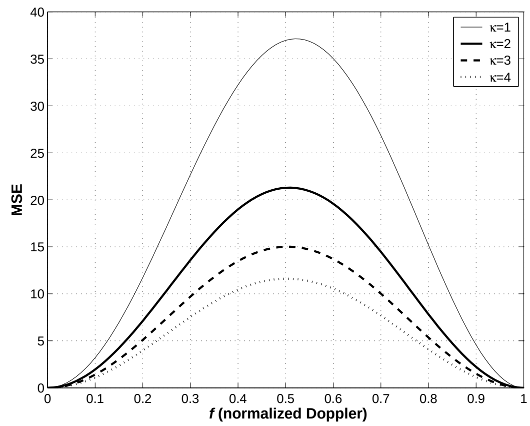

Figure 2. Mean squared error (MSE) of the Fourier basis expansion model for $\lceil WM \rceil = 1$ and $D = 2\kappa\lceil WM \rceil + 1$ basis functions, with $\kappa = \{1, 2, 3, 4\}$. The normalized doppler frequency f was varied over the interval $(0, 1)$.

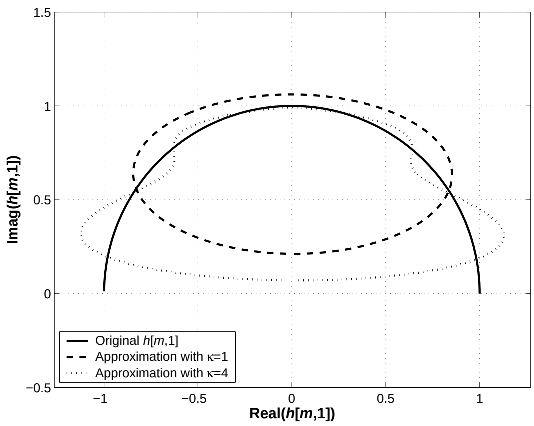

Figure 3. Trajectory of $h[m, 1]$ and its approximation by the Fourier basis expansion for $\kappa = \{1, 4\}$.

For data detection we use a time variant matched filter. The resulting code symbol estimates are demapped, deinterleaved and decoded by a BCJR decoder [1] to obtain estimates for the transmitted data bits $\hat{\chi}_k[m'']$.

We evaluate the receiver performance subject to parameter κ of the channel approximation model. The additional simulation parameters for the MC-CDMA system are chosen as follows: The spreading sequence has length $N = 64$ equal to the number of subcarriers. For data transmission the convolutional code used, is a non-systematic,

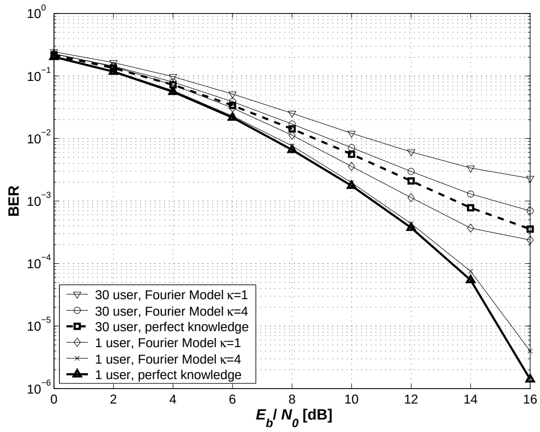

Figure 4. We compare the MC-CDMA receiver performance for the Fourier basis expansion channel model with $\kappa = \{1, 4\}$ and the perfectly known channel. The performance in the forward link is given in terms of BER versus SNR for $K = 1$ and $K = 30$ users. The spreading sequence length $N = 64$ is equal to the number of subcarriers. The data block length $M = 256$. The carrier frequency $f_C = 2\,\text{GHz}$, the Doppler bandwidth $B_D = 90\,\text{Hz}$ and the maximum carrier frequency offset is 90 Hz.

non-recursive, 4 state, rate $R_C = 1/2$ code with generator polynomial $(5, 7)_8$. We average all simulations over 2000 independent channel realizations. The QPSK symbol energy is normalized to 1, the E_b/N_0 is therefore defined as $\frac{E_b}{N_0} = \frac{1}{2R\sigma_\nu^2}\frac{P}{N}$.

We analyze the performance for perfectly known *model parameters* of the channel. This means the model parameters $\mathbf{\Gamma}$ are calculated from a perfectly known channel $\boldsymbol{\mathcal{H}}$. Applying the Fourier basis expansion model, we give the MC-CDMA receiver performance for the forward link in Fig. 4 for $\kappa = \{1, 4\}$ and for $K = 1$ and $K = 30$ users. We additionally give the performance with the perfectly known channel itself.

5. CONCLUSION

The Fourier basis expansion model allows to describe a time variant channels. Its inherent rectangular windowing results in spectral leakage. Therefore the number of Fourier basis functions given by the time bandwidth product $2\lceil WM \rceil + 1$, usually reported in literature, is not sufficient. Increasing the number of Fourier basis functions $D = 2\kappa\lceil WM \rceil + 1$ by a factor κ we can decrease the MSE of the channel approximation and the BER in the MC-CDMA forward link. The principal error floor behavior can not be removed for practical values $\kappa \leq 4$. Therefore a new basis expansion model is needed [9, 10].

6. REFERENCES

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