# ITERATIVE MULTI-USER DECODING WITH TIME-VARIANT CHANNEL ESTIMATION FOR MC-CDMA

**Thomas Zemen, Christoph F. Mecklenbräuker, Joachim Wehinger and Ralf R. Müller**

ftw. Forschungszentrum Telekommunikation Wien
Tech Gate Vienna, Donau-City Str. 1/3
A-1220 Vienna, Austria
Email: {thomas.zemen, cfm, wehinger, mueller}@ftw.at, phone: +43 1 5052830-81, fax: +43 1 5052830-99

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## Abstract

We propose an iterative receiver for a multi-carrier (MC) code division multiple access (CDMA) system in the uplink for users moving at vehicular speed. MC-CDMA is based on orthogonal frequency division multiplexing (OFDM), thus time-variant channel estimation can be performed for every subcarrier independently. The variation of a subcarrier over the duration of a data block is upper bounded by a maximum Doppler bandwidth which is determined by the maximum velocity of the users and the carrier frequency. We exploit results from the theory of time-concentrated and bandlimited sequences and apply a Slepian basis expansion for time-variant subcarrier estimation. The Slepian basis functions are determined by two parameters; the block length and the maximum Doppler bandwidth. This approach enables time-variant channel estimation *without complete knowledge* of the second-order statistics of the fading process. The performance of the iterative receiver is validated for a wide range of velocities by simulations.

## 1 Introduction

This paper deals with iterative time-variant channel estimation and multi-user detection for the uplink of a multi-carrier (MC) code division multiple access system (CDMA). In iterative receivers the soft information gained about the transmitted data symbols is used to enhance the channel estimation and data detection in consecutive iterations. It was shown in [4], [5], [10] that iterative receivers achieve performance close to the single user bound for block fading channels. In the present paper the iterative receiver concept is extended to time-variant channels and we focus on the iterative time-variant channel estimation. More details on multi-user detection and decoding and an analytic treatment of the time-variant channel estimation can be found in [13].

The variation of a wireless channel over the duration of a data block is caused by user mobility and multipath propagation. The Doppler shifts on the individual paths depend on the user's direction and its velocity $v$, the carrier frequency $f_C$, and the scattering environment. The maximum variation in time of the wireless channel is upper bounded by the maximum (one sided) normalized Doppler bandwidth

$$\nu_{\mathrm{Dmax}} = \frac{v_{\max} f_C}{c_0} T_{\mathrm{S}} \,, \qquad (1)$$

where $v_{\max}$ is the maximum supported velocity, $T_{\mathrm{S}}$ is the symbol duration, and $c_0$ denotes the speed of light.

We apply orthogonal frequency division multiplexing (OFDM) in order to transform the time-variant frequency-selective channel into a set of time-variant frequency-flat channels, the so called subcarriers. We deal with time-variant channels which vary significantly over the duration of a long block of OFDM symbols. However, for the duration of each single OFDM symbol the channel variation is small enough to be neglected. This, in other words, implies a very small inter-carrier interference. Each OFDM symbol is preceded by a cyclic prefix to avoid inter-symbol interference.

Under the assumption of small inter-carrier interference, each time-variant frequency-flat subcarrier is fully described through a sequence of complex scalars at the OFDM symbol rate $1/T_{\mathrm{S}}$. This sequence is bandlimited by $\nu_{\mathrm{Dmax}}$. In order to perform coherent multi-user detection we need to estimate a time limited snapshot of this bandlimited sequence at the receiver side. The length of these snapshots is equal to the length of a data block consisting of OFDM data symbols with interleaved OFDM pilot symbols.

We pursue a time-variant channel estimation approach that exploits the band-limitation of the time-variant subcarrier coefficients only. Differing from [3], [7] we make no detailed assumption about the shape of the power spectral density (i.e. the autocorrelation). This is due to the fact that wireless fading channels show stationary behavior for up to 70 wavelengths [9, Sec. 6.3] only. We doubt that meaningful short-term fading characteristics (second-order statistics, to begin with) can be acquired in a multiuser system when users move at vehicular speeds.

We make use of Slepian's basic result that time-limited parts (snapshots) of band-limited sequences span a low dimensional subspace [8]. The basis functions of this subspace are the discrete prolate spheroidal sequences. Using these results from the theory of time-concentrated and bandlimited sequences we represent a time-variant subcarrier through a Slepian basis expansion of low dimensionality [11].

Our contribution is the derivation of an iterative linear MMSE estimator for the Slepian basis expansion coefficients in the uplink of a multi-user MC-CDMA system. For iterative time-variant channel estimation we combine the pilot symbols with soft symbols which are supplied by a soft-in soft-out decoder, implemented by the BCJR algorithm [1].

For iterative multi-user detection we apply parallel interference cancellation (PIC), using feed back soft symbols, and individual linear MMSE filtering.

**The rest of the paper is organized as follows:** We define the notation in Section 2 and introduce the signal model for the multi-user MC-CDMA uplink in Section 3. Section 4 briefly outlines the iterative multi-user detection. The Slepian basis expansion is provided in Section 5. Based on these results the iterative multi-user channel estimator is derived. Simulation results are given in Section 6 and conclusions are drawn in Section 7.

## 2 Notation

We denote a column vector by $\boldsymbol{a}$ and its $i$-th element with $a[i]$. Equivalently, we denote a matrix by $\boldsymbol{A}$ its $i,\ell$-th element by $[\boldsymbol{A}]_{i,\ell}$. Its transpose is given by $\boldsymbol{A}^{\mathrm{T}}$, its conjugate transpose by $\boldsymbol{A}^{\mathrm{H}}$ and its upper left part with dimension $P \times Q$ by $\boldsymbol{A}_{P\times Q}$. A diagonal matrix with elements $a[i]$ is written as $\operatorname{diag}(\boldsymbol{a})$ and the $Q \times Q$ identity matrix as $\boldsymbol{I}_Q$. The absolute value of $a$ is denoted through $|a|$ and its complex conjugate by $a^*$. The largest (smallest) integer, lower (greater) or equal than $a \in \mathbb{R}$ is denoted by $\lfloor a \rfloor$ ($\lceil a \rceil$).

## 3 Signal Model for Time-Variant Frequency-Selective Channels

The MC-CDMA uplink transmission is block oriented, a data block consists of $M - J$ OFDM data symbols and $J$ OFDM pilot symbols. Every OFDM symbol is preceded by a cyclic prefix to avoid inter-symbol interference. Each user transmits symbols $b_k[m]$ with symbol rate $1/T_{\mathrm{S}}$. Discrete time is denoted by $m$. There are $K$ users in the system, the user index is denoted by $k$. Each symbol is spread by a random spreading sequence $\boldsymbol{s}_k \in \mathbb{C}^N$ with independent identically distributed (i.i.d.) elements chosen from the set $\{\pm 1 \pm \mathrm{j}\}/\sqrt{2N}$. The data symbols $b_k[m]$ result from the binary information sequence $\chi_k[m'']$ of length $2(M-J)R_C$ by convolutional encoding with code rate $R_C$, random interleaving and quadrature phase shift keying (QPSK) modulation with Gray labelling.

The $M-J$ data symbols are distributed over a block of length $M$ fulfilling

$$b_k[m] \in \{\pm 1 \pm \mathrm{j}\}/\sqrt{2} \quad \text{for} \quad m \notin \mathcal{P} \qquad (2)$$

and $b_k[m] = 0$ for $m \in \mathcal{P}$ allowing for pilot symbol insertion. The pilot placement is defined through the index set

$$\mathcal{P} = \left\{ \left\lfloor i\frac{M}{J} + \frac{M}{2J} \right\rfloor \;\middle|\; i \in \{0, \ldots, J-1\} \right\}. \qquad (3)$$

After spreading, pilot symbols $\boldsymbol{p}_k[m] \in \mathbb{C}^N$ with elements $p_k[m,q]$ are added

$$\boldsymbol{d}_k[m] = \boldsymbol{s}_k b_k[m] + \boldsymbol{p}_k[m]\,. \qquad (4)$$

The elements of the pilot symbols $p_k[m,q]$ for $m \in \mathcal{P}$ and $q \in \{0, \ldots, N-1\}$ are randomly chosen from the QPSK symbol set $\{\pm 1 \pm \mathrm{j}\}/\sqrt{2N}$, otherwise $\boldsymbol{p}_k[m] = \boldsymbol{0}_N$ for $m \notin \mathcal{P}$. Then, an $N$ point inverse DFT is performed and a cyclic prefix of length $G$ is inserted. A single OFDM symbol together with the cyclic prefix and has length $P = N + G$ chips. After parallel to serial conversion the chip stream with chip rate $1/T_{\mathrm{C}} = P/T_{\mathrm{S}}$ is transmitted over a time-variant multipath fading channel with $L$ resolvable paths. We denote the time-variant impulse response sampled at the chip-rate by $h'_k[n,\ell] = h_k(nT_{\mathrm{C}}, \ell T_{\mathrm{C}})$. We are able to treat the time-variant channel as constant for the duration of each single OFDM symbol, because the one-sided normalized Doppler bandwidth $\nu_D$ is much smaller than the normalized subcarrier bandwidth $P/N$, $\frac{\nu_D N}{P} < 0.01$. Thus $h_k[m,\ell] = h'_k[mP,\ell]$, corresponding to

$$\boldsymbol{h}_k[m] = [h_k[m,0], \ldots, h_k[m,L-1]]^{\mathrm{T}} \in \mathbb{C}^{L\times 1}$$

in vector notation. The time-variant frequency response $\boldsymbol{g}_k[m] \in \mathbb{C}^N$ with elements $g_k[m,q]$ is defined as the DFT of the time-variant impulse response $\boldsymbol{g}_k[m] = \sqrt{N}\boldsymbol{F}_{N\times L}\boldsymbol{h}_k[m]$. The unitary DFT matrix $\boldsymbol{F}_N \in \mathbb{C}^{N\times N}$ has elements $[\boldsymbol{F}_N]_{i,\ell} = 1/\sqrt{N}\,\mathrm{e}^{\frac{-\mathrm{j}2\pi i\ell}{N}}$ for $i, \ell = 0, \ldots, N-1$.

At the receive antenna the signals of all $K$ users add up. The receiver removes the cyclic prefix and performs a DFT. The received signal vector $\boldsymbol{y}[m] \in \mathbb{C}^N$ after these two operations is given by

$$\boldsymbol{y}[m] = \sum_{k=1}^{K} \operatorname{diag}\left(\boldsymbol{g}_k[m]\right)\left(\boldsymbol{s}_k b_k[m] + \boldsymbol{p}_k[m]\right) + \boldsymbol{z}[m]\,, \qquad (5)$$

where complex additive white Gaussian noise with zero mean and covariance $\sigma_z^2 \boldsymbol{I}_N$ is denoted by $\boldsymbol{z}[m] \in \mathbb{C}^N$ with elements $z[m,q]$.

## 4 Iterative Data Detection

We define the time-variant effective spreading sequences $\tilde{\boldsymbol{s}}_k[m] = \operatorname{diag}\left(\boldsymbol{g}_k[m]\right)\boldsymbol{s}_k$, and the time-variant effective spreading matrix $\tilde{\boldsymbol{S}}[m] = [\tilde{\boldsymbol{s}}_1[m], \ldots, \tilde{\boldsymbol{s}}_K[m]] \in \mathbb{C}^{N\times K}$. Using these definitions the signal model for data detection writes as

$$\boldsymbol{y}[m] = \tilde{\boldsymbol{S}}[m]\boldsymbol{b}[m] + \boldsymbol{z}[m] \quad \text{for} \quad m \notin \mathcal{P}$$

where $\boldsymbol{b}[m] = [b_1[m], \ldots, b_K[m]]^{\mathrm{T}} \in \mathbb{C}^K$ contains the stacked data symbols for $K$ users.

Figure 1 shows the structure of the iterative receiver. The receiver detects the data $\boldsymbol{b}[m]$ using the received symbol vector $\boldsymbol{y}[m]$, the spreading matrix $\tilde{\boldsymbol{S}}^{(i)}[m]$, and the feedback $\mathrm{EXT}(c_k^{(i)}[m'])$ on the code symbols at iteration $i$. We apply time-variant parallel interference cancellation and unbiased conditional linear MMSE filtering. The resulting channel values are decoded by the BCJR decoder. Please refer to [13] for more details on the detector and decoder.

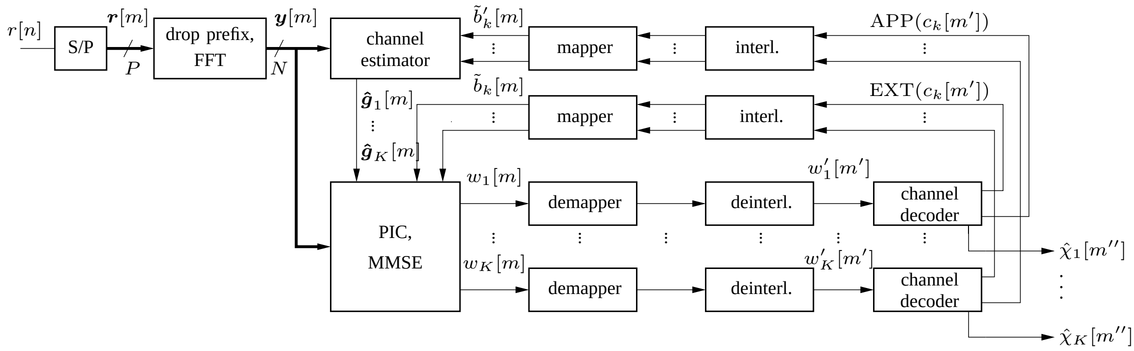


Fig. 1. Model for the MC-CDMA receiver. It performs joint iterative time-variant channel estimation and multi-user detection.

# 5 Iterative Time-Variant Channel Estimation

The performance of the iterative receiver crucially depends on the channel estimates for the time-variant frequency response $\boldsymbol{g}_k[m]$ since the effective spreading sequence directly depends on the actual channel realization. The MC-CDMA transmission takes place over $N$ (essentially) orthogonal frequency-flat time-variant subcarriers. Reflecting this we rewrite (5) as a set of equations for every subcarrier $q$ for $q \in \{0, \dots, N-1\}$,

$$y[m,q] = \sum_{k=1}^{K} g_k[m,q]\left(s_k[q]b_k[m] + p_k[m,q]\right) + z[m,q]\,. \tag{6}$$

We split the channel estimation task into two parts:

- First, we find a suitable basis expansion which describes the time-variation of $g_k[m,q]$.
- In a second step the basis expansion coefficients are estimated individually for every subcarrier but jointly for all users. The estimation is performed in an iterative manner using soft feedback symbols.

## 5.1 Slepian Basis Expansion

The coefficients of the time-variant impulse response $h_k[m,n]$ are bandlimited by $\nu_{\mathrm{Dmax}}$ (1). The same is true for $g_k[m,q]$. The Doppler spectrum for subcarrier $q$ and user $k$ is defined as

$$G_k(\nu,q) = \sum_{m=-\infty}^{\infty} g_k[m,q]\mathrm{e}^{-\mathrm{j}2\pi\nu m}\,. \tag{7}$$

where $\frac{1}{2} \leq \nu < \frac{1}{2}$. The band limitation of $G_k(\nu,q)$ to $\nu_{\mathrm{Dmax}}$ can be expressed by

$$g_k[m,q] = \int_{-\nu_{\mathrm{Dmax}}}^{\nu_{\mathrm{Dmax}}} G_k(\nu,q)\mathrm{e}^{\mathrm{j}2\pi\nu m}\mathrm{d}\,\nu\,.$$

Limiting the infinite sum in (7) to an interval of length $M$ results in the discrete Fourier transform (DFT). Truncating the DFT leads to the Fourier basis expansion described in [6] for time-variant channel estimation. However, the time windowing causes spectral leakage and the truncation of the DFT gives rise to the Gibbs phenomenon. Both effects together imply that the Fourier basis expansion suffers from high bias [11].

The theory of time-concentrated and bandlimited sequences developed by Slepian in [8] enables a better suited approach for the time-variant estimation problem. Slepian asked which sequence is most concentrated in a given frequency range $\nu_{\mathrm{Dmax}}$ and simultaneously in a certain time interval of length $M$. This optimization problem was solved for discrete time in [8].

The sequences bandlimited to $\nu_{\mathrm{Dmax}}$ and mostly concentrated in an interval of length $M$ are the discrete prolate spheroidal (DPS) sequences. The DPS sequences $u_i[m,\nu_{\mathrm{Dmax}},M]$ are defined as the real solution to

$$\sum_{\ell=0}^{M-1} \frac{\sin(2\pi\nu_{\mathrm{Dmax}}(\ell-m))}{\pi(\ell-m)} u_i[\ell,\nu_{\mathrm{Dmax}},M] = \lambda_i(\nu_{\mathrm{Dmax}},M)u_i[m,\nu_{\mathrm{Dmax}},M] \tag{8}$$

for $i \in \{0,\dots,M-1\}$ and $m \in \{-\infty,\infty\}$ [8]. We drop the explicit dependence of $u_i[m]$ on $\nu_{\mathrm{Dmax}}$ and $M$, since we consider them as fixed system parameters for the remainder of this paper. The DPS sequences are doubly orthogonal on the index sets $\{-\infty,\dots,\infty\}$ and $\{0,\dots,M-1\}$. The eigenvalues $\lambda_i$ are clustered near 1 for $i < \lceil 2\nu_{\mathrm{Dmax}}M \rceil$ and rapidly decay to zero for $i > \lceil 2\nu_{\mathrm{Dmax}}M \rceil$. Therefore, the approximate signal space dimension of time-limited snapshots of a band-limited signal is given by [8, Sec. 3.3]

$$D' = \lceil 2\nu_{\mathrm{Dmax}}M \rceil + 1\,. \tag{9}$$

For our application we are interested in $u_i[m]$ for the index set $m \in \{0,\dots,M-1\}$ only. We introduce the term Slepian sequences for the index limited DPS sequences and define the vector $\boldsymbol{u}_i \in \mathbb{R}^M$ with elements $u_i[m]$ for $m \in \{0,\dots,M-1\}$. Summarizing, the Slepian sequences span an orthogonal basis which allows to represent time-limited snapshots of band-limited sequences. We expand the sequence $g_k[m,q]$ in terms of Slepian sequences $u_i[m]$

$$g_k[m,q] \approx \tilde{g}_k[m,q] = \sum_{i=0}^{D-1} u_i[m]\psi_k[i,q]\,, \tag{10}$$

where $m \in \{0,\dots,M-1\}$ and $q \in \{0,\dots,N-1\}$. The dimension $D$ of this basis expansion fulfills $D' \leq D \leq M-1$.

## 5.2 Signal Model for Time-Variant Multi-User Channel Estimation

Substituting the basis expansion (10) for the time-variant subcarrier coefficients $g_k[m,q]$ into the system model (6) we obtain

$$y[m,q] = \sum_{k=1}^{K}\sum_{i=0}^{D-1} u_i[m]\psi_k[i,q]d_k[m,q] + z[m,q]\,, \qquad (11)$$

where $d_k[m,q] = s_k[q]b_k[m] + p_k[m,q]$.
Thus, an estimate of the subcarrier coefficients $\hat{\psi}_k[i,q]$ can be obtained jointly for all $K$ users but individually for every subcarrier $q$. We define the vector

$$\begin{aligned}\boldsymbol{\psi}_q = [&\psi_1[0,q],\ldots,\psi_K[0,q],\\ &\ldots,\psi_1[D-1,q],\ldots,\psi_K[D-1,q]]^{\mathrm{T}} \in \mathbb{C}^{KD}\end{aligned} \qquad (12)$$

containing the basis expansion coefficients of all $K$ users for subcarrier $q$. Furthermore, we introduce $\boldsymbol{y}_q = [y[0,q],\ldots,y[M-1,q]]^{\mathrm{T}} \in \mathbb{C}^{M}$ for the received symbol sequence of each single data block on subcarrier $q$. Using these definitions we write

$$\boldsymbol{y}_q = \boldsymbol{\mathcal{D}}_q\boldsymbol{\psi}_q + \boldsymbol{z}_q\,, \qquad (13)$$

where

$$\boldsymbol{\mathcal{D}}_q = [\mathrm{diag}(\boldsymbol{u}_0)\,\boldsymbol{D}_q,\ldots,\mathrm{diag}(\boldsymbol{u}_{D-1})\,\boldsymbol{D}_q] \in \mathbb{C}^{M\times KD}, \qquad (14)$$

and $\boldsymbol{D}_q \in \mathbb{C}^{M\times K}$ contains the transmitted symbols for all $K$ users on subcarrier $q$

$$\boldsymbol{D}_q = \begin{bmatrix} d_1[0,q] & \ldots & d_K[0,q] \\ \vdots & \ddots & \vdots \\ d_1[M-1,q] & \ldots & d_K[M-1,q]\end{bmatrix}. \qquad (15)$$

For channel estimation, $J$ pilot symbols in (11) are known. The remaining $M-J$ symbols are not known. We replace them by soft symbols that are calculated from the a-posteriori probabilities (APP) obtained in the previous iteration from the BCJR decoder output. This enables us to obtain refined channel estimates if the soft symbols get more reliable from iteration to iteration. For the first iteration the soft symbols $\tilde{b}'_k[m]$ for $m \notin \mathcal{P}$ are set to zero.
We define the soft symbol matrix $\tilde{\boldsymbol{D}}_q \in \mathbb{C}^{M\times K}$ according to (15) by replacing $d_k[m,q]$ with $\tilde{d}_k[m,q] = s_k[q]\tilde{b}'_k[m] + p_k[m,q]$. The soft symbols $\tilde{b}'_k[m]$ are defined according to

$$\begin{aligned}\tilde{b}'_k[m] &= \mathop{\mathbb{E}}_{b}^{(\mathrm{APP})}\{b_k[m]\} = \\ &= \frac{1}{\sqrt{2}}\left(\mathop{\mathbb{E}}_{c}^{(\mathrm{APP})}\{c_k[2m]\} + \mathrm{j}\mathop{\mathbb{E}}_{c}^{(\mathrm{APP})}\{c_k[2m+1]\}\right)\end{aligned} \qquad (16)$$

where $\mathop{\mathbb{E}}_{c}^{(\mathrm{APP})}\{c_k[m']\} = 2\mathrm{APP}\{c_k[m']\} - 1$.

Finally we define $\tilde{\boldsymbol{\mathcal{D}}}_q \in \mathbb{C}^{M\times KD}$ according to (14) by replacing $\boldsymbol{D}_q$ with $\tilde{\boldsymbol{D}}_q$. Thus, matrix $\tilde{\boldsymbol{\mathcal{D}}}_q$ contains deterministic pilot symbols and statistical information about the transmitted data symbols.

### 5.3 Linear MMSE Estimation of Basis Expansion Coefficients

We constrain the time-variant channel estimator to be linear in $\boldsymbol{y}_q$ (see [10] for the block fading case). We will omit the index $q$ in the following derivations to simplify the notation. The linear estimator can be expressed as $\hat{\boldsymbol{\psi}}_{\mathrm{LMMSE}} = \boldsymbol{A}\boldsymbol{y}$ , where the matrix $\boldsymbol{A}$ satisfies the Wiener-Hopf equation $\boldsymbol{C}_{\boldsymbol{yy}}\boldsymbol{A}^{\mathrm{H}} = \boldsymbol{C}_{\boldsymbol{y\psi}}$.
The linear MMSE estimator is then

$$\hat{\boldsymbol{\psi}}_{\mathrm{LMMSE}} = \tilde{\boldsymbol{\mathcal{D}}}^{\mathrm{H}}\left(\mathop{\mathbb{E}}_{\boldsymbol{b}}\left\{\boldsymbol{\mathcal{D}}\boldsymbol{\mathcal{D}}^{\mathrm{H}}\right\} + \sigma_z^2\boldsymbol{I}_M\right)^{-1}\boldsymbol{y}. \qquad (17)$$

We note that due to the independence of the users and the data symbols within one block, it holds

$$\mathop{\mathbb{E}}_{\boldsymbol{b}}\{b'_k[m']b^*_k[m]\} = \begin{cases} \tilde{b}'_k[m']\tilde{b}^*_k[m], & k' \neq k, m' \neq m \\ 1, & k' = k, m' = m \end{cases} \qquad (18)$$

for $k, k' \in \{1,...,K\}$ and for $m, m' \in \{0,1,...,M-1\}$.
With (18) we are able to write the expectation of the product $\mathop{\mathbb{E}}_{\boldsymbol{b}}\left\{\boldsymbol{\mathcal{D}}\boldsymbol{\mathcal{D}}^{\mathrm{H}}\right\}$ as product of expectations plus a correcting diagonal matrix $\boldsymbol{\Lambda}$ which takes (18) into account

$$\mathop{\mathbb{E}}_{\boldsymbol{b}}\left\{\boldsymbol{\mathcal{D}}\boldsymbol{\mathcal{D}}^{\mathrm{H}}\right\} = \mathop{\mathbb{E}}_{\boldsymbol{b}}\{\boldsymbol{\mathcal{D}}\}\mathop{\mathbb{E}}_{\boldsymbol{b}}\left\{\boldsymbol{\mathcal{D}}^{\mathrm{H}}\right\} + \boldsymbol{\Lambda} = \tilde{\boldsymbol{\mathcal{D}}}\tilde{\boldsymbol{\mathcal{D}}}^{\mathrm{H}} + \boldsymbol{\Lambda}. \qquad (19)$$

The elements of the diagonal matrix $\boldsymbol{\Lambda}$ are defined as

$$\boldsymbol{\Lambda}_{mm} = \sum_{k=1}^{K}\sum_{i=0}^{D-1}\frac{1}{N}u_i^2[m]\mathrm{var}\{b_k[m]\}, \qquad (20)$$

with the symbol variance

$$\mathrm{var}\{b_k[m]\} = \mathop{\mathbb{E}}_{\boldsymbol{b}}\left\{\left|b_k[m] - \mathop{\mathbb{E}}_{\boldsymbol{b}}\{b_k[m]\}\right|^2\right\} = 1 - |\tilde{b}_k[m]|^2.$$

Inserting (19) into (17) and applying the matrix inversion lemma yields

$$\hat{\boldsymbol{\psi}}_{\mathrm{LMMSE}} = \left(\tilde{\boldsymbol{\mathcal{D}}}^{\mathrm{H}}\boldsymbol{\Delta}^{-1}\tilde{\boldsymbol{\mathcal{D}}} + \boldsymbol{I}_K\right)^{-1}\tilde{\boldsymbol{\mathcal{D}}}^{\mathrm{H}}\boldsymbol{\Delta}^{-1}\boldsymbol{y}. \qquad (21)$$

where $\boldsymbol{\Delta} \triangleq \boldsymbol{\Lambda} + \sigma_z^2\boldsymbol{I}_M$. The rows of matrix $\tilde{\boldsymbol{\mathcal{D}}}$ are scaled by the diagonal matrix $\boldsymbol{\Delta}^{-1}$, taking into account the variances of the noise and of the soft symbol estimates.
After estimating $\hat{\boldsymbol{\psi}}_q$ for all $q \in \{0,\ldots,N-1\}$ an estimate for the time-variant frequency response is given by $\hat{g}'_k[m,q] = \sum_{i=0}^{D-1}u_i[m]\hat{\psi}_k[i,q]$. Further noise suppression is achieved if we exploit the correlation between the subcarriers $\hat{\boldsymbol{g}}_k[m] = \boldsymbol{F}_{N\times L}\boldsymbol{F}^{\mathrm{H}}_{N\times L}\hat{\boldsymbol{g}}'_k[m]$.

## 6 Simulation Results

The realizations of the time-variant frequency-selective channel $h'_k[n,\ell]$, sampled at the chip rate $1/T_{\mathrm{C}}$, are generated using an exponentially decaying power delay profile

$$\eta^2[\ell] = \mathrm{e}^{-\frac{\ell}{4}}/\sum_{\ell'=0}^{L-1}\mathrm{e}^{-\frac{\ell'}{4}}$$

with $L = 15$ resolvable paths, $\ell \in \{0,\ldots,L-1\}$ [2]. The discrete time indices $n$ and $\ell$ denote sampling at rate

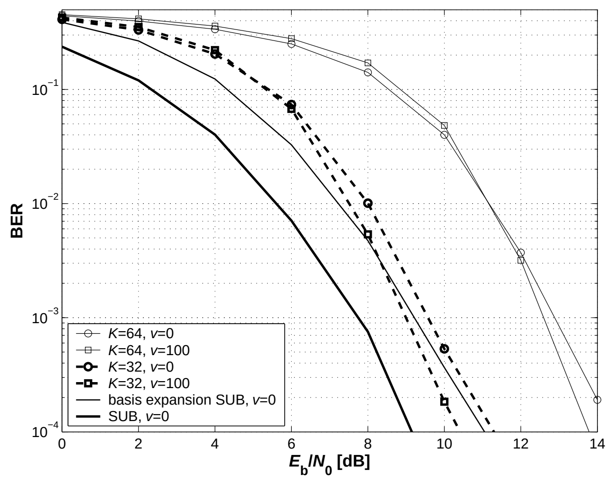


Fig. 2. MC-CDMA uplink performance in terms of BER versus SNR after $4$ iterations. The Slepian basis expansion uses $D = 3$ basis functions. The $K \in \{32, 64\}$ users are moving with $v \in \{0, 100\}$ km/h.

$1/T_{\rm C}$. The power-delay profile corresponds to a root mean square delay spread $T_{\rm D} = 4T_{\rm C} = 1\mu$s for a chip rate of $1/T_{\rm C} = 3.84 \cdot 10^6\,{\rm s}^{-1}$. The autocorrelation for every channel tap is given by $R_{h'h'}[n, \ell] = \eta^2[\ell]J_0(2\pi\nu_D Pn)$ which results in the classical Jakes spectrum. We simulate the time-variant channel with a Jakes spectrum using the model in [12] which corrects the deviations from a Rayleigh distribution of the model presented in [14] for low velocities. We emphasize that the simulation uses a time-variant channel sampled at the chip rate. Any possible effect from residual inter-carrier interference would be visible in the simulation results.

The system operates at carrier frequency $f_C = 2\,$GHz and $K \in \{32, 64\}$ users move with velocity $v \in \{0, 100\}$ km/h. These gives Doppler bandwidth $B_{\rm D} \in \{0, 190\}$ Hz and $\nu_{\rm D} \in \{0, 3.9 \cdot 10^{-3}\}$. The number of subcarriers $N = 64$ and the OFDM symbol with cyclic prefix has length $P = G+N = 79$. The data block consists of $M = 256$ OFDM symbols with $J = 60$ OFDM pilot symbols. The system is designed for $v_{\max} = 102.5\,$km/h which results in $D = D' = 3$ for the Slepian basis expansion.

For data transmission, a convolutional, non-systematic, non-recursive, $4$ state, rate $R_C = 1/2$ code with generator polynomial $(5, 7)_8$ is used. The illustrated results are obtained by averaging over $100$ independent channel realizations. The QPSK symbol energy is normalized to 1 and we define $E_b/N_0 = \frac{1}{2R_C\sigma_z^2}\frac{P}{N}\frac{M}{M-J}$ taking into account the loss due to coding, pilots and cyclic prefix. The noise variance in (17) is assumed to be known at the receiver.

In Fig. 2 we illustrate the MC-CDMA uplink performance with iterative time-variant channel estimation based on the Slepian basis expansion in terms of bit error rate (BER) versus $E_b/N_0$ after $4$ iterations. The plot also shows the single user bound (SUB) which is defined as the performance for one user $K = 1$ and a perfectly known channel $\boldsymbol{g}_k[m]$. Additionally, we plot the basis expansion SUB. This is the performance which can be achieved with the Slepian basis expansion channel-estimation algorithm for a single user.

# 7 Conclusion

We presented an iterative multi-user receiver for the uplink of a MC-CDMA system. By exploiting basic results from the theory of time-concentrated and bandlimited sequences we were able to obtain a Slepian basis expansion for time-variant channel estimation. The selection of a suitable Slepian basis, defined by $M$ and $\nu_{\rm Dmax}$, solely exploits the band-limitation of the Doppler spectrum to $\nu_{\rm Dmax}$. The details of the Doppler spectrum for $|\nu| < \nu_{\rm Dmax}$ are irrelevant. This approach enables time-variant channel estimation *with almost no* knowledge of the second-order statistics of the fading process. We presented an iterative linear MMSE estimation algorithm for the basis expansion coefficients. The consistent performance of the iterative receiver for a wide range of velocities was shown by simulations.

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