DOPPLER DIVERSITY IN MC-CDMA USING THE SLEPIAN BASIS EXPANSION MODEL

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ABSTRACT

Time-variant frequency-selective channels offer multipath diversity and Doppler diversity. MC-CDMA is able to exploit both sources of diversity, if a code with interleaving over the duration of a data block is applied and accurate channel state information is available at the receiver. A time-variant channel equalization scheme based on the Slepian basis expansion model (BEM) and a closely related pilot based channel estimation scheme based on the finite Slepian BEM was proposed by the authors in previous papers. The Slepian BEMs offer significantly reduced bias compared to the well known Fourier BEM. Exploiting the dimensionality of the Slepian BEM we propose an upper bound for the Doppler diversity of a time-variant channel. We present simulation results for a MC-CDMA forward link using an enhanced simulation model for time-variant channels with Jakes' Doppler spectrum. Applying the finite Slepian BEM for channel estimation we are able to take advantage of the additional Doppler diversity offered by a time-variant channel. In other words, the receiver performs *better* with increasing speed of the user.

1. INTRODUCTION

In MC-CDMA a data symbol is spread by a user specific spreading code to take advantage of multipath diversity. Additionally, Doppler diversity which is offered by time-variant channels, can be exploited in MC-CDMA by convolutional coding and random interleaving [1]. However, accurate time-variant channel state information is required at the receiver side to exploit both sources of diversity.

In MC-CDMA the chips resulting from the spreading operation are processed by an inverse discrete Fourier transform (DFT) to obtain an orthogonal frequency division multiplexing (OFDM) symbol. The transmission scheme is block oriented. A block consists of OFDM data symbols with interleaved OFDM pilot symbols to allow pilot-based estimation of the time-variant channel.

We deal with frequency-selective channels which vary significantly over the duration of a long block of OFDM symbols. However, for the duration of a single OFDM symbol the channel variation is small enough to be neglected. This, in other words, implies a very small inter-carrier interference (ICI). Each OFDM symbol is preceded by a cyclic prefix to avoid inter-symbol interference (ISI).

The variation in time of the wireless channel is caused by user mobility and multipath propagation. Doppler shifts on individual paths depend on v the user's velocity, f_C the carrier frequency, and the scattering environment. The maximum variation in time of the wireless channel is upper bounded by the maximum one-sided Doppler bandwidth

$$B_{\mathrm{Dmax}} = \frac{v_{\max} f_{\mathrm{C}}}{c_0},$$

where $v_{\max}$ is the maximum supported velocity, and c_0 the speed of light.

The OFDM modulation transforms a time-variant frequency-selective channel into several parallel time-variant flat-fading channels, the so called subcarriers. The variation in time of the channel coefficients of this subcarriers is bounded by B_{Dmax}, thus they can be described as band-limited sequences.

Band-limited sequences are efficiently represented through the Slepian basis expansion model (BEM) [2]. The Slepian BEM represents band-limited sequences with a minimum number of basis functions. Slepian showed in [2] that time-limited parts of band-limited sequences span a low-dimensional subspace. An orthogonal basis is spanned by the so-called discrete prolate spheroidal (DPS) sequences. These DPS sequences have a *double* orthogonality property: They are orthogonal over a finite time interval and the complete real line simultaneously. This property enables parameter estimation without the drawbacks of windowing as in the case of the Fourier BEM [2, Sec. 3.1.4]. The basis functions of the Slepian BEM are matched to the maximum variation in time of the channel, B_{Dmax}, and the length of the transmitted data block. For channel estimation with the help of a pilot pattern the Slepian BEM is biased since the orthogonality of the basis functions is lost. We apply the finite Slepian BEM which uses generalized finite discrete prolate spheroidal (FDPS) basis functions [3, 4] to overcome this problem. Generalized FDPS sequences are doubly orthogonal over a finite interval and a discrete set (which resembles the pilot pattern).

Contributions:

- We present an upper bound for the Doppler diversity of a time-variant channel based on subspace arguments and the Slepian BEM.
- We give simulation results demonstrating the ability of MC-CDMA to take advantage of Doppler diversity when the channel estimation is based on the finite Slepian BEM. In other words, the receiver performs *better* with increasing speed of the user.
- We present a simulation model [5] with enhancements for low velocities for time-variant channels with Jakes' Doppler spectrum.

The rest of the paper is organized as follows:

We present the signal model for a MC-CDMA forward link in a doubly selective channel in Sec. 2. The time-variant multi-user detector is described in Sec. 3. The time-variant channel estimator using the finite Slepian BEM is defined in Sec. 4. In Sec. 5 we discuss the Doppler diversity. The simulation results are given in Sec. 6. Finally we conclude in Sec. 7.

2. SIGNAL MODEL FOR DOUBLY SELECTIVE CHANNELS

In MC-CDMA a data symbol is spread by a user specific spreading code. The resulting chips are processed by an inverse DFT to obtain an OFDM symbol. Our transmission is block oriented, a data block consists of $M-J$ OFDM data symbols and J OFDM pilot symbols. Every OFDM symbol is preceded by a cyclic prefix to avoid ISI. We consider a channel that varies significantly over the duration of a long data block. For the duration of a single OFDM symbol the channel variation in time can be neglected.

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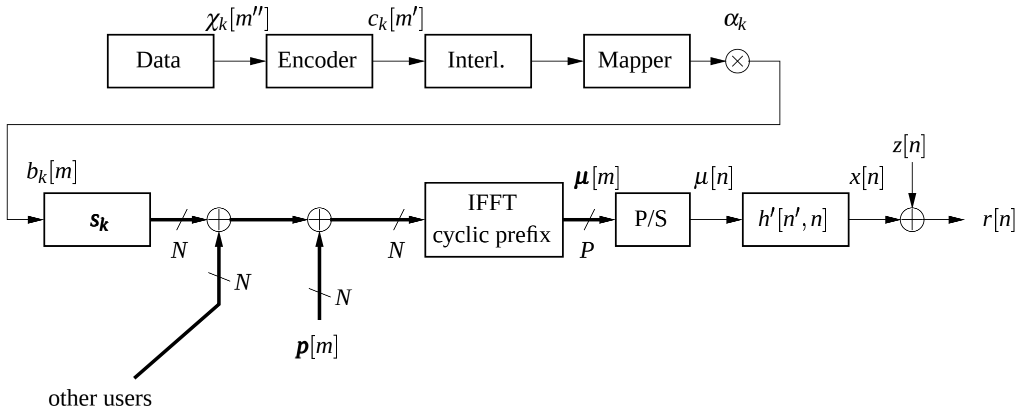

Figure 1: Model for the MC-CDMA transmitter in the forward link.

The base station transmits quaternary phase shift keying (QPSK) modulated symbols $b_k[m]$ with symbol rate $1/T_\mathrm{S}$ drawn from the alphabet $\frac{1}{\sqrt{2}}\{\pm 1 \pm \mathrm{j}\}$. Discrete time is denoted by m. There are K users in the system, the user index is denoted by k. Each symbol is spread by a random spreading sequence[1] $\boldsymbol{s}_k \in \mathbb{C}^{N\times 1}$ with elements $\frac{1}{\sqrt{2N}}\{\pm 1 \pm \mathrm{j}\}$, see Fig. 1. The data symbols $b_k[m]$ result from the binary information sequence $\chi_k[m'']$ of length $2(M-J)R_\mathrm{C}$ by convolutional encoding with code rate R_C, random interleaving and QPSK modulation with Gray labeling. The data symbols are distributed over a block of length M to fulfill $b_k[m] = 0 \;\forall\; m \in \mathscr{P}$ where the pilot placement is defined through the index set

$$\mathscr{P} = \left\{ \left\lfloor i\frac{M}{J} + \frac{M}{2J} \right\rfloor \;\middle|\; i = 0, \ldots, J-1 \right\}, \tag{1}$$

see Fig. 2.

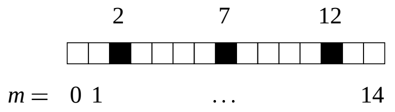

Figure 2: Example pilot pattern $\mathscr{P} = \{2, 7, 12\}$ defined by (1) for $M = 15$ and $J = 3$.

The effects of path loss and shadow fading are ignored in this paper $\alpha_k = 1 \;\forall\; k$. The spread signals of all users are added together and pilot symbols $\boldsymbol{p}[m] \in \mathbb{C}^{N\times 1}$ with elements $p[m,q]$ are inserted, fulfilling $\boldsymbol{p}[m] = \boldsymbol{0}_N \;\forall\; m \notin \mathscr{P}$. The elements of the pilot symbols $p[m,q]$ for $m \in \mathscr{P}$ and $q = 0, \ldots, N-1$ are randomly chosen from the scaled QPSK symbol set $K/\sqrt{2N}\{\pm 1 \pm \mathrm{j}\}$. Then, an N point inverse DFT is performed and a cyclic prefix of length G is inserted. A single OFDM symbol together with the cyclic prefix is represented by $\boldsymbol{\mu}[m] \in \mathbb{C}^{P\times 1}$ and has length $P = N + G$ chips. We write

$$\boldsymbol{\mu}[m] = \boldsymbol{T}_\mathrm{CP}\boldsymbol{F}_N^\mathrm{H}\left(\boldsymbol{S}\boldsymbol{b}[m] + \boldsymbol{p}[m]\right),$$

where $\boldsymbol{S} = [\boldsymbol{s}_1, \ldots, \boldsymbol{s}_K] \in \mathbb{C}^{N\times K}$, and $\boldsymbol{b}[m] = [b_1[m], \ldots, b_K[m]]^\mathrm{T} \in \mathbb{C}^{K\times 1}$ contains the stacked data symbols for K users. The cyclic prefix operation is carried out by

$$\boldsymbol{T}_\mathrm{CP} = \begin{bmatrix} \boldsymbol{I}_\mathrm{CP} \\ \boldsymbol{I}_N \end{bmatrix} \in \mathbb{R}^{P\times N}.$$

[1]Vectors are denoted by $\boldsymbol{a}$, a matrix is denoted by $\boldsymbol{A}$, $[\boldsymbol{A}]_{i,\ell}$ is its i,ℓ-th element. $\boldsymbol{A}_{P\times Q}$ is the $P\times Q$ upper left part of $\boldsymbol{A}$, $\boldsymbol{A}^\mathrm{T}$ denotes the transpose, and $\boldsymbol{A}^\mathrm{H}$ the conjugate transpose, respectively. $\mathrm{diag}(\boldsymbol{a})$ denotes a diagonal matrix with entries $a[i]$, $\mathrm{tr}(\boldsymbol{A})$ is the trace of $\boldsymbol{A}$. $\boldsymbol{I}_Q$ denotes the $Q\times Q$ identity matrix, $\boldsymbol{F}_Q$ the $Q\times Q$ unitary Fourier matrix. $\boldsymbol{1}_Q$ is a column vector with Q ones and $\boldsymbol{0}_Q$ with Q zeros. a^* is the complex conjugate of a, $\lfloor a \rfloor$ denotes the largest integer $\in \mathbb{Z}$ lower or equal than $a \in \mathbb{R}$, and $\lceil a \rceil$ the smallest integer $\in \mathbb{Z}$ greater or equal than $a \in \mathbb{R}$

It replicates the last G chips of each OFDM symbol to the front. $\boldsymbol{I}_\mathrm{CP} \in \mathbb{R}^{G\times N}$ denotes the last G rows of the identity matrix $\boldsymbol{I}_N \in \mathbb{R}^{N\times N}$. The unitary DFT matrix $\boldsymbol{F}_N \in \mathbb{C}^{N\times N}$ has elements

$$[\boldsymbol{F}_N]_{i,\ell} = \frac{1}{\sqrt{N}}\,\mathrm{e}^{\frac{-\mathrm{j}2\pi i\ell}{N}}, \qquad i,\ell = 0, \ldots, N-1. \tag{2}$$

After parallel serial conversion according to

$$\boldsymbol{\mu}[m] = [\mu[mP], \mu[mP+1], \ldots, \mu[mP+P-1]]^\mathrm{T}$$

the chip stream $\mu[n]$ with chip rate $1/T_\mathrm{C} = P/T_\mathrm{S}$ is transmitted over a time-variant multipath fading channel with L resolvable paths. The transmit filter, the time-variant channel and the matched receive filter are summarized by $h(t,\tau)$. We denote the sampled time-variant impulse response by

$$h'[n',n] = h(n'T_\mathrm{C}, nT_\mathrm{C}).$$

The channel variation in time over the duration of a single OFDM symbol is small. For an OFDM system this is directly related to small ICI [6]. The one-sided Doppler bandwidth B_D must be much smaller than the subcarrier bandwidth $\Delta f = 1/(NT_\mathrm{C})$:

$$B_\mathrm{D} = \varepsilon \Delta f,$$

where $0 \leq \varepsilon < 0.01$. Under this assumption for B_D we represent the time-variant channel through

$$h[m,n] = h'[mP, n],$$

respectively

$$\boldsymbol{h}[m] = [h(mPT_\mathrm{C}, 0), \ldots, h(mPT_\mathrm{C}, (L-1)T_\mathrm{C})]^\mathrm{T} \in \mathbb{C}^{L\times 1}$$

in vector notation. The time-variant frequency response $\boldsymbol{g}[m] \in \mathbb{C}^{N\times 1}$ with elements $g[m,q]$ is related to the time-variant impulse response via

$$\boldsymbol{g}[m] = \sqrt{N}\boldsymbol{F}_{N\times L}\boldsymbol{h}[m].$$

The receiver removes the cyclic prefix and performs a DFT. The received signal vector after these two operations is given by

$$\boldsymbol{y}[m] = \mathrm{diag}\left(\boldsymbol{g}[m]\right)\left(\boldsymbol{S}\boldsymbol{b}[m] + \boldsymbol{p}[m]\right) + \boldsymbol{z}[m], \tag{3}$$

where complex additive white Gaussian noise with zero mean and covariance $\sigma_z^2\boldsymbol{I}_N$ is denoted by $\boldsymbol{z}[m] \in \mathbb{C}^{N\times 1}$ with elements $z[m,q]$.

3. TIME-VARIANT UNBIASED LMMSE FILTER

Our receiver detects the data using the received chip sequence $\boldsymbol{y}[m]$, the spreading matrix $\boldsymbol{S}$ and the time-variant frequency response $\boldsymbol{g}[m]$ which is assumed to be known for the moment. We define the time-variant effective spreading sequences

$$\tilde{\boldsymbol{s}}_k[m] = \operatorname{diag}(\boldsymbol{g}[m])\,\boldsymbol{s}_k\,, \tag{4}$$

and the time-variant effective spreading matrix

$$\tilde{\boldsymbol{S}}[m] = [\tilde{\boldsymbol{s}}_1[m], \ldots, \tilde{\boldsymbol{s}}_K[m]] \in \mathbb{C}^{N\times K}\,,$$

to express the time-variant unbiased LMMSE filter

$$\boldsymbol{f}_k^{\mathrm{H}}[m] = \frac{\tilde{\boldsymbol{s}}_k^{\mathrm{H}}[m](\sigma_z^2\boldsymbol{I}_N + \tilde{\boldsymbol{S}}[m]\tilde{\boldsymbol{S}}^{\mathrm{H}}[m])^{-1}}{\tilde{\boldsymbol{s}}_k^{\mathrm{H}}[m](\sigma_z^2\boldsymbol{I} + \tilde{\boldsymbol{S}}[m]\tilde{\boldsymbol{S}}^{\mathrm{H}}[m])^{-1}\tilde{\boldsymbol{s}}_k[m]}\,.$$

The resulting code symbol estimates $w_k[m] = \boldsymbol{f}_k^{\mathrm{H}}[m]\boldsymbol{y}[m]$ are demapped, deinterleaved and decoded by a BCJR decoder [7] to obtain soft values for the transmitted data bits $\hat{\chi}_k[m'']$.

4. FINITE SLEPIAN BEM CHANNEL ESTIMATOR

The MC-CDMA signal model (3) describes a transmission which takes place over N parallel flat-fading channels. To reflect this we rewrite (3) as a set of equations for every subcarrier q for $q = 0, \ldots, N-1$,

$$y[m,q] = g[m,q]d[m,q] + z[m,q]\,, \tag{5}$$

where $d[m,q]$ are the elements of $\boldsymbol{d}[m] = \boldsymbol{S}\boldsymbol{b}[m] + \boldsymbol{p}[m]$. The band-limited property of $h[m,n]$ directly applies to $g[m,q]$ too. This allows to estimate the time-variant flat-fading subcarrier $g[m,q]$ with the finite Slepian BEM [4].

4.1 Finite Slepian BEM

The finite Slepian BEM expands the sequence $g[m,q]$ in terms of finite Slepian sequences $u_i[m]$

$$g[m,q] = \sum_{i=0}^{D-1} u_i[m]\,\psi_i[q]\,, \tag{6}$$

where $m = 0, \ldots, M-1$ and $q = 0, \ldots, N-1$. The finite Slepian sequences $\boldsymbol{u}_i[m] \in \mathbb{R}^{M\times 1}$ with elements $u_i[m]$ are obtained by index limiting the generalized finite discrete prolate spheroidal (FDPS) sequences $\tilde{\boldsymbol{u}}_i[m] \in \mathbb{R}^{aM\times 1}$ to $[0, M-1]$ where a is an integer parameter $a > 1$. The FDPS sequences are defined as the left singular vectors of the matrix $\mathscr{C} \in \mathbb{R}^{aM\times M}$ fulfilling

$$\mathscr{C}\mathscr{C}^{\mathrm{H}}\tilde{\boldsymbol{u}}_i = \sigma_i^2\tilde{\boldsymbol{u}}_i\,, \quad i = 0, \ldots, D-1\,.$$

The singular values are denoted by σ_i and matrix $\mathscr{C}$ is defined as

$$[\mathscr{C}]_{i,\ell} = \frac{1}{aM}\frac{\sin[\pi(2a\lceil \nu_{\mathrm{Dmax}}M\rceil + 1)(i-\ell)/(aM)]}{\sin[\pi(i-\ell)/(aM)]}$$

for $i = 0, \ldots, aM-1$ and $\ell \in \mathscr{P}$ and $[\mathscr{C}]_{i,\ell} = 0$ for $i = 0, \ldots, aM-1$ and $\ell \notin \mathscr{P}$. The normalized Doppler frequency is given by $\nu_{\mathrm{Dmax}} = B_{\mathrm{Dmax}}T_{\mathrm{S}}$. The finite Slepian sequences are orthogonal over the index set $\mathscr{P}$. The rank of $\mathscr{C}$ is $D = 2a\lceil \nu_{\mathrm{Dmax}}M\rceil + 1$ under the condition $2a\lceil \nu_{\mathrm{Dmax}}M\rceil + 1 \leq J$. For more details and background on the finite Slepian BEM please refer to [2–4, 8, 9]. The finite Slepian sequence approximate the well known Slepian sequences [2] for $\mathscr{P} = \mathscr{P}' = \{0, \ldots, M-1\}$. The approximation quality is controlled by the integer parameter a. We showed that $a = 2$ gives a sufficient approximation [3, 4].

4.2 Channel Estimator

We estimate the BEM coefficients according to

$$\hat{\psi}_i[q] = \frac{1}{\sigma_i^2}\sum_{m\in\mathscr{P}} y[m,q]b^*_{\mathrm{pilot}}[m,q]u_i[m]\,,$$

where $i = 0, \ldots, D-1$ and $q = 0, \ldots, N-1$. The estimated time-variant frequency response is given by

$$\hat{g}'[m,q] = \sum_{i=0}^{D-1} u_i[m]\,\hat{\psi}_i[q]\,.$$

Further noise suppression is obtained when we exploit the correlation between the subcarriers $\hat{\boldsymbol{g}}[m] = \boldsymbol{F}_{N\times L}\boldsymbol{F}^{\mathrm{H}}_{N\times L}\hat{\boldsymbol{g}}'[m]$. This finally allows to detect the data by inserting the channel estimates $\hat{\boldsymbol{g}}[m]$ into (4).

5. DOPPLER DIVERSITY AND THE SLEPIAN BEM

The finite Slepian sequences converge to the Slepian sequences for $\mathscr{P} = \mathscr{P}'$ and $a \to \infty$ [9]. This is also indicated by

$$\lim_{a\to\infty}[\mathscr{C}]_{i,\ell} = [\boldsymbol{C}]_{i,\ell} = \frac{\sin[2\pi(i-\ell)\nu_{\mathrm{Dmax}}]}{\pi(i-\ell)}\,, \quad i,\ell = 0, \ldots, M-1\,.$$

The eigenvectors of $\boldsymbol{C}$ fulfilling $\boldsymbol{C}\boldsymbol{u}'_i = \lambda_i\boldsymbol{u}'_i$ are the DPS sequences $u'_i[m]$ index limited to $[0, M-1]$. We call the $\boldsymbol{u}'_i$ Slepian sequences and use them to define the Slepian BEM corresponding to (6). The eigenvalues λ_i rapidly drop to zero for $i > D' = 2\nu_{\mathrm{Dmax}}M + 1$. For $\nu_{\mathrm{Dmax}}M = c$ and $M \to \infty$ this bound becomes sharp and all $\lambda_i = 1$ for $i \leq D'$ and $\lambda_i = 0$ for $i > D'$ [2]. Time-limited parts (with length M) of band-limited sequences span a subspace with approximate dimension $D' = 2\nu_{\mathrm{Dmax}}M + 1$.

Recently Ivrlac and Nossek defined a diversity measure for MIMO channels [10]. We extend this measure to time-variant flat-fading channels (respectively subcarriers). Defining $\boldsymbol{g}_q = [g[0,q], \ldots, g[M-1,q]]^{\mathrm{T}}$ we write the covariance matrix $\boldsymbol{R} = \mathbb{E}\{\boldsymbol{g}_q\boldsymbol{g}_q^{\mathrm{H}}\}$ which is independent of q if we assume the same Doppler spectrum for every channel tap.

Definition: The Doppler Diversity for a time-variant flat-fading channel is defined as

$$\Psi(\boldsymbol{R}) = \left(\frac{\operatorname{tr}(\boldsymbol{R})}{\|\boldsymbol{R}\|_{\mathrm{F}}}\right)^2\,.$$

Theorem: Exploiting properties of the DPS sequences,

$$\Psi(\boldsymbol{R}) = \left(\frac{\sum_{i=0}^{M-1}\lambda_i}{\sqrt{\sum_{i=0}^{M-1}\lambda_i^2}}\right)^2 \leq 2\nu_{\mathrm{Dmax}}M + 1$$

gives an analytic upper bound for the Doppler diversity.

Proof: Matrix $\boldsymbol{R}$ has elements $[\boldsymbol{R}]_{i,\ell} = r_{gg}[i-\ell]$. For a rectangular power density spectrum $S_{gg}(\nu) = \frac{1}{2}(\operatorname{sgn}(\nu + \nu_{\mathrm{Dmax}}) - \operatorname{sgn}(\nu - \nu_{\mathrm{Dmax}}))$, matrix $\boldsymbol{R} = 1/(2\nu_{\mathrm{Dmax}})\boldsymbol{C}$. As already noted $\boldsymbol{C}$ has $2\nu_{\mathrm{Dmax}}M + 1$ eigenvalues $\lambda_i = 1$ for large M. □

This definition gives a rigorous formulation of the observations made in [11] using the Fourier BEM [12]. Our reasoning is based on the Slepian BEM which additionally offers a strongly reduced estimation bias compared to the Fourier BEM [3, 4].

6. SIMULATION RESULTS

To generate the time-variant channel realization for the frequency-selective time-variant channel $h'[n', n]$ we use the exponentially decaying typical urban power-delay profile $\eta[\ell]$ from COST 259

[13], $\eta[\ell] = 10^{-\frac{2\ell}{10}} / \sum_{\ell'=0}^{L-1} \left| 10^{-\frac{2\ell'}{10}} \right|^2$ with $L = 15$ resolvable paths, $\ell = 0, \ldots, L-1$. The autocorrelation for every channel tap is given by

$$R_{h'h'}[\tilde{n}, \ell] = \eta[\ell] J_0(2\pi\tilde{n}\nu_{\mathrm{D}}/P).$$

Independent realizations of the following simulation model (a modified version of [5]) are used for every channel tap:

$$h[n] = \frac{1}{\sqrt{2}}(h_c[n] + \mathrm{j}h_s[n]) \tag{7}$$

$$h_c[n] = \frac{2}{\sqrt{A}} \sum_{i=1}^{A} \cos(\psi_i) \cdot \cos(2\pi\nu_{\mathrm{D}} n \cos\alpha_i + \phi_i) \tag{8}$$

$$h_s[n] = \frac{2}{\sqrt{A}} \sum_{i=1}^{A} \sin(\psi_i) \cdot \cos(2\pi\nu_{\mathrm{D}} n \cos\alpha_i + \phi_i) \tag{9}$$

with

$$\alpha_i = \frac{2\pi i - \pi + \theta}{4A}, \quad i = 1, 2, \ldots, A, \tag{10}$$

where θ, ϕ_i, and ψ_i are independent and uniformly distributed over $[-\pi, \pi)$ for all i. We use independent starting phases ϕ_i for every path which is contrary to the model in [5] where a common starting phase ϕ is used. With our modification the channel coefficients keep their Rayleigh distribution in the limit $\nu_{\mathrm{D}} = 0$ which is equivalent to a block fading channel. We fix the number of interfering paths to $A = 20$, see [5] for more details.

The system operates at carrier frequency $f_{\mathrm{C}} = 2\,\mathrm{GHz}$, the $K = 32$ users move with velocity $v \in \{0, 50, 100\}$ km/h which gives $B_{\mathrm{D}} \in \{0, 93, 185\}$ Hz and $\nu_{\mathrm{D}} \in \{0, 0.0019, 0.0038\}$. The number of subcarriers $N = 64$ and the OFDM symbol with cyclic prefix has length of $P = G + N = 79$. The chip rate is $1/T_{\mathrm{C}} = P/T_{\mathrm{S}} = 3.84$ Mcps and the data block has length $M = 256$ OFDM symbols with $J = 10$ OFDM pilot symbols. The system is designed for $v_{\max} = 100$ km/h which results in $D = 5$ for the finite Slepian BEM when $a = 2$.

For data transmission, a convolutional, non-systematic, nonrecursive, 4 state, rate $R_{\mathrm{C}} = 1/2$ code with generator polynomial $(5,7)_8$ is used. The illustrated results are obtained by averaging over 2000 independent channel realizations. The mean received QPSK symbol energy is normalized to 1 and the E_b/N_0 is defined as

$$\frac{E_b}{N_0} = \frac{1}{2R_{\mathrm{C}}\sigma_z^2} \frac{P}{N} \frac{M}{M-J}. \tag{11}$$

In Fig. 3 we illustrate the forward link MC-CDMA receiver performance in term of bit error rate (BER) versus E_b/N_0. The plot also shows the single user bound (SUB) which is defined as the performance for one user $K = 1$ and a perfectly known channel $\boldsymbol{g}[m]$.

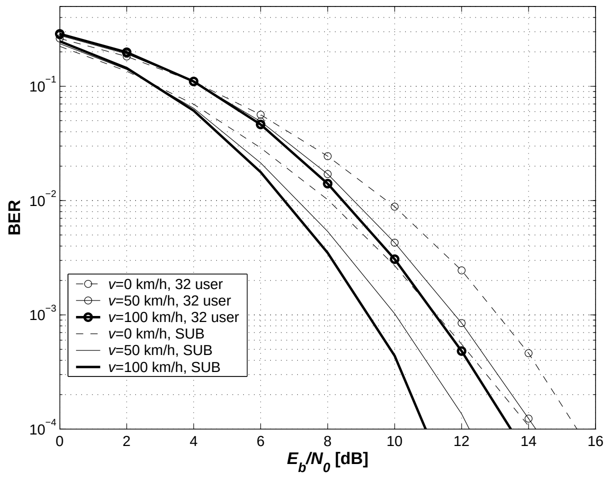

Figure 3: Forward link MC-CDMA receiver performance in terms of BER versus SNR for users moving with $v \in \{0, 50, 100\}$ km/h. The system is designed for $v_{\max} = 100$ km/h, we chose $a = 2$ which results in $D = 5$ basis functions for the finite Slepian BEM. There are $K = 32$ users in the system.

7. CONCLUSION

We show that the finite Slepian BEM is very suitable to model a time-variant frequency-selective channel for the duration of a data block. The finite Slepian BEM is designed according to three system parameters: the maximum Doppler bandwidth B_{Dmax}, the block length M, and the pilot pattern $\mathscr{P}$. A MC-CDMA system employing the finite Slepian BEM for channel equalization and estimation can take advantage of the Doppler diversity which is offered by a time-variant channel.

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