

# MINIMUM-ENERGY BANDLIMITED TIME-VARIANT CHANNEL PREDICTION WITH DYNAMIC SUBSPACE SELECTION

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## ABSTRACT

In current cellular communication systems the time-selective fading process is highly oversampled. We exploit this fact for time-variant flat-fading channel prediction by using dynamically selected predefined low dimensional subspaces spanned by discrete prolate spheroidal (DPS) sequences. The DPS sequences in each subspace exhibit a subspace-specific bandwidth matched to a certain Doppler frequency range. Additionally, DPS sequences are most energy concentrated in a time interval matched to the channel observation interval. Both properties enable the application of DPS sequences for minimum-energy (ME) bandlimited prediction. The dimensions of the predefined subspaces are in the range from one to five for practical communication systems. The subspace used for ME bandlimited prediction is selected based on a probabilistic bound on the reconstruction error.

By contrast, time-variant channel prediction based on non-orthogonal complex exponential basis functions needs Doppler frequency estimates for each propagation path which requires high computational complexity. We compare the performance of this technique under the assumption of perfectly known complex exponentials with that of ME bandlimited prediction augmented with dynamic subspace selection. In particular we analyze the mean square prediction error of the two schemes versus the number of discrete propagation paths.

## 1. INTRODUCTION

In mobile communication systems channel state information at the transmitter proves to be beneficial for increasing the system capacity. In a time-division duplex (TDD) system channel state information can be obtained while a data block is received and used for the subsequent transmission period by exploiting channel reciprocity. However, for moving users at vehicular speed the channel state information gets outdated rapidly and thus appropriate prediction is necessary.

In mobile communication systems the Doppler bandwidth is much smaller than the actual channel bandwidth. Thus the time-selective fading process is highly oversampled. Time-limited snapshots of the sampled fading process span a subspace with small dimension. The same subspace is also spanned by time-limited discrete prolate spheroidal (DPS) sequences [1]. The energy concentration of the DPS sequences in time is matched to the length of the observation interval, i.e. the length of the data block. The bandlimitation is chosen according to the Doppler bandwidth of

the time-selective fading process. Slepian [2] showed that time-limited snapshots of a bandlimited sequence can be continued uniquely in the minimum-energy (ME) bandlimited sense. We use this property for ME bandlimited prediction of time-variant flat-fading channels [3].

In [4, Sec. 12.7] a solution to the prediction problem is shown using a linear MMSE (Wiener) filter. This filter can be closely approximated by a reduced-rank maximum likelihood (ML) filter according to [5]. For both predictors full knowledge of the Doppler power spectral density is required. By contrast the ME bandlimited predictor [3] needs knowledge of the Doppler bandwidth only. We show in [6] that ME bandlimited prediction is equivalent to reduced-rank maximum likelihood prediction for a process with flat Doppler spectrum. Using analytic performance results we prove that the prediction error is strongly dependent on the Doppler bandwidth, while the actual shape of the Doppler spectrum is of minor importance [6].

The ME bandlimited predictor in [3] utilizes the Doppler-bandwidth estimator proposed in [7]. The estimator in [7] assumes a Doppler spectrum according to Clarke's model [8]. Thus specular Rice components lead to biased estimates as well as channels with a small number of specular propagation paths. The estimator proposed in [9] is less sensitive to deviations from Clarke's model, however requires large observation intervals and an SNR larger than 30 dB. In this paper we develop a subspace selection procedure for ME bandlimited prediction that avoids the need for an explicit Doppler bandwidth estimate.

## Contributions of the Paper

- We use a predefined set of subspaces. The DPS sequences in each subspace exhibit a subspace-specific bandwidth matched to a certain Doppler frequency range. The energy concentration of the DPS sequences is matched to the channel observation time interval. We obtain probabilistic bounds on the reconstruction error of each subspace using the method from [10]. The subspace with the smallest reconstruction error is used for channel prediction.
- We present numerical performance results in terms of mean square error versus Doppler-bandwidth for ME bandlimited prediction with dynamic subspace selection. The results are compared with those of a Wiener predictor [4, Sec. 12.7] and a predictor that is derived based on the assumption that the channel is composed by a finite number of specular paths [11].

## Notation

We denote a column vector by  $\mathbf{a}$  and its  $i$ -th element with  $a[i]$ . Similarly, we denote a matrix by  $\mathbf{A}$  and its  $(i, \ell)$ -th element by  $[\mathbf{A}]_{i, \ell}$ . The transpose of  $\mathbf{A}$  is given by  $\mathbf{A}^T$ , its con-

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jugate transpose by  $\mathbf{A}^H$ . A diagonal matrix with elements  $a[i]$  is written as  $\text{diag}(\mathbf{a})$  and the  $Q \times Q$  identity matrix as  $\mathbf{I}_Q$ . The absolute value of  $a$  is denoted by  $|a|$  and its complex conjugate by  $a^*$ . The largest (smallest) integer that is lower (greater) or equal than  $a \in \mathbb{R}$  is denoted by  $\lfloor a \rfloor$  ( $\lceil a \rceil$ ).

## Organization of the Paper

We introduce the signal model in Section 2. ME bandlimited prediction is reviewed in Section 3. In Section 4 we describe the dynamic subspace selection method and Monte-Carlo simulation results are presented in Section 5. Finally, concluding remarks are provided in Section 6.

## 2. SIGNAL MODEL FOR TIME-VARIANT FLAT-FADING CHANNELS

We consider a time division duplex (TDD) communication system transmitting data in blocks of length  $M$  over a time-variant channel. The symbol duration is much longer than the delay spread of the channel, i.e.  $T_S \gg T_D$ . Hence we assume the channel as frequency-flat. Discrete time at rate  $R_S = 1/T_S$  is denoted by  $m$ . The channel incorporates the transmit filter, the transmit antenna, the physical channel, the receive antenna, and the receive matched filter. The data symbols  $b[m]$  are randomly and evenly drawn from a symbol alphabet with constant modulus, i.e.  $|b[m]| = 1$ . The discrete-time signal at the matched filter output

$$y'[m] = h[m]b[m] + n'[m] \quad (1)$$

is the superposition of the data symbol multiplied by the sampled time-variant channel weight  $h[m]$ , and complex white Gaussian noise  $n'[m]$  with variance  $\sigma^2$ .

We assume an error-free decision feedback structure [12, 13]. Thus we are able to obtain noisy channel observations using the error-free data symbol estimates  $\hat{b}[m] = b[m]$ :

$$\begin{aligned} y[m] &= y'[m]\hat{b}[m]^* \\ &= h[m] + n'[m]\hat{b}[m]^* = h[m] + n[m]. \end{aligned} \quad (2)$$

Notice that  $n[m]$  has the same statistical properties as  $n'[m]$ . The signal-to-noise ratio (SNR) is  $\text{SNR} = 1/\sigma^2$ .

The transmission is block oriented. A data block corresponds to the time interval  $\mathcal{I}_M = \{0, \dots, M-1\}$ . The noisy channel observations  $y[m]$ ,  $m \in \mathcal{I}_M$ , obtained during a single data block are used to predict the channel weight up to  $N$  symbols into the future.

The electromagnetic field at the receiver is the superposition of the contribution of the individual fields of  $P$  impinging wave fronts. Each wave front is conceived as originating from a specific scatterer. For a user moving with velocity  $v$  the time-variant fading process  $\{h[m]\}$  will be bandlimited by the one-sided normalized Doppler bandwidth  $\nu_D = v f_C T_S / c_0$  where  $f_C$  is the carrier frequency and  $c_0$  stands for the speed of light.

Without loss of generality  $\{h[m]\}$  is a zero-mean, circularly symmetric, unit-variance (due to power control) process. We assume a time-variant block-fading channel model. Hence the fading process  $\{h[m]\}$  with correlation function  $R_h[k] = \mathbf{E}\{h^*[m]h[m+k]\}$  is wide-sense stationary over the limited time interval  $\mathcal{I}_{M+N}$  (c.f. Section 5.1).

## 3. MINIMUM-ENERGY BANDLIMITED PREDICTION

In this section we review the concept of ME bandlimited prediction of a time-variant flat-fading channel [3, 6].

### 3.1 Channel Estimation

The channel coefficients for a single block of length  $M$  are collected in the vector  $\mathbf{h} = [h[0], h[1], \dots, h[M-1]]^T$  and the covariance matrix is defined as  $\mathbf{\Sigma}_h = \mathbf{E}\{\mathbf{h}\mathbf{h}^H\}$  with elements  $[\mathbf{\Sigma}_h]_{\ell, m} = R_h[\ell - m]$ . The noisy observation vector  $\mathbf{y} = [y[0], y[1], \dots, y[M-1]]^T$  is used for channel prediction. Its covariance matrix reads  $\mathbf{\Sigma}_y = \mathbf{\Sigma}_h + (1/\text{SNR})\mathbf{I}_M$ .

We consider a subspace-based approximation which expands the vector  $\mathbf{h}$  in terms of  $D$  orthonormal basis vectors  $\mathbf{u}_i = [u_i[0], u_i[1], \dots, u_i[M-1]]^T$ ,  $i \in \{0, \dots, D-1\}$ :

$$\mathbf{h} \approx \mathbf{U}\boldsymbol{\gamma} = \sum_{i=0}^{D-1} \gamma_i \mathbf{u}_i. \quad (3)$$

In this expression  $\mathbf{U} = [\mathbf{u}_0, \dots, \mathbf{u}_{D-1}]$  and  $\boldsymbol{\gamma} = [\gamma_0, \dots, \gamma_{D-1}]^T$  contains the basis expansion coefficient. The least square estimate of  $\boldsymbol{\gamma}$  simplifies to  $\hat{\boldsymbol{\gamma}} = \mathbf{U}^H \mathbf{y}$  due to the orthogonality of the basis functions.

The reconstruction error per data block is defined as

$$z = \frac{1}{M} \|\mathbf{h} - \hat{\mathbf{h}}\|^2 = \frac{1}{M} \|\mathbf{U}^H \mathbf{n}\|^2 + \frac{1}{M} \|\mathbf{V}^H \mathbf{h}\|^2 \quad (4)$$

where  $\hat{\mathbf{h}} = \mathbf{U}\hat{\boldsymbol{\gamma}}$ ,  $\mathbf{V} = [\mathbf{u}_D, \dots, \mathbf{u}_{M-1}]$  contains the basis vectors spanning the subspace orthogonal to the signal subspace  $\mathbf{U}$  and the noise vector is defined as  $\mathbf{n} = [n[0], n[1], \dots, n[M-1]]^T$ . The mean square reconstruction error writes  $\text{MSE} = \mathbf{E}\{z\}$ .

We seek basis vectors  $\mathbf{u}_i$  minimizing the reconstruction error per data block. Discrete prolate spheroidal (DPS) sequences  $\{u_i[m]\}$  time limited to  $\mathcal{I}_M$  form such basis vectors for a fading process with flat Doppler spectrum [1]. The properties of DPS sequences are analyzed by Slepian in [2]. DPS sequences are bandlimited to the frequency range  $[-\nu_D, \nu_D]$  and simultaneously most concentrated in  $\mathcal{I}_M$ . They are defined as

$$\sum_{\ell=0}^{M-1} \frac{\sin(2\pi\nu_D(\ell - m))}{\pi(\ell - m)} u_i[\ell] = \lambda_i u_i[m], \quad (5)$$

for  $m \in \mathbb{Z}$  where  $\mathbb{Z}$  denotes the set of integers. The sequences  $\{u_i[m]\}$  and the eigenvalues  $\lambda_i$  depend on the Doppler bandwidth  $\nu_D$  and the observation interval length  $M$ . We omit this dependence to keep the notation simple.

Both, the DPS sequences  $\{u_i[m]\}$  and their restriction on  $\mathcal{I}_M$  form orthogonal sets. The eigenvalues  $\lambda_i$  decay exponentially for  $i \geq D'$ . The essential subspace dimension is defined as

$$D' = \lceil 2\nu_D M \rceil + 1. \quad (6)$$

The vectors  $\mathbf{u}_i$  satisfy the eigenvalue decomposition  $\mathbf{C}\mathbf{u}_i = \lambda_i \mathbf{u}_i$ , where

$$[\mathbf{C}]_{\ell, m} = \frac{\sin(2\pi\nu_D(\ell - m))}{\pi(\ell - m)}, \quad \ell, m \in \mathcal{I}_M. \quad (7)$$

Knowing  $\mathbf{u}_i$  we can continue the sequence  $\{u_i[m]\}$  over  $\mathbb{Z}$  in the ME bandlimited sense [2] by evaluating (5).

The subspace dimension minimizing the MSE for a given SNR is found to be [14]

$$D = \underset{\tilde{D} \in \{1, \dots, M\}}{\text{argmin}} \left( \frac{1}{2\nu_D M} \sum_{i=\tilde{D}}^{M-1} \lambda_i + \frac{\tilde{D}}{M} \frac{1}{\text{SNR}} \right). \quad (8)$$

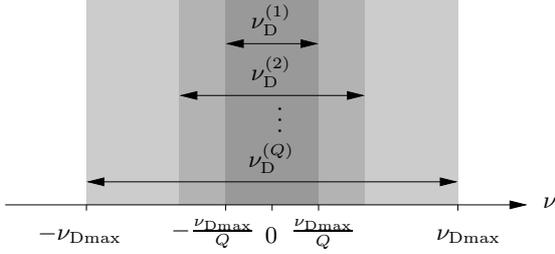


Figure 1: Symmetric bandlimiting regions with (one sided) bandwidth  $\nu_D^{(q)} = q/Q \nu_{D\max}$ ,  $q \in \{1, \dots, Q\}$  used to define a set of  $Q$  subspaces. The  $q$ -th subspace is spanned by the columns of  $\mathbf{U}_q$ .

### 3.2 Channel Prediction

So far we treated the channel estimation problem for a channel observed over a time interval  $\mathcal{I}_M$ . We used orthogonal basis vectors that result from time limiting an infinite sequence to the interval  $\mathcal{I}_M$ . The DPS sequences are most energy concentrated in this interval.

However, the main interest of this paper lies on channel prediction. Slepian points out [2, Sec. 3.1.4] that there are infinitely many ways to choose the channel samples  $h[m]$ ,  $m \in \mathbb{Z} \setminus \mathcal{I}_M$  such that the infinite sequence  $\{h[m]\}$  is bandlimited. However, there exists a unique way to extend a bandlimited sequence in the sense of a ME continuation. This is achieved by using the DPS sequences  $\{u_i[m]\}$ .

We can express the ME bandlimited prediction of the fading process for any  $m \in \mathbb{Z}$  as

$$\hat{h}[m] = \mathbf{f}[m]^T \hat{\boldsymbol{\gamma}} = \sum_{i=0}^{D-1} \hat{\gamma}_i u_i[m], \quad (9)$$

where  $\mathbf{f}[m] = [u_0[m], \dots, u_{D-1}[m]]^T$ .

For a processes with flat spectrum the mean square prediction error per symbol  $\text{MSE}[m]$  can be expressed by  $\text{MSE}[m] = \mathbb{E}\{|h[m] - \hat{h}[m]|^2\} = 1 - \sum_{i=0}^{D-1} \lambda_i |u_i[m]|^2$ . For a general Doppler power spectral density (mismatched case) this expression provides a lower bound (for more details see [6]).

## 4. DYNAMIC SUBSPACE SELECTION

We are interested in a low-complexity channel predictor. To this end we develop a dynamic subspace selection scheme for ME bandlimited prediction that does not need an explicit Doppler bandwidth estimate. Firstly, we define a set of subspaces. The orthogonal basis vectors spanning each subspace are calculated once and then kept fixed. Secondly, we propose a subspace selection method based on the observation of a single data block.

### 4.1 Subspace Definition

We define the maximum Doppler bandwidth  $\nu_{D\max} = v_{\max} f_C T_S / c_0$  as system parameter given by the maximum user velocity  $v_{\max}$ . We define a set of  $Q$  subspaces with (one sided) Doppler bandwidth  $\nu_D^{(q)} = q/Q \nu_{D\max}$ ,  $q \in \{1, \dots, Q\}$ , see Fig. 1. The DPS sequences  $\{u_i[m, \nu_D^{(q)}]\}$  corresponding to a Doppler bandwidth  $\nu_D^{(q)}$  are calculated according to (5). We define the subspace  $\mathbf{U}_q = [\mathbf{u}_0(\nu_D^{(q)}), \dots, \mathbf{u}_{D_q}(\nu_D^{(q)})]$ , where the vectors  $\mathbf{u}_i(\nu_D^{(q)})$  are the sequences  $\{u_i[m, \nu_D^{(q)}]\}$  time-limited to  $\mathcal{I}_M$ . The dimension of the subspace  $D_q$  spanned by  $\mathbf{U}_q$  grows with increasing  $q$  due to the increasing spectral support  $2\nu_D^{(q)}$  (c.f. (6) and (8)).

### 4.2 Subspace Selection

In [10] an information theoretic subspace selection scheme is proposed. This method uses the observable data error

$$x_q = \frac{1}{M} \|\mathbf{y} - \hat{\mathbf{h}}_q\|^2 \quad (10)$$

where

$$\hat{\mathbf{h}}_q = \mathbf{U}_q \mathbf{U}_q^H \mathbf{y}, \quad (11)$$

to obtain a bound on the reconstruction error

$$z_q = \frac{1}{M} \|\mathbf{h} - \hat{\mathbf{h}}_q\|^2 \quad (12)$$

which cannot be observed directly. For the subspace selection  $\mathbf{h}$  is considered deterministic. We adapt the results from [10] to complex valued variables. The reconstruction error  $z_q$  is a sample of a random variable  $Z_q$  which is distributed as [10, Lemma 1]

$$\frac{2M}{\sigma^2} \left( Z_q - \frac{1}{M} \|\mathbf{V}_q^H \mathbf{h}\|^2 \right) \sim \chi_{2D_q}^2 \quad (13)$$

where  $\chi_{2D_q}^2$  is a Chi-square random variable of order  $2D_q$ . Assuming  $\|\mathbf{V}_q^H \mathbf{h}\|^2$  is known, the reconstruction error is bounded with probability  $p_1$  according to  $\underline{z}'_q \leq z_q \leq \overline{z}'_q$ , where  $\underline{z}'_q = D_q \sigma^2 / M + 1/M \|\mathbf{V}_q^H \mathbf{h}\|^2 - G_q(p_1, \sigma, 2D_q)$  and  $\overline{z}'_q = D_q \sigma^2 / M + 1/M \|\mathbf{V}_q^H \mathbf{h}\|^2 + G_q(p_1, \sigma, 2D_q)$ . The term  $G_q(p_1, \sigma, 2D_q)$  is calculated by solving  $p_1 = F(2D_q + 2G_q M / \sigma^2, 2D_q) - F(2D_q - 2G_q M / \sigma^2, 2D_q)$  numerically for  $G_q$ . The Chi-square cumulative distribution function with  $n$  degrees of freedom is denoted by  $F(x, n)$ .

The data error  $x_q$  is a sample of a random variable  $X_q$  which is distributed as [10, Lemma 2]

$$\frac{2M}{\sigma^2} X_q \sim \chi_{2(M-D_q)}^2. \quad (14)$$

Because  $2(M - D_q)$  is large we can invoke the Central Limit Theorem to approximate  $X_q$  with a Gaussian random variable. The term  $1/M \|\mathbf{V}_q^H \mathbf{h}\|^2$  is bounded with probability  $p_2$ , i.e.  $\underline{B}_q \leq 1/M \|\mathbf{V}_q^H \mathbf{h}\|^2 \leq \overline{B}_q$ . The bounds  $\underline{B}_q$  and  $\overline{B}_q$  are given in [10, Theorem 1]. Finally, we use the upper bound

$$\overline{z}_q = \frac{D_q}{M} \sigma^2 + \overline{B}_q + G_q(p_1, \sigma, 2D_q) \geq \overline{z}'_q \quad (15)$$

on the reconstruction error to select the appropriate subspace  $q$  spanned by the columns of  $\mathbf{U}_q$ ,  $q = \text{argmin}_q \overline{z}_q$ . The chosen subspace  $\mathbf{U}_q$  is used for ME bandlimited prediction.

## 5. MONTE-CARLO SIMULATIONS

### 5.1 Physical Wave Propagation Channel Model

We simulate the fading process  $\{h[m]\}$  using physical wave propagation principles [8, 15]. The electromagnetic field at the receiver is the superposition of the contribution of the individual fields of  $P$  impinging plane waves. Each plane wave is conceived as originating from a specific scatterer. Under these assumption the channel weight is of the form

$$h[m] = \sum_{p=0}^{P-1} a_p e^{j2\pi f_p T_S m} = \sum_{p=0}^{P-1} a_p e^{j2\pi \nu_p m}. \quad (16)$$

Here  $f_p$  is the Doppler shift of wave  $p$ . For easier notation we define the normalized Doppler frequency as  $\nu_p = f_p T_S$ . Notice that  $|\nu_p| \leq \nu_D < 1/2$ . The gain and phase shift of

path  $p$  are embodied in the complex weight  $a_p \in \mathbb{C}$ . We model the random parameter sets  $a_p$  and  $\nu_p$ ,  $p \in \{0, \dots, P-1\}$  as independent. The random variables in each set are independent and identically distributed. The path angles  $\alpha_p$  are uniformly distributed over  $[-\pi, \pi)$  and the Doppler shift per path  $\nu_p = \cos(\alpha_p)\nu_D$ . The path weights are defined as  $a_p = 1/\sqrt{P}[\cos(\psi_p) + j\sin(\psi_p)]$  where  $\psi_p$  are uniformly distributed over  $[-\pi, \pi)$ . Under the above assumptions, the covariance function of  $h[m]$  converges to  $R_h[k] = J_0(2\pi\nu_D k)$ , for  $P \rightarrow \infty$  where  $J_0$  is the zeroth order Bessel function of the first kind [8].

We assume a time-variant block-fading channel model comprised of  $P$  paths. Hence the random path parameters  $a_p$  and  $\nu_p$  are assumed to be constant over a block of  $M + N$  symbols. However, the path parameters  $a_p$  and  $\nu_p$  change independently from block to block therefore the short-time Doppler spectrum changes as well.

We note that the over-idealized simulation models from Jakes [16] or Zheng [17] are not suitable for the evaluation of channel prediction algorithms. This is because a symmetric distribution of the scatterers with equidistant spacing is assumed in [16, 17]. However real-world channels will not show equidistantly spaced scatterers. Prediction algorithms assuming a finite number of specular paths [18] show optimistically biased performance due to this over-idealized scatterer distribution.

## 5.2 Simulation Parameters

The symbol duration  $T_S = 20.57 \mu\text{s}$  is chosen according to the system parameters considered in [1]. The speed of the receiver varies in the range  $0 \leq v \leq v_{\max} = 100 \text{ km/h} = 27.8 \text{ m/s}$ . The carrier frequency is  $f_C = 2 \text{ GHz}$ . This results in a Doppler bandwidth range  $0 \leq B_D \leq 180 \text{ Hz}$ . Thus, the normalized Doppler bandwidth ranges  $0 \leq \nu_D \leq \nu_{D\max} = 3.8 \cdot 10^{-3}$ . The channel is observed over  $M = 256$  symbols. We are interested in the prediction error at a prediction horizon of  $m - M + 1 \in \{32, 128\}$  symbols. At speed  $v_{\max}$  the prediction horizon  $\{32, 128\}$  relates to a distance of  $\{\lambda/8, \lambda/2\}$  where  $\lambda = c_0/f_C$  denotes the wavelength. For all simulations the SNR is 10 dB. All simulation results are averaged over 1000 independent channel realizations. The probabilities  $p_1$  and  $p_2$  for the subspace selection are chosen as  $p_1 = p_2 = \int_{-\alpha}^{\alpha} (1/\sqrt{2\pi})e^{-x^2/2} dx$  with  $\alpha = 8$ , i.e.  $p_1 = p_2 \approx 1 - 10^{-15}$ .

## 5.3 Choice of the Number of Subspaces $Q$

We use the Cramer Rao lower bound (CRLB) for frequency estimation of a single complex exponential in white Gaussian noise [4, Sec. 15.10] to obtain a suitable choice on the number of subspaces  $Q$ . In the case of multiple paths  $P$  the energy per paths will be reduced by  $1/P$  and we can utilize

$$\theta = \frac{1}{\nu_{D\max}} \sqrt{\frac{6P\sigma^2}{(2\pi)^2 M(M^2 - 1)}} \leq \frac{\sqrt{\text{var}(\hat{\nu})}}{\nu_{D\max}} \quad (17)$$

as lower bound on the relative frequency resolution  $\sqrt{\text{var}(\hat{\nu})}/\nu_{D\max}$ . In Fig. 2 we plot  $\theta$  versus the number of paths  $P$  for an SNR  $\in \{0, 10, 20\}$  dB. Fig. 2 documents that the lower bound on the frequency resolution is in the order of  $0.01\nu_{D\max}$  to  $0.1\nu_{D\max}$  for  $P = 20$  propagation paths. We choose to partition the range of the Doppler bandwidth  $0 \leq \nu_D \leq \nu_{D\max}$  into  $Q = 10$  intervals, see Fig. 1.

## 5.4 Complexity

The complexity of the proposed ME bandlimited predictor with dynamic subspace selection is mainly determined by the

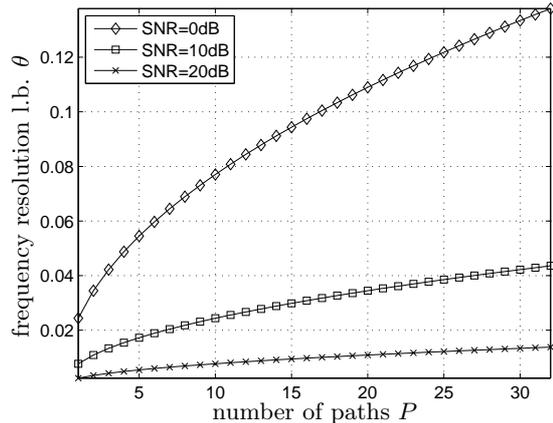


Figure 2: Lower bound on the frequency resolution (17) versus number of paths  $P$ . The SNR  $\in \{0, 10, 20\}$  dB.

complexity of projecting the observation vector  $\mathbf{y}$  on all  $Q$  subspaces  $\mathbf{U}_q$  in (11). This operation requires  $2(\sum_{q=1}^Q D_q)M$  complex multiply accumulate instructions. The calculation of the lower bound on the reconstruction error involves simple arithmetics or look-up tables of which the complexity and can be neglected.

A predictor based on complex exponentials needs Doppler shift estimates for all  $P$  paths. Most methods for Doppler shift estimation rely on an eigenvalue decomposition of the channel's sample covariance matrix [19]. The complexity of the eigenvalue decomposition grows with  $PM^2$ . Hence the overall complexity of complex-exponential-based predictors is much higher than the one of the ME bandlimited predictor with dynamic subspace selection.

## 5.5 Simulation Results

Monte Carlo simulations have been performed to contrast the performance of the three following predictors: The Wiener predictor [4, Sec. 12.7], the ME bandlimited predictor and a predictor derived based on the specular path model (16) [11]. The later predictor knows the number of paths and their Doppler shift. The MSE of all three predictors are reported in Fig. 3 and Fig. 4 for two different prediction horizons  $m - M + 1 \in \{32, 128\}$ .

The ME bandlimited predictor with dynamic subspace selection (denoted 'ME bandlimited') performs better than a Wiener predictor (denoted 'Wiener predictor') which is designed using the long-term Clarke [8] Doppler power spectral density. Comparing the ME bandlimited predictor to a predictor based on complex exponentials (denoted 'compl. exponential') we can see the following results: For a short prediction horizons  $m - M + 1 = 32$  (Fig. 3) the predictor based on complex exponentials performs better for  $P = 2$  and  $\nu_D M > 0.8$  only. For a longer prediction horizon  $m - M + 1 = 128$  (Fig. 4) the crossover point for  $P = 2$  is at  $\nu_D M = 0.4$  and for  $P = 4$  at  $\nu_D M = 0.8$ .

## 6. CONCLUSION

We presented a minimum-energy (ME) bandlimited predictor with dynamic subspace selection. Each subspace is matched to a certain Doppler bandwidth. The dimensions of the predefined subspaces are in the range from one to five for practical communication systems. The subspace applied for ME bandlimited prediction is selected based on a probabilistic bound on the reconstruction error. The ME bandlimited predictor with dynamic subspace selection performs better

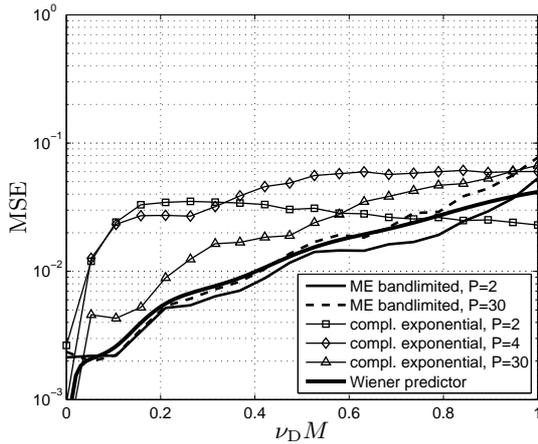


Figure 3: Mean square prediction error  $MSE[m]$  at prediction horizon  $m - M + 1 = 32$ . We compare the minimum-energy (ME) bandlimited predictor with dynamically selected subspace ('ME bandlimited'), the Wiener predictor ('Wiener predictor') and a predictor using perfectly known complex exponential basis functions ('compl. exponential').

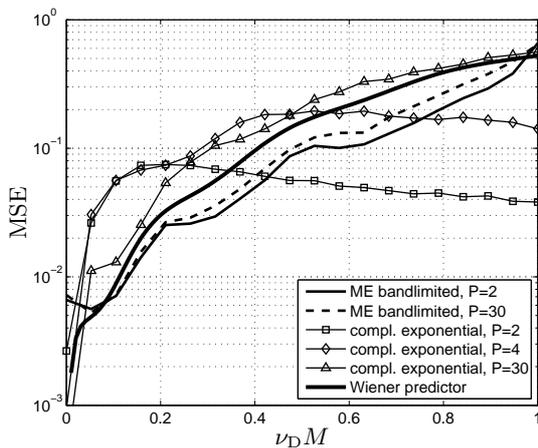


Figure 4: Mean square prediction error  $MSE[m]$  at prediction horizon  $m - M + 1 = 128$ .

than a predictor based on complex exponentials for channels with more than ten propagation paths.

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