Time-Variant Channel Prediction using Time-Concentrated and Band-Limited Sequences

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***Abstract*— We present the basic methodology for minimum-energy bandlimited prediction of time-variant flat Rayleigh-fading channels. This predictor is based on a subspace spanned by time-concentrated and bandlimited sequences. The concept of time-variant channel *estimation* using time-concentrated and band-limited sequences was introduced recently by Zemen et al. We extend this concept to the problem of time-variant channel *prediction*. Slepian showed that discrete prolate spheroidal (DPS) sequences can be used to calculate the minimum-energy bandlimited continuation of a finite sequence. DPS sequences are optimal for a time-variant channel with flat Doppler spectrum. We generalize the concept of time-concentrated and band-limited sequences to arbitrary Doppler spectra approaching closely the lower prediction error limit defined by the Wiener filter. In practical systems detailed channel covariance information is not available. We design a set of subspaces spanned by DPS sequences with fixed time-concentration but growing bandwidth. The best DPS subspace is selected dynamically for each observation interval using a Doppler bandwidth estimate allowing for low complexity channel prediction. The performance of the new predictor is compared to that of the Wiener filter by means of Monte Carlo simulations.**

I. Introduction

In mobile communication systems channel state information at the transmitter proves to be beneficial for increasing the system capacity. In a time-division duplex (TDD) system channel state information can be obtained during the uplink transmission period and used for the subsequent downlink transmission by exploiting channel reciprocity. However, for moving users at vehicular speed the channel state information gets outdated rapidly and thus appropriate prediction is necessary.

In this paper we present the basic methodology of minimum-energy bandlimited prediction of time-variant flat-fading channels. Time-variant channel *estimation* using time-concentrated and band-limited sequences was introduced in [1]. We extend the concepts from [1] to the problem of time-variant channel *prediction* generalizing in this way earlier results by Slepian on the minimum-energy bandlimited continuation of a finite sequence [2]. The same method can be applied on a per-subcarrier basis in an orthogonal frequency division multiplexing (OFDM) based system operating in time-variant frequency-selective channels.

We model the time-variant flat-fading channel as the superposition of P propagation paths. Each path is characterized by its distinct complex weight and Doppler shift. Classic approaches for time-variant channel prediction use a finite number of noisy channel observations to identify the parameters of all P paths:

- Firstly the Doppler shift of each path is identified [3]–[6] using e.g. ESPRIT [7].
- Secondly the complex weight of each path is estimated in the minimum mean square error (MMSE) sense [4].
- Thirdly, future channel values are predicted, based on the above estimators.

In this paper we will describe a minimum-energy bandlimited channel prediction method. This method avoids the problems associated with a per-path Doppler shift estimation based on a time-limited snapshot of noisy channel observations.

Contributions of the Paper

- Slepian [2] showed that discrete prolate spheroidal (DPS) sequences are optimal for the prediction of a time-variant process with flat Doppler spectrum in the sense of a minimum energy band-limited extension. We generalize the concept of time-concentrated and bandlimited sequences to arbitrary Doppler spectra.
- In practical systems detailed channel covariance information is not available. We design a set of subspaces spanned by DPS sequences with fixed time concentration and growing bandwidth. The best DPS subspace is chosen dynamically for each observation interval based on a Doppler bandwidth estimate. This algorithm allows low complexity prediction of the time-variant channel.

Notation

We denote a column vector by $\boldsymbol{a}$ and its i-th element with $a[i]$. Equivalently, we denote a matrix by $\boldsymbol{A}$ and its (i, ℓ)-th element by $[\boldsymbol{A}]_{i,\ell}$. The transpose of $\boldsymbol{A}$ is given by $\boldsymbol{A}^{\mathrm{T}}$, its conjugate transpose by $\boldsymbol{A}^{\mathrm{H}}$ and its upper left part with dimension $P \times Q$ by $\boldsymbol{A}_{P\times Q}$. A diagonal matrix with elements $a[i]$ is written as $\mathrm{diag}(\boldsymbol{a})$ and the $Q \times Q$ identity matrix as $\boldsymbol{I}_Q$. The absolute value of a is denoted by $|a|$ and its complex conjugate by a^*. The largest (smallest) integer, lower (greater) or equal than $a \in \mathbb{R}$ is denoted by $\lfloor a \rfloor$ ($\lceil a \rceil$).

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Organization of the Paper

We introduce the signal model in Section II and explain the channel model in Section III. Minimum-energy bandlimited prediction is presented in Section IV. In Section V we present a comparison of the proposed prediction method with a method based on complex exponential basis functions by means of Monte Carlo simulations. In Section VI a dynamic subspace selection method using a Doppler bandwidth estimate is derived and simulation results are given. Finally, concluding remarks are provided in Section VII.

II. Signal Model for Time-Variant Flat-Fading Channels

We consider a time division duplex (TDD) communication system transmitting data in blocks of length M over a time-variant flat-fading channel. The symbol duration is much longer than the delay spread of the channel, i.e. $T_{\mathrm{S}} \gg T_{\mathrm{D}}$. Discrete time at rate $R_{\mathrm{S}} = 1/T_{\mathrm{S}}$ is denoted by m. The channel incorporates the transmit filter, the transmit antenna, the physical channel, the receive antenna, and the receive matched filter. The data symbols $b[m]$ are randomly and evenly drawn from a symbol alphabet with constant modulus, i.e. $|b[m]| = 1$. The discrete-time signal at the matched filter output

$$y'[m] = h[m]b[m] + z'[m] \tag{1}$$

is the superposition of the data symbol multiplied with the sampled time-variant channel $h[m]$ and additive complex white Gaussian noise $z'[m]$ with zero mean and variance $\sigma_{z'}^2$.

We assume an error-free decision feedback structure [8]–[10]. Thus we are able to obtain noisy channel observations

$$y[m] = y'[m]\tilde{b}[m]^{\mathrm{H}} = h[m]+z'[m]\tilde{b}[m]^{\mathrm{H}} = h[m]+z[m] \tag{2}$$

using the data symbol estimates $\tilde{b}[m]$, where $z[m]$ has the same statistical properties than $z'[m]$.

We assume Rayleigh fading and power control. Without loss of generality $\{h[m]\}$ is a zero-mean, circularly symmetric, unit-variance wide-sense stationary process with covariance function $R_h[k] = \mathsf{E}\{h^*[m]h[m+k]\}$. The signal-to-noise ratio (SNR) $\mathrm{SNR} = 1/\sigma_z^2$.

The noisy channel observations $y[m]$, $m \in \{0, \ldots, M-1\}$ are used to predict the time-variant flat-fading channel N symbols into the future for $m \in \{M, \ldots, M+N-1\}$.

III. Physical Wave Propagation Channel Model

We model the fading process using physical wave propagation principles [11]. The impinging wave-fronts at the receive antenna originate from P scatterers. The contributions of individual paths sum up as

$$h[m] = \sum_{p=0}^{P-1} a_p \mathrm{e}^{\mathrm{j}2\pi f_p T_{\mathrm{S}} m} = \sum_{p=0}^{P-1} a_p \mathrm{e}^{\mathrm{j}2\pi \nu_p m}. \tag{3}$$

Here f_p is the Doppler shift of path p. For easier notation we define the normalized Doppler frequency as $\nu_p = f_p T_{\mathrm{S}}$. The gain and phase shift of path p are embodied in the complex weight $a_p \in \mathbb{C}$. The one-sided normalized Doppler bandwidth is

$$\nu_{\mathrm{D}} = \frac{v f_{\mathrm{C}}}{c_0} T_{\mathrm{S}} \geq |\nu_p|, \tag{4}$$

where v denotes the user velocity, f_{C} is the carrier frequency and c_0 stands for the speed of light. We model the random parameter sets a_p and ν_p, $p \in \{0, \ldots, P-1\}$ as independent. The random variables in each set are independent and identically distributed. For $P \geq 20$ the Rayleigh fading assumption for $h[m]$ is realistic due to the central limit theorem.

We assume a time-variant block-fading channel model. Hence the random path parameters a_p and ν_p are assumed to be constant over a block of $M+N$ symbols. The fading process is wide-sense stationary over the duration of a single data block. However from block to block the fading is non-stationary [12].

IV. Minimum-Energy Bandlimited Prediction

Noisy channel observations $y[m]$, $m \in \{0, \ldots, M-1\}$, are used for channel prediction. The observation covariance matrix is given by

$$\boldsymbol{\Sigma}_{\boldsymbol{y}} = \boldsymbol{\Sigma}_{\boldsymbol{h}} + \frac{1}{\mathrm{SNR}} \boldsymbol{I}_M. \tag{5}$$

The channel coefficients for a single block of length M are collected in the vector

$$\boldsymbol{h} = [h[0], h[1], \ldots, h[M-1]]^{\mathrm{T}} \tag{6}$$

and the covariance matrix is defined as

$$\boldsymbol{\Sigma}_{\boldsymbol{h}} = \mathsf{E}\{\boldsymbol{h}\boldsymbol{h}^{\mathrm{H}}\} \tag{7}$$

with elements

$$[\boldsymbol{\Sigma}_{\boldsymbol{h}}]_{\ell,m} = R_h[\ell - m]. \tag{8}$$

We consider a subspace-based channel description which approximate the sequence $h[m]$ by a linear combination of D orthogonal basis sequences $u_i[m]$, $i \in \{0, \ldots, D-1\}$:

$$h[m] \approx \hat{h}[m] = \sum_{i=0}^{D-1} u_i[m] \hat{\gamma}_i, \tag{9}$$

for $m \in \{0, \ldots, M-1\}$. The least square estimate of the basis expansion coefficients γ_i simplifies to

$$\hat{\gamma}_i = \sum_{m=0}^{M-1} u_i^*[m] y[m] \tag{10}$$

due to the orthogonality of the basis functions.

We want to use a set of basis functions $u_i[m]$ that minimize the mean square error (MSE) per observation interval. The MSE per symbol is defined as

$$\mathrm{MSE}[m] = \mathsf{E}\left\{\left|h[m] - \hat{h}[m]\right|^2\right\} \tag{11}$$

and the MSE per observation interval reads

$$\mathrm{MSE}_M = \frac{1}{M} \sum_{m=0}^{M-1} \mathrm{MSE}[m]. \tag{12}$$

It is shown in [1], [13], [14] that MSE_M can be described as the sum of a square bias term and a variance term:

$$\mathrm{MSE}_M = \mathrm{bias}_M^2 + \mathrm{var}_M \,. \tag{13}$$

In general, if the second order statistic $\boldsymbol{\Sigma_h}$ of the fading process is known, the Karhunen-Loève expansion [15] provides the optimum basis functions in terms of minimum MSE_M. The basis functions of the Karhunen-Loève subspace are defined by

$$\boldsymbol{\Sigma_h} \boldsymbol{u}_i = \lambda_i' \boldsymbol{u}_i \tag{14}$$

for $i \in \{0, \ldots, D-1\}$ where $\boldsymbol{u}_i \in \mathbb{C}^M$ has elements $u_i[m]$. The eigenvalues λ_i' are sorted in descending order. The summands in (13) are given by

$$\mathrm{var}_M = \frac{D}{M} \frac{1}{\mathrm{SNR}} \,, \tag{15}$$

and

$$\mathrm{bias}_M^2 = \frac{1}{M} \sum_{i=D}^{Q-1} \lambda_i' \,, \tag{16}$$

where Q denotes the rank of the covariance matrix $\boldsymbol{\Sigma_h}$.

The subspace dimension that minimizes MSE_M for a given SNR is found to be [13]

$$D = \underset{D' \in \{1, \ldots, Q\}}{\operatorname{argmin}} \left(\frac{1}{M} \sum_{i=D'}^{Q-1} \lambda_i' + \frac{D'}{M} \frac{1}{\mathrm{SNR}} \right) . \tag{17}$$

Thus, the optimal subspace dimension D depends on the SNR.

So far we treated the channel estimation problem for a channel that is observed for a time interval of length M. However the main interest of this paper lies on channel prediction. Slepian explained in [2, Section 3.1.4] that there are infinitely many ways to choose the channel coefficients $h[M], h[M+1], \ldots$ and $h[-1], h[-2], \ldots$ such that the sequence $\{h[m]\}$ is band-limited. However, there exists a unique way to extend a band-limited sequence in the sense of a minimum energy continuation, which we will use for time-variant channel prediction.

Consider a process $h[m]$ with flat Doppler spectrum in the interval $[-\nu_\mathrm{D}, \nu_\mathrm{D}]$ which is observed over the finite time interval $m \in \{0, \ldots, M-1\}$. In this case, the entries of $\boldsymbol{\Sigma_h}$ read

$$[\boldsymbol{\Sigma_h}]_{\ell,m} = \frac{1}{2\nu_\mathrm{D}} \frac{\sin(2\pi\nu_\mathrm{D}(\ell - m))}{\pi(\ell - m)} \,. \tag{18}$$

The sequences $u_i[m]$ calculated from (14) coincide with the discrete prolate spheroidal (DPS) sequences truncated to $m \in \{0, \ldots, M-1\}$ [2].

Slepian showed that the basis functions $u_i[m]$ can be extended for $m \geq M$ in the minimum-energy sense according to

$$\sum_{\ell=0}^{M-1} \frac{\sin(2\pi\nu_\mathrm{D}(\ell - m))}{2\pi\nu_\mathrm{D}(\ell - m)} u_i[\ell, \nu_\mathrm{D}] = \lambda_i(\nu_\mathrm{D}) u_i[m, \nu_\mathrm{D}] \tag{19}$$

for $i \in \{0, \ldots, M-1\}$. In the remainder we will drop the parameter ν_D and use the notation $u_i[m]$ and λ_i when ν_D is clear from the context. The eigenvalues $\lambda_i(\nu_\mathrm{D})$ are clustered near $1/(2\nu_\mathrm{D})$ for $i \leq \lceil 2\nu_\mathrm{D} M \rceil$ and decay rapidly for $i > \lceil 2\nu_\mathrm{D} M \rceil$. For practical communication systems, e.g. with block length $M = 256$ and normalized Doppler bandwidth $\nu_\mathrm{D} = 0.004$ the subspace dimension D ranges between three to five only.

It is known [16], that the sequences which span the time-concentrated and band-limited subspace of a band-limited process with arbitrary Doppler spectrum possess the double orthogonality property of the DPS sequences, too. We conjecture a minimum energy continuation of basis sequences $u_i[m]$ for $m \geq M$ for arbitrary band-limited stationary processes $\{h[m]\}$:

$$\sum_{\ell=0}^{M-1} R_h[\ell - m] u_i[\ell] = \lambda_i' u_i[m] \,, \tag{20}$$

where $u_i[m]$ for $m \in \{0, \ldots, M-1\}$ are the elements of the ith eigenvector $\boldsymbol{u}_i$ obtained in (14) with $\boldsymbol{\Sigma_h}$ given in (7).

Using (20) we can extend the sequence $u_i[m]$ for $m \geq M$ and exploit the low-rank description (9) for predicting future coefficients of the time-variant channel.

In [17, Section 12.7] a solution to the prediction problem is shown using a linear MMSE filter. We prove that minimum-energy bandlimited prediction is identical to reduced-rank maximum likelihood (ML) filtering in [18]. Reduced-rank ML filtering itself provides a close approximation of Wiener filtering.

V. Comparison with a Predictor based on Complex Exponential Basis Functions

The time-variant channel predictor described in Section IV uses time-concentrated and band-limited sequences in order to span the channel subspace. Classical channel prediction algorithms describe the channel subspace using complex exponential basis functions. We compare the performance of both algorithms by means of Monte Carlo simulations.

A. *Predictor using Complex Exponential Basis Functions*

In the method proposed in [3] the path parameters a_p and ν_p in (3) are identified in order to enable channel prediction. We review here shortly the method, so that we are able to compare it with our minimum-energy bandlimited predictor.

For a limited observation interval of length M we can rewrite (3) in vector matrix notation according to

$$\boldsymbol{h} = \begin{bmatrix} 1 & 1 & \ldots & 1 \\ w_0 & w_1 & \ldots & w_{P-1} \\ \vdots & \vdots & & \vdots \\ w_0^{M-1} & w_1^{M-1} & \ldots & w_{P-1}^{M-1} \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_{P-1} \end{bmatrix} = \boldsymbol{W}\boldsymbol{a} \,, \tag{21}$$

where $w_p = \mathrm{e}^{\mathrm{j}2\pi\nu_p}$. In [3] ESPRIT [7] is used to estimate the Doppler shift ν_p of each single propagation path. In this paper we assume that all ν_p, $p \in \{0, \ldots, P-1\}$ are known exactly.

The complex weight vector $\boldsymbol{a} = [a_1, \ldots, a_P]$ is estimated according to

$$\hat{\boldsymbol{a}} = \left(\boldsymbol{W}^\mathrm{H} \boldsymbol{W} + \sigma_z^2 \boldsymbol{I}_P \right)^{-1} \boldsymbol{W}^\mathrm{H} \boldsymbol{y} \tag{22}$$

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in [3]. Finally, the time-variant channel is predicted via

$$\hat{h}[m] = \sum_{p=0}^{P-1} \hat{a}_p e^{j2\pi\nu_p m} \tag{23}$$

for $m \in \{M, \ldots, M+N-1\}$.

B. Monte-Carlo Simulations

The actual realization of the time-variant flat-fading channel $h[m]$ is calculated according to a sum of sinusoid model following Clarkes model, see [1, Appendix] and [19]. We simulate $P = 20$ distinct paths. The covariance function of $h[m]$ is

$$R_h[\tilde{m}] = \mathrm{J}_0(2\pi\nu_\mathrm{D}\tilde{m}), \tag{24}$$

where J_0 is the zeroth order Bessel function of the first kind. All simulation results are averaged over 500 independent channel realizations.

The symbol duration $T_\mathrm{S} = (64+15)/(3.84 \cdot 10^6\,\mathrm{s}^{-1}) = 20.57\,\mu\mathrm{s}$ is chosen according to the system parameters described in [1]. The speed of the mobile user is varied in the range $0 < v < v_\mathrm{max} = 100\,\mathrm{km/h} = 27.8\,\mathrm{m/s}$, which results in a range of Doppler bandwidths $0 \leq B_\mathrm{D} \leq 180\,\mathrm{Hz}$ or normalized to the symbol duration $0 \leq \nu_\mathrm{D} \leq \nu_\mathrm{Dmax} = 3.8 \cdot 10^{-3}$. The channel is observed over $M = 256$ symbols and predicted over $N = 256$ symbols. For all simulations $\mathrm{SNR} = 10\,\mathrm{dB}$.

In Fig. 1 we plot the MSE per symbol $\mathrm{MSE}[m]$ with m ranging in the interval $m \in \{0, \ldots, M-1, M, \ldots, M+N-1\}$ for two velocities $v \in \{10, 100\}$ km/h. These velocities relate to time-bandwidth products $\nu_\mathrm{D}M \in \{0.1, 1\}$. The MSE per symbol $\mathrm{MSE}[m]$ for $m \in \{0, \ldots, M-1\}$ is the estimation MSE. For $m \in \{M, \ldots, M+N-1\}$ $\mathrm{MSE}[m]$ is the prediction MSE.

We compare three predictors. The lower bound is given by the Wiener filter [17, Section 12.7]. The minimum-energy bandlimited predictor (denoted 'time-concentr. subsp.') achieves a performance extremely close to the Wiener filter. The predictor based on complex exponentials (denoted 'compl. exp.') performs significantly worse even though the Doppler shift of each path is known perfectly.

The typical behavior depicted in Fig. 1 is also documented in [3] and [5]. With increasing $\nu_\mathrm{D}M$ the prediction error increases faster with discrete time $m \geq M$. We know from Section IV that the band-limited sequence of channel samples is extended in the sense of minimum energy. This explains why the prediction error increases with increasing time index m.

The channel predictor based on complex exponentials has perfect knowledge of the Doppler shift per path ν_p and uses a linear MMSE estimate of the path parameters a_p. It performs worse than the minimum-energy bandlimited predictor. This is not unexpected since time-limited complex exponentials with frequencies confined in an interval $[-\nu_\mathrm{D}, \nu_\mathrm{D}]$ span approximately the same subspace as the $u_i[m]$ (see also Fig. 4 in [1]). The number of complex exponential basis functions equal the number of paths P. In this example we used $P = 20$ paths. In

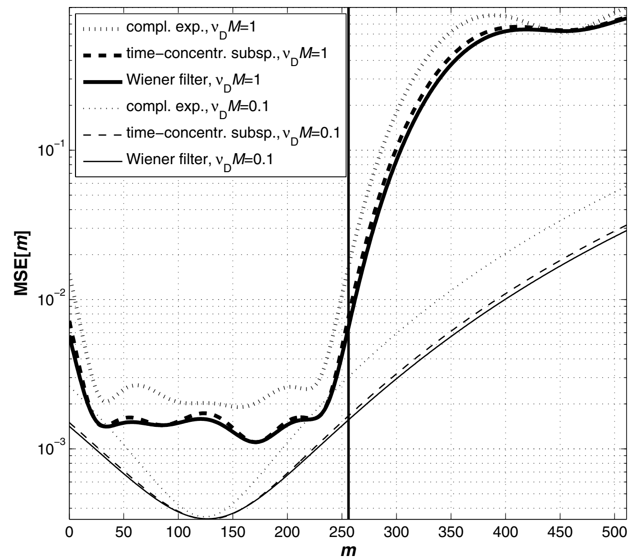


Fig. 1. Mean square error per symbol $\mathrm{MSE}[m]$ versus discrete time m. We plot the performance of three predictors: The Wiener filter (as lower bound), the minimum-energy bandlimited predictor (denoted 'time-concentr. subsp.') and a predictor based on complex exponentials (denoted 'compl. exp.'). The user moves at velocity $v \in \{10, 100\}$ km/h which corresponds to a time-bandwidth product of $\nu_\mathrm{D}M \in \{0.1, 1\}$ for an observation interval with length $M = 256$.

comparison the minimum-energy bandlimited predictor uses a subspace of dimension $D = 5$ only (for $v = 100\,\mathrm{km/h}$).

In practical situations neither the Doppler shift of each of the P paths nor the covariance function of the time-variant process $\{h[m]\}$ is known exactly. In [1] we assume in such a situation a maximum Doppler bandwidth ν_Dmax and design a DPS subspace accordingly. The performance for $\nu_\mathrm{D} < \nu_\mathrm{Dmax}$ is suboptimal but the analysis in [1] shows that the performance loss for channel *estimation* is small.

Due to the fast increase of $\mathrm{MSE}[m]$ for $m \geq M$ the usage of a single subspace is not sufficient for channel *prediction*. In the next Section we will design a small set of subspaces with fixed time-concentration in the interval $\{0, \ldots, M-1\}$ and with growing band-limitation $0 < \nu_{\mathrm{D}1} < \nu_{\mathrm{D}2} < \ldots < \nu_\mathrm{Dmax}$. We select the predefined subspaces dynamically for each data block by using a Doppler-bandwidth estimate.

VI. Dynamic Subspace Selection

We are interested in a low complexity implementation. To this end we select a set of subspaces which can be computed beforehand. The subspace dimension $D(\nu_\mathrm{Dmax})$ for the maximum Doppler bandwidth ν_Dmax is calculated from (17) to be $D(\nu_\mathrm{Dmax}) = 5$ for the simulation setting used in Section V.

We propose to define $D(\nu_\mathrm{Dmax}) = 5$ subspaces. The largest subspace will have Doppler bandwidth $\nu_{\mathrm{D}5} = \nu_\mathrm{Dmax}$ and $D(\nu_\mathrm{Dmax}) = 5$ dimension. For the subspace with index four we chose the largest Doppler bandwidth $\nu_{\mathrm{D}4}$ such, that the subspace dimension is reduced by one. Generally we can write

$$\nu_{\mathrm{D}i} = \max_{\nu_{\mathrm{D}}} D(\nu_{\mathrm{D}}) = i\,. \tag{25}$$

We follow this procedure until the subspace with index one is spanned by one sequence only. The individual subspaces have a time-bandwidth product $\nu_{\mathrm{D}}M \in \{0.01, 0.21, 0.54, 0.95, 1\}$ for the simulation setting from Section V.

In order to select the subspace with the smallest possible Doppler bandwidth we use the Doppler bandwidth estimator proposed in [20]. We reduce the noise level by projecting the channel observations on the largest subspace spanned by $u_i[m, \nu_{\mathrm{Dmax}}]$ for $i \in \{0, \ldots, D(\nu_{\mathrm{Dmax}})\}$. We define

$$\boldsymbol{f}[m, \nu_{\mathrm{D}}] = \begin{bmatrix} u_0[m, \nu_{\mathrm{D}}] \\ \vdots \\ u_{D(\nu_{\mathrm{D}})-1}[m, \nu_{\mathrm{D}}] \end{bmatrix} \in \mathbb{R}^{D(\nu_{\mathrm{D}})} \tag{26}$$

and calculate the time-variant channel estimates for the observation interval $m \in \{0, \ldots, M-1\}$ according to

$$\hat{h}[m] = \boldsymbol{f}[m, \nu_{\mathrm{Dmax}}]^{\mathrm{H}} \sum_{\ell=0}^{M-1} \boldsymbol{f}[\ell, \nu_{\mathrm{Dmax}}]^* y[\ell]\,. \tag{27}$$

Using $\hat{h}[m]$ we obtain an estimate of the Doppler bandwidth

$$\hat{\nu}_{\mathrm{D}} = \frac{1}{2\pi} \sqrt{\frac{2/(M-1)\sum_{n=0}^{M-2} |\hat{h}[m] - \hat{h}[m+1]|^2}{1/M \sum_{n=0}^{M-1} |\hat{h}[m]|^2}}\,. \tag{28}$$

A Doppler bandwidth estimate $\hat{\nu}_{\mathrm{D}}$ is calculated for each observation interval. Based on this estimate we choose the subspace with the smallest Doppler-bandwidth that contains $\hat{\nu}_{\mathrm{D}}$. The chosen subspace is used for minimum-energy bandlimited channel prediction.

We plot the performance of minimum-energy bandlimited prediction with a dynamically selected subspace in Fig. 2. The user moves with velocity $v = 100$ km/h which corresponds to a time-bandwidth product of $\nu_{\mathrm{D}}M = 1$. Due to noisy Doppler bandwidth estimates $\hat{\nu}_{\mathrm{D}}$ either the subspace with $\nu_{\mathrm{D}5}M = 1$ or $\nu_{\mathrm{D}4}M = 0.95$ is chosen. As can be seen in Fig. 2 the performance of the adaptive predictor denoted 'adapt. DPS subspace' lies in between the performance of the predictor based on a fixed DPS subspace with $\nu_{\mathrm{D}4}M = 0.89$ and $\nu_{\mathrm{D}5}M = 1$.

In Fig. 3 a similar result is shown for $v = 10$ km/h resulting in $\nu_{\mathrm{D}}M = 0.1$. In this case either the subspace with $\nu_{\mathrm{D}1}M = 0.01$ or $\nu_{\mathrm{D}2}M = 0.21$ is chosen. Again the performance curve for the adaptive-DPS-subspace-based predictor is in between those of two fixed-DPS-subspace-based predictors.

VII. Conclusion

In this paper we present a new minimum-energy bandlimited prediction method for a time-variant process with arbitrary power spectral density. The predictor is based on doubly orthogonal time-concentrated and band-limited sequences. We obtain time-concentrated and band-limited sequences for arbitrary Doppler spectra by generalizing results from Slepian [2].

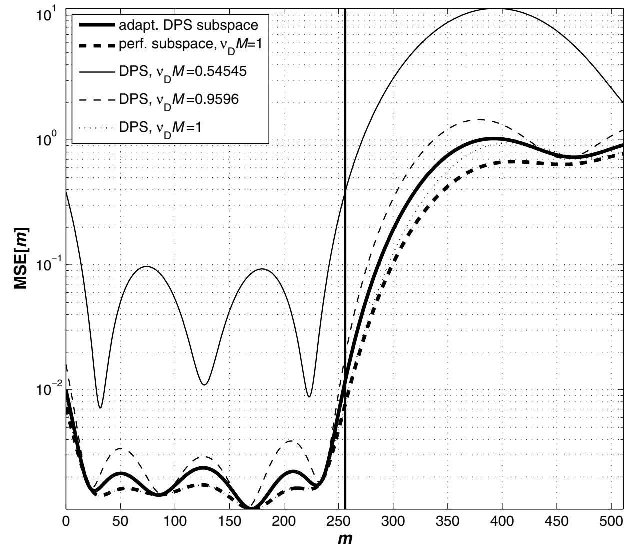
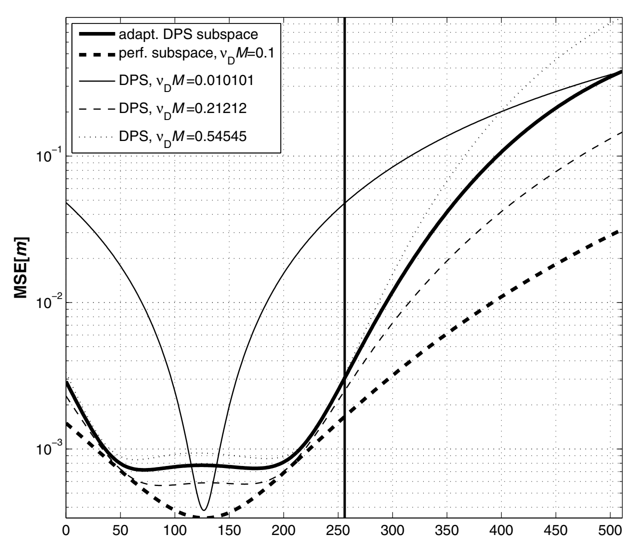


Fig. 2. Mean square error per symbol $\mathrm{MSE}[m]$ versus discrete time m for minimum-energy bandlimited prediction. We plot $\mathrm{MSE}[m]$ for the predictor using full covariance information (denoted 'perf. subspace') as lower bound and compare it with the adaptive-DPS-subspace-based prediction scheme (denoted 'adapt. DPS subspace') and with three fixed-DPS-subspace-based prediction schemes (denoted 'DPS'). The user moves at $v = 100$ km/h, which yields a time-bandwidth product $\nu_{\mathrm{D}}M = 1$.



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Accurate estimation of the channels covariance function, or equivalently its Doppler power spectral density, can be costly and we have shown that this is not required because the subspace spanned by time-limited snapshots of bandlimited sequences is of extremely low dimensionality. For current mobile communication systems, this dimension ranges between three and five only. We conclude that the predictor performance primarily depends on the Doppler bandwidth, but is almost indifferent to other features of the Doppler spectrum.

We exploit these observations in order to design a set of subspaces spanned by DPS sequences with fixed time-concentration but growing Doppler bandwidth. The appropriate DPS subspace is selected dynamically for each data block based on a Doppler bandwidth estimate. The selected DPS subspace is used for minimum-energy bandlimited prediction.

For typical packet lengths and number of propagation paths minimum-energy bandlimited prediction performs close to an estimator based on complex exponentials. However a Doppler bandwidth estimate is needed only enabling low complexity time-variant channel prediction.

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