TIME-VARIANT CHANNEL PREDICTION USING TIME-CONCENTRATED AND BANDLIMITED SEQUENCES - ANALYTIC RESULTS

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Abstract We use time-concentrated and bandlimited sequences for minimum-energy prediction of time-variant flat-fading channels. The time-concentration of these sequences is matched to the length of the observation interval and the band-limitation is determined by the Doppler bandwidth of the time-selective fading process. We prove in this paper that minimum-energy bandlimited prediction is identical to reduced-rank maximum likelihood filtering (which is a close approximation of Wiener filtering). In wireless communication systems the time-selective fading process is highly over-sampled. Thus the essential dimension of the subspace spanned by time-concentrated and bandlimited sequences is small. The prediction error mainly depends on the Doppler bandwidth, while the actual shape of the Doppler spectrum is of minor importance. Analytic performance results prove this fact for a practical wireless communication system.

1. Introduction

The Doppler bandwidth in wireless communication systems is much smaller than the actual channel bandwidth. Thus time-selective fading processes are highly oversampled. We exploit this basic fact for minimum-energy bandlimited prediction of a time-variant flat-fading channel by using a low dimensional subspace spanned by time-concentrated and bandlimited sequences [1]. The time-concentration of these sequences is matched to the length of the observation interval and the band-limitation is chosen according to the Doppler bandwidth of the time-selective fading process. In this paper we present analytic performance results and highlight the tight interconnection of minimum-energy bandlimited prediction to linear minimum mean square error (MMSE) channel prediction [2, Section 12.7].

Prediction of continuous-time channels is treated in [3–5]. In this paper we deal with sampled discrete-time channels which are important for practical implementations using digital signal processing techniques.

We model the time-variant flat-fading channel as a superposition of P individual propagation paths. Each path exhibits a distinct complex weigth and Doppler shift.

The normalized Doppler bandwidth $\nu_{\mathrm{D}} \ll 1/2$ because the Doppler bandwidth is much smaller than the overall channel bandwidth. Therefor the time-variant channel process can be described using a subspace with small dimension [6]. In other words, the detailed shape of the Doppler spectrum is of minor importance. Only the Doppler bandwidth ν_{D} and the observation interval length M are the prime parameters that define the subspace dimension

$$D' = \lceil 2\nu_{\mathrm{D}} M \rceil + 1. \tag{1}$$

This result is utilized for time-variant channel *estimation* in [7]. In the present paper we extend the concept to time-variant channel *prediction*.

Contributions

- We present analytic performance results for minimum-energy bandlimited prediction of time-variant flat-fading channels.
- We show that minimum-energy bandlimited prediction is equivalent to reduced-rank maximum likelihood (ML) filtering. Reduced-rank ML filtering itself is a close approximation of linear MMSE (Wiener) filtering. This result allows to clarify the strictly limited prediction horizon of linear prediction techniques.

Notation

We denote a column vector by $\boldsymbol{a}$ and its i-th element with $a[i]$. Equivalently, we denote a matrix by $\boldsymbol{A}$ and its (i,ℓ)-th element by $[\boldsymbol{A}]_{i,\ell}$. The transpose of $\boldsymbol{A}$ is given by $\boldsymbol{A}^{\mathrm{T}}$, its conjugate transpose by $\boldsymbol{A}^{\mathrm{H}}$ and its upper left part with dimension $P \times Q$ by $\boldsymbol{A}_{P\times Q}$. A diagonal matrix with elements $a[i]$ is written as $\mathrm{diag}(\boldsymbol{a})$ and the $Q \times Q$ identity matrix as $\boldsymbol{I}_Q$. The absolute value of a is denoted by $|a|$ and its complex conjugate by a^*. The largest (smallest) integer, lower (greater) or equal than $a \in \mathbb{R}$ is denoted by $\lfloor a \rfloor$ ($\lceil a \rceil$).

Organization of the Paper

The signal and the channel model are described in Section 2 and 3 respectively. Minimum-energy bandlimited prediction is reviewed in Section 4. We present analytic performance results in Section 5. In Section 6 we show that minimum-energy bandlimited prediction is identical to reduced-rank ML prediction. We compare the minimum-energy predictor with a Wiener predictor in Section 7 and conclude in Section 8.

2. Signal Model for Time-Variant Flat-Fading Channels

We consider a time division duplex (TDD) communication system transmitting data in blocks of length M over a time-variant flat-fading channel. The symbol duration is much longer than the delay spread of the channel, i.e. $T_{\mathrm{S}} \gg T_{\mathrm{D}}$. Discrete time at rate $R_{\mathrm{S}} = 1/T_{\mathrm{S}}$ is denoted by m. The channel incorporates the transmit filter, the transmit antenna, the physical channel, the receive antenna, and the receive matched filter. The data symbols $b[m]$ are drawn from a symbol alphabet with constant modulus, $|b[m]| = 1$. The discrete-time signal at the matched filter output

$$y'[m] = h[m]b[m] + z'[m] \tag{2}$$

is given by the data symbol multiplied with the sampled time-variant channel $h[m]$ plus additive complex white Gaussian noise $z'[m]$ with zero mean and variance $\sigma_{z'}^2$.

We assume an error free decision feedback structure [8–10], thus we are able to obtain noisy channel observations

$$y[m] = y'[m]\tilde{b}[m]^{\mathrm{H}} = h[m] + z'[m]\tilde{b}[m]^{\mathrm{H}} = h[m] + z[m] \tag{3}$$

using the data symbol estimates $\tilde{b}[m]$, where $z[m]$ has the same statistical properties than $z'[m]$.

We assume Rayleigh fading and power control. Without loss of generality $\{h[m]\}$ is a zero-mean, unit variance wide-sense stationary process with correlation function $R_h[k] = \mathbb{E}\{h^*[m]h[m+k]\}$. The signal-to-noise ratio (SNR) is defined as $\mathrm{SNR} = 1/\sigma_z^2$.

The noisy channel observations $y[m]$, $m \in \{0,\ldots,M-1\}$ shall be used to predict the time-variant flat-fading channel N symbols into the future for $m \in \{M,\ldots,M+N-1\}$.

3. Physical Wave Propagation Channel Model

We model the fading process using physical wave propagation principles [11]. The impinging wave-fronts at the receive antenna originate from P scatterers. The individual paths sum up as

$$h[m] = \sum_{p=0}^{P-1} a_p \mathrm{e}^{\mathrm{j}2\pi f_p T_{\mathrm{S}} m} = \sum_{p=0}^{P-1} a_p \mathrm{e}^{\mathrm{j}2\pi \nu_p m}\,. \tag{4}$$

Here f_p is the Doppler shift of path p. For easier notation we define the normalized Doppler frequency as $\nu_p = f_p T_{\mathrm{S}}$. The gain and phase shift of path p are embodied in the complex weight $a_p \in \mathbb{C}$. The one-sided normalized Doppler bandwidth is given by

$$\nu_{\mathrm{D}} = \frac{v f_{\mathrm{C}}}{c_0} T_{\mathrm{S}} \geq |\nu_p|\,, \tag{5}$$

where v denotes the user's velocity, f_{C} is the carrier frequency and c_0 stands for the speed of light. We model the random parameter sets a_p and ν_p for $p \in \{0,\ldots,P-1\}$ as independent. The random variables in each set are independent and identically distributed. For $P \geq 20$ the Rayleigh fading assumption for $h[m]$ is realistic due to the central limit theorem.

We assume a time-variant block-fading channel model. In this model the random path parameters a_p and ν_p are assumed to be constant over $M+N$ symbols. The fading process is wide-sense stationary over the duration of a single data block. However from block to block the fading is non-stationary [12].

4. Minimum-Energy Bandlimited Prediction

The channel coefficients for a single block of length M are collected in the vector $\boldsymbol{h} = [h[0], h[1], \ldots, h[M-1]]^{\mathrm{T}}$ and the covariance matrix is defined as $\boldsymbol{\Sigma}_{\boldsymbol{h}} = \mathbb{E}\{\boldsymbol{h}\boldsymbol{h}^{\mathrm{H}}\}$ with elements $[\boldsymbol{\Sigma}_{\boldsymbol{h}}]_{\ell,m} = R_h[\ell - m]$. The noisy observation vector $\boldsymbol{y} = [y[0], y[1], \ldots, y[M-1]]^{\mathrm{T}}$ is used for channel prediction. Its covariance matrix reads

$$\boldsymbol{\Sigma}_{\boldsymbol{y}} = \boldsymbol{\Sigma}_{\boldsymbol{h}} + 1/\mathrm{SNR}\,\boldsymbol{I}_M\,. \tag{6}$$

We consider a subspace-based approximation which expands the vector $\boldsymbol{h}$ in terms of D orthogonal basis functions $u_i[m]$, $i \in \{0, \ldots, D-1\}$ represented in vector notation $\boldsymbol{u}_i = [u_i[0], u_i[1], \ldots, u_i[M-1]]^{\mathrm{T}}$:

$$\boldsymbol{h} \approx \hat{\boldsymbol{h}} = \boldsymbol{U}_D \hat{\boldsymbol{\gamma}}\,, \tag{7}$$

where $\boldsymbol{U}_D = [\boldsymbol{u}_0, \ldots, \boldsymbol{u}_{D-1}]$ and $\hat{\boldsymbol{\gamma}} = [\hat{\gamma}_0, \ldots, \hat{\gamma}_{D-1}]^{\mathrm{T}}$ contains the basis expansion coefficient estimates. The least square estimate for the basis expansion coefficient vector $\hat{\boldsymbol{\gamma}}$ simplifies to

$$\hat{\boldsymbol{\gamma}} = \boldsymbol{U}_D^{\mathrm{H}} \boldsymbol{y} \tag{8}$$

due to the orthogonality of the basis functions.

The mean square error (MSE) per symbol is defined as

$$\mathrm{MSE}[m] = \mathbb{E}\left\{\left|h[m] - \hat{h}[m]\right|^2\right\} \tag{9}$$

and the MSE per observation interval reads [13]

$$\mathrm{MSE} = \frac{1}{M}\sum_{m=0}^{M-1} \mathrm{MSE}[m] = \frac{1}{M}\mathbb{E}\{|\boldsymbol{h} - \hat{\boldsymbol{h}}|^2\}\,. \tag{10}$$

We want to use basis functions $u_i[m]$, $m \in \{0, \ldots, M-1\}$ that minimize the MSE per observation interval for large P. Slepian showed [6] that discrete prolate spheroidal (DPS) sequences time limited to $m \in \{0, \ldots, M-1\}$ are such basis functions for a fading process with flat Doppler spectrum [7]. The DPS sequences are defined as

$$\sum_{\ell=0}^{M-1} \frac{\sin(2\pi\nu_{\mathrm{D}}(\ell - m))}{2\pi\nu_{\mathrm{D}}(\ell - m)} u_i[\ell] = \lambda_i u_i[m]\,. \tag{11}$$

DPS sequences are bandlimited to the frequency range $[-\nu_{\mathrm{D}}, \nu_{\mathrm{D}}]$ and simultaneously most concentrated in a time interval of length M. The time-concentrated and bandlimited sequences $\{u_i[m]\}$ are both orthogonal on the infinite set of integers $\mathbb{Z}$

$$\sum_{m \in \mathbb{Z}} u_i^*[m] u_\ell[m] = 0 \quad i \neq \ell \tag{12}$$

and on the finite set $\{0, \ldots, M-1\}$

$$\boldsymbol{u}_i^{\mathrm{H}} \boldsymbol{u}_\ell = 0 \quad i \neq \ell\,. \tag{13}$$

The eigenvalues λ_i decay exponentially for $i > \lceil 2\nu_{\mathrm{D}} M \rceil$.

The conjecture that (11) can be generalized to bandlimited processes with arbitrary power spectral density

$$S_h(\nu) = \sum_{k=-\infty}^{\infty} R_h(k) \mathrm{e}^{-\mathrm{j}2\pi k\nu} \tag{14}$$

is established in [1], see also [14, 15]. Bandlimitation is expressed as $S_h(\nu) \equiv 0$ for $\nu_{\mathrm{D}} < |\nu| < 1/2$. Furthermore we require $S_h(\nu)$ to be bounded away from zero, $S_h(\nu) > 0$, for $|\nu| \leq \nu_{\mathrm{D}}$, which ensures full rank of $\boldsymbol{\Sigma}_{\boldsymbol{h}}$.

The generalized time-concentrated and bandlimited sequences $\{u_i[m]\}$ are defined through the eigenvalue equation

$$\sum_{\ell=0}^{M-1} R_h[\ell - m] u_i[\ell] = \lambda_i u_i[m]\,, \tag{15}$$

for $m \in \{-\infty, \ldots, \infty\} = \mathbb{Z}$ and $i \in \{0, \ldots, M-1\}$. From [16,17] we can assume that the double orthogonality of these sequences is maintained for any bandlimited power spectral density $S_h(\nu)$. Similar conclusions can be drawn for the exponential decay of the eigenvalues based on [18, 19].

So far we treated the channel estimation problem for a channel that is observed for a time interval with length M. We used orthogonal basis functions that result from time limiting a infinite sequence to the interval $m \in \{0, \ldots, M-1\}$. The sequences are energy concentrated in this interval.

However the main interest of this paper lies on channel prediction. Slepian explained in [6, Section 3.1.4] that there are infinitely many ways, the channel samples $h[m]$, $m \in \mathbb{Z} \setminus \{0, \ldots, M-1\}$ can be chosen such, that the infinite sequence $\{h[m]\}_{m\in\mathbb{Z}}$ is bandlimited. However, there exists a unique way to extend a bandlimited sequence in the sense of a minimum-energy continuation. This is achieved by using the time-concentrated and bandlimited sequences $\{u_i[m]\}$.

The double orthogonality property of $\{u_i[m]\}$ enables a way for evaluating (15): The vectors $\boldsymbol{u}_i$ satisfy the Karhunen-Loève identity [20], i.e. the eigenvalue decomposition

$$\boldsymbol{\Sigma}_{\boldsymbol{h}} = \boldsymbol{U}\boldsymbol{\Lambda}\boldsymbol{U}^{\mathrm{H}}, \tag{16}$$

where $\boldsymbol{U} = \boldsymbol{U}_M$ and $\boldsymbol{\Lambda} = \boldsymbol{\Lambda}_M = \mathrm{diag}(\lambda_0, \ldots, \lambda_{M-1})$. Knowing $\boldsymbol{u}_i$ we can obtain the sequence $\{u_i[m]\}$ for $m \geq M$ and $m < 0$ in the minimum energy sense by evaluating (15).

Finally, we can express the minimum-energy bandlimited prediction of a time-variant channel for $m \in \mathbb{Z}$ as

$$\tilde{h}[m] = \boldsymbol{f}[m]^{\mathrm{T}} \boldsymbol{U}_D^{\mathrm{H}} \boldsymbol{y} = \boldsymbol{f}[m]^{\mathrm{T}} \hat{\boldsymbol{\gamma}}, \tag{17}$$

where

$$\boldsymbol{f}[m] = \begin{bmatrix} u_0[m] \\ \vdots \\ u_{D-1}[m] \end{bmatrix}. \tag{18}$$

The subspace dimension minimizing the MSE per observation interval for a given SNR is found to be [21]

$$D = \underset{D' \in \{1,\ldots,Q\}}{\mathrm{argmin}} \left(\frac{1}{M} \sum_{i=D'}^{Q-1} \lambda_i + \frac{D'}{M} \frac{1}{\mathrm{SNR}} \right), \tag{19}$$

where $Q = \mathrm{rank}(\boldsymbol{\Sigma}_{\boldsymbol{h}}) = \min(M, P)$.

In [22] a similar method is used for the prediction of *time concentrated* signals. However the signals of interest in this paper are wide-sense stationary. Therefore they can *not* be time concentrated.

In the next section we present analytic performance results for minimum-energy bandlimited time-variant channel prediction.

5. Analytical Derivation of the Prediction Error

The MSE per symbol (9) of the minimum-energy bandlimited predictor can be described as the sum of a square bias and a variance term

$$\mathrm{MSE}[m] = \mathrm{bias}^2[m] + \mathrm{var}[m]. \tag{20}$$

The expression for square bias and variance developed in [13, Sec. 6] and [7] can be extended for minimum-energy bandlimited prediction. All equations in this section are valid for $m \in \mathbb{Z}$.

We define the instantaneous frequency response of the minimum energy predictor according to

$$H[m,\nu] = \boldsymbol{f}^T[m] \sum_{\ell=0}^{M-1} \boldsymbol{f}^*[\ell] \mathrm{e}^{-\mathrm{j}2\pi\nu(m-\ell)}, \tag{21}$$

where $|\nu| < 1/2$. The instantaneous error characteristic of the basis expansion is defined as [13, Sec. 6.1.4]

$$E[m,\nu] = |1 - H[m,\nu]|^2. \tag{22}$$

The square bias per symbol $\mathrm{bias}^2[m]$ of the minimum-energy bandlimited predictor can be expressed as the integral over the instantaneous error characteristic $E[m,\nu]$ multiplied by the power spectral density of $h[m]$,

$$\mathrm{bias}^2[m] = \int_{-\frac{1}{2}}^{\frac{1}{2}} E[m,\nu] S_h(\nu) \mathrm{d}\nu\,. \tag{23}$$

The minimum-energy bandlimited prediction variance can be expressed as

$$\mathrm{var}[m] \approx \sigma_z^2 \boldsymbol{f}^H[m]\boldsymbol{f}[m]. \tag{24}$$

Note that the equations in this section stay valid if the sequences $\{u_i[m]\}$ are not matched to the power spectral density $S_h(\nu)$. Thus equations (20)-(24) enable the evaluation of the prediction error for the mismatched case too.

6. Relation to the Wiener Filter

In [2, Section 12.7] a solution to the prediction problem is shown using a linear MMSE (Wiener) filter. This filter can be closely approximated by a reduced-rank maximum likelihood (ML) filter according to [23]. We show now that a reduced-rank ML predictor coincides with a minimum-energy bandlimited predictor. A first result in the same direction but for the noiseless case and for flat Doppler spectrum only is shown in [24].

With the definition

$$\boldsymbol{r}_h[m] = [R_h[m], R_h[m-1], \ldots, R_h[m-(M-1)]]^{\mathrm{T}} \tag{25}$$

for $m > M-1$ the $\ell = m-(M-1)$ step Wiener predictor is of the form

$$\tilde{h}[m] = \boldsymbol{r}_h[m]^{\mathrm{H}} \boldsymbol{\Sigma}_{\boldsymbol{y}}^{-1} \boldsymbol{y}. \tag{26}$$

The minimum MSE per symbol is given by

$$\mathrm{MSE}[m] = 1 - \boldsymbol{r}_h[m]^{\mathrm{H}} \boldsymbol{\Sigma}_{\boldsymbol{y}}^{-1} \boldsymbol{r}_h[m]\,. \tag{27}$$

We can closely approximate (26) with a reduced-rank ML filter using a subspace of $\boldsymbol{\Sigma_h}$ (see (16)) with an optimum subspace dimension D:

$$\hat{h}[m] = \boldsymbol{r}_h[m]^{\mathrm{H}} \boldsymbol{U}_D \boldsymbol{\Lambda}_D^{-1} \boldsymbol{U}_D^{\mathrm{H}} \boldsymbol{y}\,. \tag{28}$$

The optimum dimension D is chosen according to (19) so that the noise variance is taken into account [23].

In order to show the similarity between a reduced-rank ML filter (28) and minimum-energy bandlimited prediction (17) we first write (15) as

$$u_i[m] = \lambda_i^{-1} \sum_{\ell=0}^{M-1} R_h[m-\ell]^* u_i[\ell] = {\lambda_i}^{-1} \boldsymbol{r}_h[m]^{\mathrm{H}} \boldsymbol{u}_i\,, \tag{29}$$

where we use the fact that $R_h[k] = R_h[-k]^*$. Secondly, inserting (29) in (18) yields

$$\boldsymbol{f}[m]^{\mathrm{T}} = \boldsymbol{r}_h[m]^{\mathrm{H}} \boldsymbol{U}_D \boldsymbol{\Lambda}_D^{-1} \tag{30}$$

for $m \geq M$. Inserting the expression for $\boldsymbol{f}[m]$ in (30) and $\hat{\boldsymbol{\gamma}}$ in (8) into the minimum-energy bandlimited prediction equation (17) we obtain

$$\hat{h}[m] = \boldsymbol{f}[m]^{\mathrm{T}} \hat{\boldsymbol{\gamma}} = \boldsymbol{r}_h[m]^{\mathrm{H}} \boldsymbol{U}_D \boldsymbol{\Lambda}_D^{-1} \boldsymbol{U}_D^{\mathrm{H}} \boldsymbol{y}\,, \tag{31}$$

which is identical to (28). Thus the reduced-rank ML predictor coincides with the minimum-energy bandlimited predictor which uses a subspace spanned by time-concentrated and bandlimited sequences.

We can conclude that the general subspace for the prediction problem is spanned by time-concentrated bandlimited sequences. Thus the prediction horizon of linear prediction methods is inherently limited. Furthermore we can draw the conclusion that minimizing the MSE of the predicted sequence is equivalent to minimizing its energy.

7. Performance Comparison

The minimum-energy bandlimited predictor (17) allows to reduce complexity with respect to the linear MMSE filter (26). Yet second order statistics must be known in detail. This means the exact shape of the power spectral density must be known, opposed to knowing the Doppler bandwidth only. In wireless communication systems detailed second order statistics are difficult to acquire due to the short time-interval over which the channel can be assumed to be stationary (in the wide sense) [12]. However, the small dimension of the time-concentrated and bandlimited channel subspace allows to pursuit a new approach. In [7] a subspace with predefined time-concentration and band-limitation was used for time-variant channel *estimation*. Such a subspace achieves consistent performance for a large velocity range. A similar approach is not sufficient for the *prediction* problem due to the fact that with increasing prediction horizon the mean square prediction error increases strongly [1]. In this section we provide analytic performance results for minimum-energy bandlimited prediction using different amounts of information about the time-variant flat-fading channel.

7.1 Channel Model and System Assumption

The time-variant flat-fading channel $h[m]$ is assumed to conform to Clark's model. The correlation function of $h[m]$ is

$$R_h[k] = \mathrm{J}_0(2\pi\nu_{\mathrm{D}}k)\,, \tag{32}$$

where $\mathrm{J}_0(\cdot)$ is the zeroth order Bessel function of the first kind. The correlation function defined in (32) corresponds to Clark's spectrum

$$S_h(\nu) = \frac{1}{\pi\nu_{\mathrm{D}}\sqrt{1-\left(\frac{\nu}{\nu_{\mathrm{D}}}\right)^2}} \qquad \text{for } |\nu| < \nu_{\mathrm{D}}\,, \tag{33}$$

and $S_h(\nu) = 0$ elsewhere.

The symbol duration $T_{\mathrm{S}} = 20.57\,\mu\mathrm{s}$ is chosen according to the system parameters considered in [7]. The speed of the receiver varies in the range $0 < v < v_{\max} = 100\,\mathrm{km/h} = 27.8\,\mathrm{m/s}$. This results in a Doppler bandwidth range $0 \leq B_{\mathrm{D}} \leq 180\,\mathrm{Hz}$. The normalized Doppler bandwidth is in the range $0 \leq \nu_{\mathrm{D}} \leq \nu_{\mathrm{Dmax}} = 3.8 \cdot 10^{-3}$. The channel is observed over $M = 256$ symbols. We are interested in the prediction error at a prediction horizon $m - M + 1 \in \{32, 64\}$ symbols. At speed $v_{\max}$ the prediction horizon $\{32, 64\}$ relates to a distance of $\{\lambda/8, \lambda/4\}$. For all simulations the SNR is $10\,\mathrm{dB}$.

7.2 Simulation Results

In Figure 1 and Figure 2 we use the MSE of the linear MMSE (Wiener) predictor as lower bound and plot $\mathrm{MSE}[m, \nu_{\mathrm{D}}]$ (27) for $m - M + 1 \in \{32, 64\}$. Secondly we plot the MSE of the minimum-energy predictor (20) calculating the integral (23) numerically (denoted 'Clark subsp.'). This result coincides with a reduced-rank ML predictor as explain in Section 6. The steps in the curve result from increasing the dimension D with increasing Doppler bandwidth, see (1).

Thirdly we assume no knowledge about the detailed shape of the Doppler spectrum and assume exact knowledge of the Doppler bandwidth only. This assumption leads to a subspace spanned by DPS sequences, the resulting MSE is labeled 'DPS subsp.' [1]. The dimension switching points for this case are slightly suboptimal.

We can see that the MSE for the minimum-energy predictor ('Clark subsp.') is larger than that achieved with the Wiener filter. If we assume knowledge of the Doppler bandwidth only the MSE is increased again ('DPS subsp.'). However the MSE changes by several orders of magnitude with increasing Doppler bandwidth. Thus for a practical implementation the minimum-energy predictor based on DPS sequences achieves close to optimum results while needing a Doppler-bandwidth estimate only.

8. Conclusion

In this paper analytic performance results for minimum-energy bandlimited prediction have been presented. We showed that minimum-energy bandlimited prediction is identical to reduced-rank ML filtering, which is a close approximation of a Wiener filter. This equivalence allows the conclusion that the subspace underlying any linear prediction problem is energy-concentrated. This fact inherently limits the prediction horizon of linear prediction methods. Furthermore we can conclude that minimizing the MSE of the predicted sequence is equivalent to minimizing the energy of the predicted sequence.

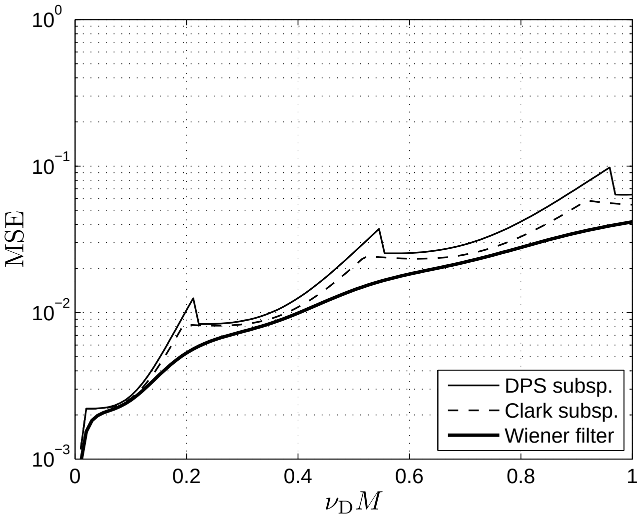

Figure 1: Mean square prediction error $\mathrm{MSE}[m,\nu_\mathrm{D}]$ at prediction horizon $m-M+1=32$ for a receiver moving with $v=0\ldots27.8\,\mathrm{m/s}$ corresponding to a normalized Doppler bandwidth $\nu_\mathrm{D}=0\ldots3.8\cdot10^{-3}$. The SNR equals 10 dB. We compare a linear MMSE predictor (denoted 'Wiener') with the minimum-energy bandlimited predictor using the exact subspace (denoted 'Clark subsp.') and an approximation thereof spanned by DPS sequences (denoted 'DPS subsp').

We provided a performance analysis for minimum-energy bandlimited prediction using either full information about the channel covariance function or the Doppler bandwidth only. Numeric evaluation of the prediction error in the mismatched case show that knowledge of the detailed power spectral density is not crucial. Only the Doppler bandwidth of the fading process has a strong impact on the prediction performance. Thus, for a practical implementation a Doppler bandwidth estimate is needed only and the minimum-energy predictor using DPS sequences achieves close to optimum performance.

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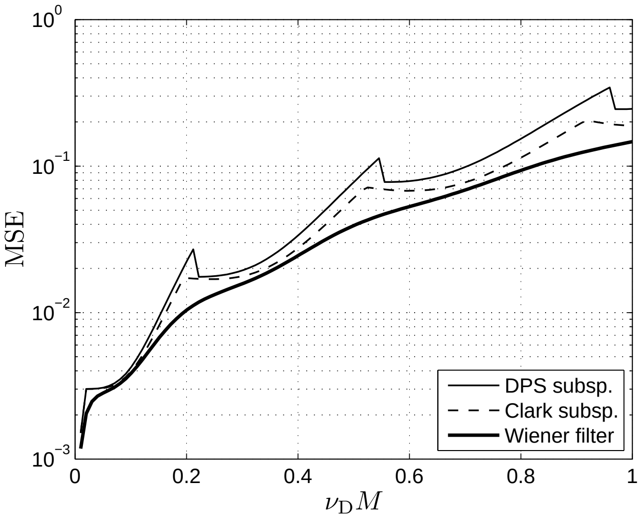

Figure 2: Mean square prediction error $\mathrm{MSE}[m, \nu_{\mathrm{D}}]$ at prediction horizon $m - M + 1 = 64$ for a receiver moving with $v = 0 \ldots 27.8\,\mathrm{m/s}$ corresponding to a normalized Doppler bandwidth $\nu_{\mathrm{D}} = 0 \ldots 3.8 \cdot 10^{-3}$. The SNR equals 10 dB. We compare a linear MMSE predictor (denoted 'Wiener') with the minimum-energy bandlimited predictor using the exact subspace (denoted 'Clark subsp.') and an approximation thereof spanned by DPS sequences (denoted 'DPS subsp').

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