Iterative Detection and Channel Estimation for MC-CDMA

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Abstract-Multi-carrier code division multiple access (MC-CDMA) systems are under intense investigation for high bit rate wireless communication systems. Their equalization is based on the fast Fourier transform, allowing for an efficient implementation. Iterative receivers with joint detection and decoding have been shown to achieve very good performance for direct sequence (DS)-CDMA systems. We apply this concept to MC-CDMA, the multiuser detector is implemented as parallel interference canceller with post-minimum mean squared error filtering. In this contribution a new pilot based channel estimation scheme based on random time domain sequences is developed. The presented simulation results for a multi path scenario show, that in a fully loaded system with 64 users the single user bound can be approached up to 1 dB. A bit error rate (BER) of 10^{-3} is reached already at an E_b/N_0 of 12 dB with a 4 state, rate 1/2 convolutional code.

I. INTRODUCTION

In this work we present an iterative receiver for the uplink of a multi-carrier code division multiple access (MC-CDMA) system. MC-CDMA is based on orthogonal frequency division multiplexing (OFDM) [1], the spreading codes to distinguish each user are applied in the frequency domain; the chips are therefore transmitted over different subcarriers.

Iterative receivers with joint detection and decoding achieve very good performance in DS-CDMA systems [2]. The strong similarities between DS-CDMA and MC-CDMA motivates the adoption of this iterative approach. In MC-CDMA inter symbol interference (ISI) is handled by inserting a cyclic prefix (CP), the equalization of each user's channel is done in the frequency domain. These properties allow one to reduce the computational complexity of the iterative receiver, and therefore enable the use of MC-CDMA for high bit rate transmission systems.

Channel estimation is a crucial part of the receiver structure as we will see later. We present a new pilot based channel estimation scheme for a MC-CDMA system with K users and spreading factor N, which can be also used for overloaded systems (K/N > 1) and achieves performance close to the single user bound with a low percentage of pilot symbols.

We present the signal model for the multi-user scenario in section II. The parallel interference cancellation (PIC) and

minimum mean squared error (MMSE) filter for multiuser detection is developed in section III and in section IV the decoding stage is presented. We develop a novel pilot based channel estimation scheme for the iterative MC-CDMA receiver in section V. Finally we give the simulation results in section VI and summarize this work with concluding remarks in section VII.

II. SIGNAL MODEL

The block structure of the individual MC-CDMA transmitters is shown in Fig. 1. Each user transmits quadrature phase shift keying (QPSK) modulated symbols $b_k(m)$ in blocks of length M. Each symbol is spread by a random spreading sequence s_k of length N, and each chip is transmitted over an individual subcarrier. The number of subcarriers is equal to the length of the spreading sequence. The elements of the spreading sequence are randomly chosen from the QPSK constellation set $(\{\pm 1 \pm j\})/(\sqrt{2N})$ satisfying

$$\sum_{n=0}^{N-1} |s_k(n)|^2 = 1 \ \forall k.$$

The first J QPSK symbols in each block are used as pilot symbols. The remaining M - J data symbols result from the convolutionally encoded, randomly interleaved and QPSK modulated binary information sequence γ_k of length 2(M - J)R by applying Gray labelling. The code rate is denoted by R.

We assume the channel to remain constant over M symbols i.e. block fading characteristic. The multipath fading channel $h_k(n)$ has a delay spread of L chips. The resulting intersymbol interference (ISI) is handled by insertion of a cyclic prefix (CP) after the inverse fast fourier transformation (IFFT) as in conventional OFDM [3]. The length of an OFDM symbol in chip time after insertion of the cyclic prefix with length G is denoted by P = N + G, and must satisfy

$$G = P - N \ge L. \tag{1}$$

The $N \times 1$ vector resulting from the spreading operation $s_k b_k(m) a_k$, is mapped into a $P \times 1$ vector

$$\boldsymbol{u}_k(m) := [u(mP), u(mP+1), \dots, u(mP+P-1)]^T$$

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Fig. 1. Model for the MC-CDMA transmitter.

according to

$$\boldsymbol{u}_k(m) = \boldsymbol{T}_{cp} \boldsymbol{F}^{\mathcal{H}} \boldsymbol{s}_k b_k(m) a_k$$

The amplitude of user k is denoted by a_k . Here all a_k are equal, which is the worst case for an interference cancellation scheme. Matrix F is the unitary $N \times N$ Fourier matrix with elements

$$F_{i,k} = \frac{1}{\sqrt{N}} e^{\frac{-j2\pi ik}{N}}, \quad i,k = 0...N-1,$$

 $\boldsymbol{F}^{\mathcal{H}}$ is the inverse Fourier matrix, $(\cdot)^{\mathcal{H}}$ denotes the Hermitian transpose. The cyclic prefix operation is performed by $\boldsymbol{T}_{cp} = [\boldsymbol{I}_{cp}^T, \boldsymbol{I}_N^T]^T$. It replicates the last *G* chips of each OFDM symbol to the front. \boldsymbol{I}_{cp} denotes the last *G* rows of the $N \times N$ identity matrix \boldsymbol{I}_N [4].

The resulting signal at the receiver input from user k without noise is

$$x_k(n) = u_k(n) \star h_k(n), \tag{2}$$

where $h_k(n)$ represents the combined effect of the channel, the transmit, and the receive filter and \star denotes the convolution operator. The complete received signal in the presence of additive white complex Gaussian noise (AWGN) v(n) with variance σ_n^2 can be written as

$$r(n) = \sum_{k=1}^{K} x_k(n) + v(n).$$

Following the lines of [4], [5] we convert the serial representation of (2) to a more convenient matrix-vector form. We define the $P \times 1$ vector

$$\boldsymbol{x}_k(m) := [x_k(mP), x_k(mP+1), \dots, x_k(mP+P-1)]^T$$

and equivalently v(m). Let $H_{k,0}$ be the $P \times P$ lower triangular Toeplitz channel matrix with first column $[h_k(0), \ldots, h_k(L-1), 0, \ldots, 0]^T$ and first row $[h_k(0), 0, \ldots, 0]^T$ and let $H_{k,1}$ be the $P \times P$ upper triangular Toeplitz channel matrix with first column $[0, \ldots, 0]^T$ and first row $[0, \ldots, 0, h_k(L-1), \ldots, h_k(0)]$. We can write (2) as

$$\boldsymbol{x}_k(m) = \boldsymbol{H}_{k,0}\boldsymbol{u}_k(m) + \boldsymbol{H}_{k,1}\boldsymbol{u}_k(m-1) + \boldsymbol{v}(m)$$

where the second term represents the ISI between two consecutive OFDM symbols.

At the receiver the CP of length G is removed, and an FFT is performed on the remaining $N \times 1$ vector. This can

be represented by the matrix $\mathbf{R}_{cp} := [\mathbf{0}_{N \times G}, \mathbf{I}_N]$ which removes the first G entries of a $P \times 1$ vector when the product $\mathbf{R}_{cp} \mathbf{x}_k(m)$ is formed. As long as (1) holds,

$$\boldsymbol{R}_{cp}\boldsymbol{H}_{k,1}=\boldsymbol{0},$$

which indicates that the ISI between two OFDM symbols is eliminated.

The received signal for user k can finally be written as:

$$y_{k}(m) = FR_{cp}x_{k}(m) = FR_{cp}H_{k,0}u_{k}(m) + FR_{cp}v(m)$$

$$= FR_{cp}H_{k,0}T_{cp}F^{\mathcal{H}}s_{k}b_{k}(m) + FR_{cp}v(m)$$

$$= F\bar{H}_{k}F^{\mathcal{H}}s_{k}b_{k}(m) + FR_{cp}v(m)$$
(3)

where \bar{H}_k is the overall circulant channel matrix that can be decomposed as

$$\bar{\boldsymbol{H}}_k = \boldsymbol{F}^{\mathcal{H}} \operatorname{diag}(\boldsymbol{g}_k) \boldsymbol{F}$$

where $\boldsymbol{g}_k = \sqrt{N} \boldsymbol{F} \boldsymbol{h}_k$. Therefore we can write (3) as

$$\boldsymbol{y}_k = \operatorname{diag}(\boldsymbol{g}_k)\boldsymbol{s}_k b_k(m) + \bar{\boldsymbol{v}}(m)$$

where we model $\bar{\boldsymbol{v}}(m) = \boldsymbol{F} \boldsymbol{R}_{cp} \boldsymbol{v}(m)$ as white again. We define the effective spreading sequence

$$\tilde{\boldsymbol{s}}_k = \operatorname{diag}(\boldsymbol{g}_k) \boldsymbol{s}_k \tag{4}$$

and can finally represent the multi-user system through

$$\boldsymbol{y}(m) = \tilde{\boldsymbol{S}}\boldsymbol{b}(m) + \bar{\boldsymbol{v}}(m)$$

where $\tilde{S} = [\tilde{s}_1, \tilde{s}_2, \dots, \tilde{s}_K]$ is the effective spreading matrix and $\boldsymbol{b}(m) = [b_1(m), b_2(m), \dots, b_K(m)].$

III. DATA DETECTION

Our receiver detects the data $\boldsymbol{b}(m)$ using the received chip sequence $\boldsymbol{y}(m)$, the effective spreading matrix $\tilde{\boldsymbol{S}}^{(i)}$, and the fedback extrinsic information $\text{EXT}(c_k^{(i)}(l))$ on the code symbols at iteration step *i* as shown in Fig. 2.

The frequency selective (multipath) nature of our channel implies to build a filter which is matched to the effective spreading sequence $\tilde{s}_k^{(i)}$. For the moment, it is only of interest that our channel estimator supplies an estimate \hat{h}_k for the channel impulse response for every user. The general optimization problem is therefore reduced to the estimation of



Fig. 2. Model for the MC-CDMA joint channel-estimation and decoding multi-user receiver.

 $\boldsymbol{b}(m)$ only. To cancel the multi-access interference (MAI), we perform soft cancelling for user k

$$\tilde{\boldsymbol{y}}_{k}^{(i)}(m) = \boldsymbol{y}(m) + \tilde{\boldsymbol{s}}_{k}^{(i)}\tilde{\boldsymbol{b}}_{k}^{(i)}(m) - \tilde{\boldsymbol{S}}^{(i)}\tilde{\boldsymbol{b}}^{(i)}(m).$$

Vector $\tilde{\boldsymbol{b}}^{(i)}(m)$ contains the soft bit estimates that are computed from the extrinsic information supplied by the decoding stage (see Section IV). The mapping for the used QPSK alphabet is given by

$$\tilde{b}_{k}^{(i)}(m) = \varphi(c_{k}^{(i)}(2m)) + j\varphi(c_{k}^{(i)}(2m+1)).$$
(5)

where $\varphi(\cdot) = 2 \operatorname{EXT}(\cdot) - 1$. The $\tilde{\boldsymbol{y}}_k^{(i)}(m)$ are further cleaned from noise and MAI with a successive MMSE-filter

$$\boldsymbol{z}_{k}^{(i)}(m) = (\boldsymbol{f}_{k}^{(i)})^{\mathcal{H}} \boldsymbol{\tilde{y}}_{k}^{(i)}(m)$$

to obtain an estimate of the transmitted symbol $b_k(m)$. An unbiased MMSE filter for the MC-CDMA system can be found similarly to the MMSE detector given in [2], [6]. To simplify notation we omit the iteration index $(\cdot)^{(i)}$ for the filter. It has the form

$$oldsymbol{f}_k^{\mathcal{H}} = rac{ ilde{oldsymbol{s}}_k^{\mathcal{H}} (\sigma_v^2 oldsymbol{I} + ilde{oldsymbol{S}} V oldsymbol{ ilde{oldsymbol{S}}}^n)^{-1}}{ ilde{oldsymbol{s}}_k^{\mathcal{H}} (\sigma_v^2 oldsymbol{I} + ilde{oldsymbol{S}} V oldsymbol{ ilde{oldsymbol{S}}}^{\mathcal{H}})^{-1} ilde{oldsymbol{s}}_k}$$

Matrix V denotes the error covariance matrix

$$\boldsymbol{V} = E\{(\boldsymbol{b}(m) - \tilde{\boldsymbol{b}}(m))(\boldsymbol{b}(m) - \tilde{\boldsymbol{b}}(m))^{\mathcal{H}}\}$$

with diagonal elements

$$V_{j,j} = E\{1 - |\tilde{b}_k^{(i)}(m)|^2\}$$

that are constant during iteration i, the other elements are assumed to be zero. In this case we calculate the variance from all symbols in the block belonging to user k and call the filter unconditional.

IV. DECODING

The iterative receiver feeds back soft values on code bits $c_k(l)$ to get better detection results and better channel estimates. The soft feedback values are computed from the so-called a-posteriori probabilities (APP) and the extrinsic information (EXT) of the code symbols though mapping to QPSK symbols (7), (5), see also [2], [7]. A soft input soft output (SISO) decoder for binary convolutional codes, implemented using the BCJR algorithm [8], supplies these measures. The input values to the decoder are the so called channel values $w_k(l)$ derived from the MMSE-filter outputs after demapping and deinterleaving and the estimated noise variances $\hat{\sigma}_{v,k}^2 = \frac{1}{2M} \sum_{l=1}^{2M} |w_k(l)|^2 - 1$. The APP for the code symbol being +1 when $w_k(l)$ is observed is given by $\text{APP}(c_k(l)) = \Pr[c_k(l) = +1|w_k(l)]$. The link between APP and EXT is established via $\text{APP}(c_k(l)) \propto \text{EXT}(c_k(l))p(w_k(l)|c_k(l) = +1)$, where the last expression denotes the channel transition function, which will be formulated as conditional Gaussian pdf

$$p(w_k(l)|c_k(l) = +1) \propto \exp\left(-\frac{|w_k(l) - 1|^2}{2\hat{\sigma}_{v,k}^2}\right)$$

V. CHANNEL ESTIMATION

The importance of having a good channel estimate is made obvious by (4), since the matched filter to the effective spreading sequence depends directly on the quality of the channel estimate [9]. For MC-CDMA various blind channel estimation schemes have been proposed in literature. All these schemes suffer from an inherent phase ambiguity [10], so some sort of pilot symbols have to be introduced anyway. Furthermore, the popular blind subspace method limits the maximum number of users in the system to $K \leq N - L$, see [11]–[14].

Therefore we propose a new pilot based channel estimation scheme that can be applied to overloaded systems K > N and allows for coherent detection. To achieve good estimates we use a random time domain pilot sequence that is J symbols long and unique for every user and subcarrier. This was inspired by equivalent approaches for DS-CDMA in [2], [15] and the analysis in [4]. The J resulting chip sequences $p_k(m)$, which build the OFDM pilot symbols for user k (see Fig. 1), are randomly chosen in the same way as the spreading sequences s_k (see first paragraph in section II).

An estimate of the K user channels \hat{g}_k in the frequency domain in the least squares sense can be obtained joint for all users but individually for every subcarrier q.

$$\begin{pmatrix} \hat{g}_1(q) \\ \hat{g}_2(q) \\ \vdots \\ \hat{g}_K(q) \end{pmatrix} = \boldsymbol{P}_q^{\#} \begin{pmatrix} y_q(0) \\ y_q(1) \\ \vdots \\ y_q(J-1) \end{pmatrix}$$
(6)

 $\hat{g}_k(q)$ denotes the channel coefficient for user k and subcarrier q. The received signal on subcarrier q and time index m is given by $y_q(m)$, $(\cdot)^{\#}$ denotes the pseudo inverse. In matrix

$$\boldsymbol{P}_{q} = \begin{pmatrix} p_{1,q}(0) & p_{2,q}(0) & \cdots & p_{K,q}(0) \\ p_{1,q}(1) & p_{2,q}(1) & \cdots & p_{K,q}(1) \\ \vdots & \vdots & \ddots & \vdots \\ p_{1,q}(J-1) & p_{2,q}(J-1) & \cdots & p_{K,q}(J-1) \end{pmatrix}$$

the pilot spreading coefficient for user k at time index m and subcarrier q is given by $p_{k,q}(m)$.

The channel impulse response in the time domain h_k possesses only L taps and can be estimated by

$$\hat{\boldsymbol{h}}_k = rac{1}{\sqrt{N}} \boldsymbol{F}_{N imes L}^{\mathcal{H}} \hat{\boldsymbol{g}}_k$$

which also reduces the noise in the channel estimates. $F_{N \times L}$ is a partial Fourier matrix with only the first L columns taken from F. The estimates \hat{h}_k are supplied to the PIC and MMSE detector. In the first iteration the missing energy from $\hat{h}_k(n), n = L + 1 \dots N - 1$ (assumed to be zero) is compensated by scaling with $\sqrt{N/L}$, i.e.

$$\hat{\boldsymbol{h}}_{k}^{(1)} = rac{1}{\sqrt{N}} \boldsymbol{F}_{N imes L}^{\mathcal{H}} \hat{\boldsymbol{g}}_{k}^{(1)} \sqrt{rac{N}{L}}$$

In the second iteration the APP information

$$\tilde{b'}_k(m) = \varphi(c_k(2m)) + j\varphi(c_k(2m+1)) \tag{7}$$

from the decoder is mapped to QPSK symbols and used as additional pilots to further refine the channel estimates [16]

$$\begin{pmatrix} \hat{g}_1(q) \\ \hat{g}_2(q) \\ \vdots \\ \hat{g}_K(q) \end{pmatrix} = \begin{pmatrix} \mathbf{P}_q \\ \mathbf{Q}_q \end{pmatrix}^{\#} \begin{pmatrix} y_q(0) \\ y_q(1) \\ \vdots \\ y_q(M-1) \end{pmatrix}, \quad (8)$$

matrix Q_q is defined according to

$$\boldsymbol{Q}_{q} = \begin{pmatrix} s_{1}(q)\tilde{b'}_{1}(J) & \cdots & s_{K}(q)\tilde{b'}_{K}(J) \\ s_{1}(q)\tilde{b'}_{1}(J+1) & \cdots & s_{K}(q)\tilde{b'}_{K}(J+1) \\ \vdots & \ddots & \vdots \\ s_{1}(q)\tilde{b'}_{1}(M-1) & \cdots & s_{K}(q)\tilde{b'}_{K}(M-1) \end{pmatrix}.$$

In subsequent iterations each column of $\begin{pmatrix} P_q \\ Q_q \end{pmatrix}$ is further normalized to $\sqrt{2M/N}$.



Fig. 3. Power delay profile for typical urban according to COST 259, with L=15, cf. [17].

VI. SIMULATION RESULTS

To demonstrate the performance of the system we use the power-delay-profile (PDP) typical urban (TU) from COST 259 (see Fig. 3) [17], the chip rate is chosen to be 3.84 Mcps as in UMTS and the delay spread L = 15. The channel impulse response is normalized so that

$$\sum_{n=0}^{L-1} E(|h_k(n)|^2) = 1 \ \forall k.$$

The number of iterations is limited to 10. The single-user bound (SUB) is taken as a reference for the multi-user receiver performance. In this context it is defined as the receiver performance with K = 1 and perfect channel knowledge. The spreading sequence has a length N = 64 equal to the number of subcarriers. The complete OFDM symbol with cyclic prefix has length of P = G + N = 79. The convolutional code used is a non-systematic, non-recursive, 4 state, rate R = 1/2 code with generator polynomial $(5,7)_8$. All simulation are averaged over 50 independent channel realizations.

The receiver performance in terms of bit error rate (BER) versus signal to noise ratio (SNR) for load $\beta = K/N = 1$ is given in Fig. 4. The SNR is defined as

$$\frac{E_b}{N_0} = \frac{1}{R\sigma_v^2} \frac{P}{N} \frac{M}{M-J}$$

The BER decreases after each iteration and converges up to 1 dB towards the SUB with increasing SNR after 6 iterations. For a moderately overloaded system with load $\beta = 1.25$ the results can be seen in Fig. 5. The bit rate per user is 44.8 kbit/s, the net bit rate per cell is 2.87 Mbit/s for 64 users and 3.58 Mbit/s for 80 users.

A comparison with other non iterative receiver structures can be given the following way: In the first iteration only the pilot symbols are used for channel estimation. This estimates are used in the multi-user detector. The PIC does not remove any interference during the first iteration. The BER curve of the first iteration is therefore comparable with the performance of a non iterative MMSE multi-user detectors with imperfect channel knowledge. The improvements of this new iterative structure (given the same coding, spreading and PDP) is clearly seen in Fig. 4 and Fig. 5 between iteration 1 and iteration 7. In a fully loaded system and even in an overloaded



Fig. 4. Receiver performance in terms of BER versus SNR for K = 64 users after iteration 1 to 10, N = 64 spreading length and subcarrier count, L = 15 channel length and length of cyclic prefix G = 15, J/M = 20/256 = 7.8% pilot symbols.



Fig. 5. Receiver performance in terms of BER versus SNR for K = 80 users after iteration 1 to 10, N = 64 spreading length and subcarrier count, L = 15 channel length and length of cyclic prefix G = 15, J/M = 20/256 = 7.8% pilot symbols.

system the reduction in BER is in the orders of 3 magnitudes after 7 iterations.

VII. CONCLUSIONS

In this work we have presented an iterative receiver structure with joint detection and decoding and channel estimation for the uplink of an MC-CDMA system. The new channel estimation scheme, which is based on random time domain pilot sequences, allows to achieve very good performance in this iterative receiver architecture with a low percentage of pilot symbols per transmitted data block. The computational complexity of the MC-CDMA receiver is smaller than that of comparable systems for DS-CDMA, because the ISI is removed through insertion of the CP, and channel equalization is done in the frequency domain. The complexity for the channel estimation is growing by $\mathcal{O}(K^3)$ in MC-CDMA but in comparable DS-CDMA systems for UMTS TDD [2] it grows with $\mathcal{O}(L^3K^3)$. A BER of 10^{-3} in a fully loaded system, suitable for voice communication, is already reached at an E_b/N_0 of 12 dB. In an overloaded system with load $\beta = 1.25$ the same BER can be achieved with an E_b/N_0 of 13 dB.

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K=80, N=64, M=256, J=20, L=15, G=15, NoF=50